FLAGS OF ALL COMMERCIAL NATIONS.

Nystrom's P Book   J. B. LIPPINCOTT & CO. PHILADELPHIA. 1861   T. Sinclair's lith.

Pocket-Book

OF

MECHANICS

AND

Engineering.

CONTAINING A MEMORANDUM OF FACTS

AND CONNECTION

OF

Practice and Theory.

BY

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PREFACE.

Let every Engineer make his own Pocket-Book as he proceeds in study and practice, it will then suit his particular business. The present work was compiled in this way continually during the Author's professional career. It was originally not intended for publication, but had grown so large in manuscript as to be inconvenient for the pocket, which circumstance combined with repeated requests to publish it, placed it before the public, first in the year 1854. Since that time he has travelled in Europe for nearly five years, collecting such information for the Pocket-Book as to follow up the progress of Engineering profession, and no expense has been spared in attaining that object.

The Author hopes that the introduction of Algebraical formulas instead of written rules will be favourably received. Written rules adopted by English Authors and Engineers are indeed excellent;—but formulas are better,—because they not only tell what is to be done, but at a glance, impress the mind with the complete operation. If all the formulas in this book were explained in words, it would be far too large for the pocket. It is avoided to impose upon Engineers to carry elementary explanations in their pockets which belongs to schoolboys and apprentices.

It is not necessary to understand Algebra for the use of the formulas,—only practise the insertion of numerical values, and perform the arithmetical operations indicated by the particular formula used. The Author has furnished the formulas ready to receive what is given, and refund what is required.

Should there be any letters in formulas not clearly explained refer then to the examples.

Mr. Nystrom, the Author of this Pocket-Book, has been connected with me since 1849, as Constructor and Draughtsman, in which capacity he has given the greatest satisfaction. During that time my attention was constantly called to the readiness and accuracy of his calculations, which were made by means of a Pocket-Book then in manuscript. I have frequently requested him to publish it, and am now gratified by receiving its pages in print.

In it there is a Drawing of my Propeller, illustrating the expanding pitch first adopted by me, and now generally used. This is the principle upon which Propellers have been constructed, under my direction by Mr. Nystrom.

R. F. LOPER.

March, 1854.

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INTRODUCTION.

Quantity is that which can be increased or diminished by augments or abatements of homogeneous parts. Quantities are of two essential kinds, *Geometrical* and *Physical.*

1st, *Geometrical* quantities are those which occupy space; as *lines, surfaces, solids, liquids, gases,* &c.

2nd, *Physical* quantities are those which exist in the time but occupy no space, they are known by their character and action upon geometrical quantities; as *attraction, light, heat, electricity* and *magnetism, colours, force, power,* &c., &c.

To obtain the magnitude of a quantity we compare it with a part of the same, this part is imprinted in our mind as a *unit,* by which the whole is measured and conceived. No quantity can be measured by a quantity of another kind, but any quantity can be compared with any other quantity, and by such comparison arises what we call *calculation* or *Mathematics.*

MATHEMATICS.

Mathematics is a science by which the comparative value of quantities are investigated; it is divided into:

1st, **Arithmetic,**—that branch of Mathematics, which treats of the nature and property of numbers; it is subdivided into *Addition, Subtraction, Multiplication, Division, Involution, Evolution and Logarithms.*

2nd, **Algebra,**—that branch of Mathematics which employs letters to represent quantities, and by that means performs solutions without knowing or noticing the *value* of the quantities. The subdivisions of Algebra are the same as in Arithmetic.

3rd, **Geometry,**—that branch of Mathematics which investigates the relative property of quantities that occupies space; its subdivisions are *Longemetry, Planemetry, Stereometry, Trigonometry, and Conic Sections.*

4th, **Differential-calculs,**—that branch of Mathematics, which ascertains the mean effect, produced by group of continued variable causes.

5th, **Integral-calculs,**—the contrary of Differential, or that branch of Mathematics which investigates the nature of a continued variable cause, that has produced a known effect.

ARITHMETIC.

The art of manœuvering numbers, and to investigate the relationship of quantities.

Figures—1, 2, 3, 4, 5, 6, 7, 8, 9. Arabic dignets, about nine hundred years old.

Ciphers—0, 0, 0. Sometimes called noughts, it is the beginning of figures and things.

Number is the expression of one or more figures and ciphers.

Integer is a whole number or unit.

Fraction is a part of a number or unit.

When figures are joined together in a number, the relative dignity expressed by each figure, depends upon its position to the others. Thus,

Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Hundreds. Tens. Units.
674,385 ;	496,345 ;	695,216 ;	505,310 :	685,	3 6 7 ;

Notation is the setting down of any number by figures and ciphers.

Numeration is the reading of any number in words, from the expression of figures and ciphers.

Characters which describe the operation by numbers (significations).

= Equality, as 6=6, reads 6 is equal to 6.

+ *Plus*, Addition, ...	$3+6=9$
— *Minus*, Subtraction, ...	$6-2=4$
× Multiplication, ...	$3\times4=12$
÷ or : Division, ...	$15:5=3$
√ Square root, ...	$\sqrt{9}=3$
∛ Cube root, ...	$\sqrt[3]{8}=2$
> Greater, ...	$8>4$
< Less, ...	$6<9$

ROMAN NOTATION.

The Romans expressed their numbers by various repetitions and combinations of seven letters in the alphabet; as,

1=I.
2=II.
3=III.
4=IV.
5=V.
6=VI.
7=VII.
8=VIII.
9=IX
10=X.
20=XX.
30=XXX.
40=XL.
50=L.
60=LX.
70=LXX.
80=LXXX.
90=XC.
100=C.
500=D, or LↃ.

As often as a character is repeated, so many times is its value repeated.

A less character before a greater diminishes its value, as IV=I from V, or 1 substracted from 5=4.

A less character after a greater increases its value, as XI=X+I, or 10+1=11.

1,000=M, or CↃ.

For every Ↄ annexed, this becomes 10 times as many.

2,000=MM, or IIↃↃↃ.
5,000=$\overline{\text{V}}$, or LↃↃ·
6,000=$\overline{\text{VI}}$, or MMM.
10,000=$\overline{\text{X}}$, or CↃↃ.
50,000=$\overline{\text{L}}$. or LↃↃↃ·
60,000=$\overline{\text{LX}}$, or MMMↃ.
100,000=$\overline{\text{C}}$, or CↃↃↃ.
1,000,000=$\overline{\text{M}}$, or CↃↃↃↃ.
2,000,000=$\overline{\text{MM}}$, or MMↃↃↃ.

A bar, thus, — over any number, increases it 1000 times.

EXAMPLES.—1854, MDCCCLIV.

524,365, $\overline{\text{DXXIV}}$CCCLXV.

An imperfection in the Roman Notation consists in that, there is no signification for the cipher as in the Arabic notation.

ALGEBRA.

In **Algebra** we employ certain *characters* or *letters* to represent quantities. These characters are separated by *signs*, which describe the operations; and by that means, simplify the solution.

1. Whatever the value of *any* quantity may be, it can be represented by a character, as a. Another quantity of the same kind, but of different value, being represented by b. The sum of these two quantities is of the same kind but of different value.

For **Addition** we have the algebraical sign $+$, (plus) which, when placed between quantities, denotes they shall be added; as $a+b$, reads in the algebraical language, "a plus b," or a is to be added to b.

Another algebraical sign $=$, (Equal) denotes that quantities which are placed on each side of this sign, are *equal*. Let the sum of a and b be denoted by the letter c; then we have,

$$a+b=c.$$

This composition is called an *algebraical equation*. The quantity on each side of the equal sign is called a *member*, as $a+b$, is one member, and c, the other. When one of the members contains only one quantity, that member is generally placed on the first side of the equal sign, and its value commonly unknown; but the value of the quantities in the other member being given, as $a=4$, and $b=5$, then the practical mode, to insert numerical values in algebraical equations, will appear; as,

Equation, $c=a+b$,
$4+5=9$, the value of c.

2. The sum of three quantities a, b, and c, is equal to d, then

Equation, $d=a+b+c$,
$4+5+9=18$, the value of d.

3. For **Subtraction** we have the algebraical sign, $-$, (minus) which, when placed before a quantity, denotes it is to be subtracted as, $a-b$, reads in the algebraical language "a minus b," or from a, subtract b. Let the difference be denoted by the letter c; and $a=8$. $b=3$

Equation, $c=a-b$,
$8-3=5$, the value of c.

4. From the sum of a and b, subtract c, and the result will be d; then,

Equation, $d=a+b-c$,
$8+3-5=6$, the value of d.

5. When two equal quantities are to be added, as $a+a$, it is the same as to take one of them twice, and is marked thus $2a$. The number 2 is called the coefficient of the quantity a. If there are more than *two* equal quantities to be added, the coefficient denotes how many there are of them; as,

Equation, - - - - $a+a=2a$,
" $a+a+a=3a$,
" $a+a+a+a=4a$,
&c., *&c.*

When the quantities are separated by the signs, *plus*, or *minus*, they are called *terms*.

6. **Multiplication.**—When a quantity a, is to be multiplied by another quantity b, then a and b are called *factors*; and separated by no sign as ab; which denotes that a is to be multiplied by b; but when the values of a and b are expressed by numbers, they are separated by the sign $\times$ (Multiplication); the result from Multiplication is called the product. Let $a=8$, and $b=6$, and the product of a and b, to be c, then,

Equation, $c=ab$,
$8\times6=48$, the value of c.

7. The product of a and b, is to be multiplied by c, and the latter product will be equal to d; then,

Equation, $d=abc$,
$8\times6\times48=2304$, the value of d.

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8. The sum of a and b, is to be multiplied by c, and the product will be d; then,

$$\textit{Equation,}\quad d=c\,(a+b),$$
$$48\,(8+6)=672 \text{ the value of } d.$$

When the sum of two or more quantities is to be multiplied by another quantity, the sum is to be enclosed in parentheses, and denotes itself to be one factor. The other factor is to be placed on the outside of the parentheses, as seen in the preceding example.

9. To the product of a and c, add b, and the result will be d; then,

$$\textit{Equation,}\quad d=\ ac+b,$$
$$8\times48+6=390 \text{ the value of } d.$$

Be particular to distinguish the two Examples 8, and 9.

10. The sum of a and b, to be multiplied by the sum of a and c; the product will be d; then,

$$\textit{Equation,}\quad d=(a+b)\,(a+c),$$
$$(8+6)\,(8+48)=784.$$

11. The sum of c and b, to be multiplied by the difference of c and a; the result will be d; then,

$$\textit{Equation,}\quad d=(c+b)\,(c-a),$$
$$(48+6)\,(48-8)=2160.$$

12. **Division.**—When a quantity a, is to be separated into b equal parts, the numbers of parts or b, is called the *divisor*, and the value of each part, is called the *quotient*. The sum of the parts or the whole quantity a, is called the *dividend*; a and b, is separated by the sign : (Division); as $a:b$, reads in the algebraical language, "a divided by b." Let the *quotient* be denoted by the letter c; and $a=18$, $b=6$, then,

$$\textit{Equation,}\quad c=a:b,$$
$$18:6=3 \text{ the quotient } c.$$

In Algebra it is found more convenient to set up Division as a fraction, then it will appear as,

13. Divide a, by c, and the quotient will be b. Then,

$$\textit{Equation,}\quad b=\frac{a}{c},$$
$$\frac{18}{3}=6 \text{ the quotient } b.$$

14. The product of a and b, to be divided by c; and the product will be d. Then,

$$\textit{Equation,}\quad d=\frac{ab}{c},$$
$$\frac{18\times6}{3}=36.$$

15. The sum of d and b, to be multiplied by c, and the product divided by a; then the result will be e.

$$\textit{Equation,}\quad e=\frac{c\,(d+b)}{a},$$
$$\frac{3\,(36+6)}{18}=7.$$

16. From the product of a and c, subtract $3b$; divide the remainder by the difference of a, and c; the result will be h.

$$\textit{Equation,}\quad h = \frac{ac-3b}{a-c},$$

$$\frac{18\times3-3\times6}{18-3} = 2.4.$$

An old man said to a smart boy, "How old are you?" to which he replied.—"To seven times my father's age add yours, divide the sum by double the difference of yours and his, and the result will be my age."

Letters will denote,
a = the old man's age,
b = the father's age,
c = the boy's age. Then,

$$\textit{Equation,}\quad c = \frac{7b+a}{2\,(a-b)} \text{ the boy's age.}$$

Now for any number of years of the old man and the father, will be a corresponding age of the boy; suppose,

$a = 73$ years the age of the old man,
$b = 57$ years the father's age.
Require the boy's age.

$$c = \frac{7\times57+73}{2\,(73-57)} = 14\tfrac{3}{4} \text{ years.}$$

PROPORTION.

THE *relative value* of two quantities, is obtained by dividing one into the other, and the quotient is called the *ratio* of their relationship. If the ratio of two quantities is equal to the ratio of two other quantities, they are said to be in the same proportion; as,

$$a : b = c : d,$$

reads in the algebraical language "a is to b as c is to d."—a, b, c, and d, are called *terms*, of which a is the first, b the second, c the third, and d the fourth *term*. The first and fourth are called "the *outer terms*," and the second and third, "the *inner terms*." The whole is called an "*analogy*."

A property in the nature of analogies is, that the product of the outer terms ad, is equal to the product of the inner bc. Suppose $a = 4$, $b = 9$, $c = 12$, $d - 27$.

$$4 : 9 = 12 : 27,$$
$$ad = bc,\quad 4\times27 = 9\times12.$$

If any one of the four quantities are unknown, its value can be calculated by the other three; as,

$$a = \frac{bc}{d} = \frac{9\times12}{27} = 4,$$

$$b = \frac{ad}{c} = \frac{4\times27}{12} = 9,$$

$$c = \frac{ad}{b} = \frac{4\times27}{9} = 12.$$

$$d = \frac{bc}{a} = \frac{9\times12}{4} = 27.$$

To Alternate a Proportion.

If $a : b = c : d,$
then $a : c = b : d,$
and $ad = bc.$

To Inverse a Proportion.

If $a : b = c : d,$
then $b : a = d : c,$
and $bc = ad.$

To Multiply a Proportion.

If $a : b = c : d,$
then $na : nb = nc : nd,$
and $\frac{a}{n} : \frac{b}{n} = \frac{c}{n} : \frac{d}{n}.$

To Reduce a Proportion.

If $a : b = c : d,$
then $na : nb = mc : md,$
and $\frac{a}{n} : \frac{b}{n} = \frac{c}{m} : \frac{d}{m}.$

Compared Proportions.

If $a : b = c : d,$
and $c : d = e : f$
then $a : b = e : f.$

Continued Proportion.

If $a : b = c : d = e : f,$
then $af = be = cd,$
and $ad = bc, cf = de.$

To Compound Proportions.

If $a : d = c : d,$
and $e : f = g : h,$
then $ae : df = cg : dh.$

To Compare Proportions.

If $a : b = c : d,$
and $e : f = g : h,$
then $\frac{a}{e} : \frac{b}{f} = \frac{c}{g} : \frac{d}{h}.$

To Combine a Proportion.

If $a : b = c : d,$
then $(a+b) : b = (c+d) : d,$
and $d\,(a+b) = b\,(c+d).$

To Combine a Proportion Inversely.

If $a : b = c : d,$
then $a : b = (a+c) : (b+d),$
and $a\,(b+d = b\,(a+c).$

To Dissolve a Proportion.

If $a : b = c : d,$
then $(a-b) : b = (c-d) : d,$
and $d\,(a-b) = b\,(c-d).$

To Dissolve a Proportion Inversely.

If $a : b = c : d,$
then $a : b = (a-c) : (b-d),$
and $a\,(b-d) = b\,(a-c).$

If $a : b = d : c,$
then $a : b = \frac{1}{c} : \frac{1}{d},$
and $\frac{1}{a} : \frac{1}{b} = c : d.$
and $ac = bd.$

To Find the mean Proportion.

$a : x = x : b$
then $x = \sqrt{ab}\,,$
x is the mean Proportion.

Proportion of Square and Square Root.

If $a : b = c : d,$
then $a^2 : b^2 = c^2 : d^2$
and $\sqrt{a} : \sqrt{b} = \sqrt{c} : \sqrt{d}.$

Proportion of any Power or Root.

If $a : b = c : d,$
then $a^n : b^n = c^n : d^n,$
and $\sqrt[n]{a} : \sqrt[n]{b} = \sqrt[n]{c} : \sqrt[n]{d}.$

SIMPLE INTEREST.

Interest is a profit on money which is lent for a certain time.

Letters will denote.

c = the standing capital, or lent money.
r = interest on the capital c,
p = per cent. on 100 in the certain time.

Analogy, $c : r = 100 : p$.

If p is the per cent. on 100, in one year, then t = time in years for the standing capital c, and the interest r.

Analogy, $c : r = 100 : pt$.

From this analogy we obtain the equations,

Interest, $$r = \frac{cpt}{100}, \quad \cdots \quad 1,$$

Per cent., $$p = \frac{100\,r}{tc}, \quad \cdots \quad 2,$$

Capital, $$c = \frac{100\,r}{pt}, \quad \cdots \quad 3,$$

Time in years, $$t = \frac{100\,r}{cp}, \quad \cdots \quad 4.$$

Now for any question in Simple Interest, there is one equation which gives the answer. If the time is given in *months, weeks*, or *days*, multiply the 100 correspondingly by 12, 52, 365.

Example 1. What is the interest on \$3789.35, for 3 years and five months, at 6 per cent. per annum?

$t = 3\times12+5 = 41$ months, from the Equation 1, we have,

Interest, $$r = \frac{3789.35\times6\times41}{12\times100} = 776.81 \text{ Dollars.}$$

Example 2. A capital c = \$469.78, gave an interest $r = 150.72$ dollars, in a time t = 4 years and 7 months. Require the per centage per annum?

$t = 4\times12+7 = 55$ months, from Equation 2, we have,

Per cent., $$p = \frac{12\times100\times150.72}{469.78\times55} = 7 \text{ per cent.}$$

Example 3. What capital is required to give an interest $r = 345$ Dollars in 6 years, at 5 per cent. per annum? From the Equation 3, we have,

Capital, $$c = \frac{100\times345}{5\times6} = \$1150.$$

Example 4. A capital c = \$2365 shall stand until the interest will be $r = 550$ Dollars, at $p = 6$ per cent. per annum. How long must the capital stand?

From the Equation 4, we have,

Time, $$t = \frac{100\times550}{2365\times6} = 3.876 \text{ years.}$$

$12\times0.876 = 10.512$ months, $4\times0.512 = 2.048$ weeks, the time t = 3 years, 10 months, and 2 weeks.

REBATE OR DISCOUNT.

Rebate or **Discount** is an allowance on money, which is paid before due. a = amount of money to be paid in the time t. By agreement the amount is paid with a capital c, at the beginning of the time t, but discounted a Rebate r, at p per cent, so that the interest on the capital c, at p per cent., should be equal to the Rebate r, in the time t. $a = c + r$.

Rebate, $r = \frac{apt}{100+pt}$, ... 5,

Capital, $c = \frac{100\,a}{100+pt}$, ... 6,

Per cent., $p = \frac{100\,(a-c)}{ct}$, . . . 7,

Time, $t = \frac{100\,(a-c)}{cp}$, ... 8,

Amount, $a = \frac{c}{100}\,(100+pt)$, . . . 9,

Amount, $a = \frac{r}{pt}(100+pt)$, . . . 10.

Now for any question in Rebate or Discount, there is one equation that will give the answer.

Example 5. A sum of money, a = 78460 dollars is to be paid after 3 years and 6 months, but by agreement payment is to be made at the present time. What will be the Rebate at 7 per cent.

Rebate, $r = \frac{78460 \times 7 \times 3.5}{100+7 \times 3.5} = \$15439.91.$

FELLOWSHIP.

Fellowship, or *partnership*, is a rule by which companies ascertain each fellow's profit or loss, by their stock. Each fellow's part in the stock is called his *share*. The sum of shares is called the *stock*.

Fellowships are of two kinds, *Simple* and *Double*.

Simple Fellowship, when there is no regard to the time, the shares or stock is employed.

Letters will denote,

A = *share* of either one fellow.
a = profit or loss on the share A.
S = stock or the sum of the shares.
s = gain or loss on the stock S.

Then $A : a = S : s$.

Share, $A = \frac{aS}{s}$, ... 11,

Profit or loss, $a = \frac{As}{S}$, ... 12,

Stock, $S = \frac{As}{a}$, ... 13,

Gain or loss, $s = \frac{aS}{A}$, ... 14.

Example 1. A person had invested $A = \$11645$, in a stock $S = \$64800$, which gave a gain of $s = 13864$. What will be the profit of the person's share?

Profit, $$a = \frac{11645 \times 13864}{64800} = \$2491.45$$

Double Fellowship. When the different shares are employed at a different length of time, each share is multiplied by its time employed, and the product is the *effect* of the share.

Letters will denote,

t = time for the employed share A.
T = meantime for the employed stock S.
e = effect of the share A.
a = profit of the effect e.
E = effect of the stock.
s = gain of the effect E. Then,

$$e : a = E : s.$$

Effect of A,	$e = \frac{aE}{s}$,	15,	*Time,*	$t = \frac{aE}{As}$,	19,
Profit of e,	$a = \frac{es}{E}$,	16,	*Share,*	$A = \frac{aE}{ts}$,	20,
Effect of S,	$E = \frac{es}{a}$,	17,	*Meantime,*	$T = \frac{es}{aS}$,	21,
Gain of E,	$s = \frac{aE}{e}$,	18,	*Stock,*	$S = \frac{es}{aT}$,	22.

Example 2. A canal is to be dug, and requires an effect $E = 76850$ (men and days) to be accomplished; after that it will give a gain $s = 12390$ Dollars. An employer has $A = 168$ laborers. How many days must those laborers be employed at the canal, that the employer will obtain a profit $a = \$5000$?

Time, $$t = \frac{5000 \times 76850}{168 \times 12390} = 184.6 \text{ days.}$$

PERMUTATION.

Permutation is to arrange a number of things in every possible position. It is commonly used in *games.*

Example 1. How many different values can be written by the three ciphers 1, 2, 3.
$1 \times 2 \times 3 = 6$ different values, namely,

123, 132, 213, 231, 312, 321.

With any three different ciphers can be written six different values. Any three things can be placed in 6 different positions.

Example 2. How many names can be written by the three syllables *mo, ta, la?* The answer is,—Motala, Molata, Tamola, Talamo, Lamota, Latamo.

Example 3. How many words can be written by the five syllables, *mul, tip, li, ca, tion?*

$1 \times 2 \times 3 \times 4 \times 5 = 120$ words, the answer.

$n-p$	
1	1
2	2
3	6
4	24
5	120
6	720
7	5040
8	40320
9	362880
10	3628800
11	39916800
12	479001600
13	6227020800
14	87178291200.

The accompanying table shows the permutation of different numbers of things up to 14; which will be convenient in the next coming examples in combination

COMBINATION.

Combination is to arrange a less number of things out of a greater, in every possible position. It is commonly used in *games*.

Example 1. How many different numbers can be set up by the nine ciphers, 1, 2, 3, 4, 5, 6, 7, 8, 9, and three ciphers in each number?

$$\frac{9\times8\times7}{1\times2\times3} = 84 \text{ different numbers.}$$

Example 2. How many different variations can a player obtain his cards, when the set contains 52 cards, of which he receives 8 at a time?

$$\frac{52\times51\times50\times49\times48\times47\times46\times45}{1\times\ 2\times\ 3\times\ 4\times\ 5\times\ 6\times\ 7\times\ 8} = 752538150 \text{ variations.}$$

If they are four players, and $pr.4 = 24$, they can play $24\times752538150 = 18{,}060{;}915{,}600$ different plays.

If it takes half an hour for each play, and they play 8 hours per day, it will take

$$\frac{18060915600}{2\times8} = 1128807225 \text{ days} = 3{;}092{,}622 \text{ years.}$$

ALLIGATION.

Alligation is to mix together a number of different things of different price or value, and ascertain the mean value of the mixture; or from a given mean value of a mixture ascertain the proportion and value of each ingredient.

Let the different things be a, b, c, and d, &c., their respective price or value per unit, z, y, x, and w, &c.

$A = a+b+c+d$ &c., the sum of the things.

$P =$ mean value or price per unit of A. Then,

$$AP = az+by+cx+dw+\&c, \quad \cdots\cdots \quad 1,$$

and

$$P = \frac{az+by+cx+dw+\&c.}{A}, \quad \cdots\cdots \quad 2,$$

Example 1. If 3 gallons of wine at \$1.37 per gallon, 2 at \$2.18, and 5 at \$1.75, be mixed together, what is a gallon worth of the mixture?

$$A = 3+2+5 = 10 \text{ gallons.}$$

$$P = \frac{3\times1.37+2\times2.18+5\times1.75}{10} = \$1.72 \text{ per gallon.}$$

Alligation of two ingredients a and b, with their respective prices or value per unit, z and y, $z > P > y.$ $A = a + b.$

$$a : b = (P - y) : (z - P) \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 3,$$

$$a = \frac{b\,(P - y)}{(z - P)}, \quad \text{and } a = \frac{A\,(P - y)}{(z - y)}, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 4, \,\&\, 5,$$

Example 2. A Silver-smith will mix two sorts of silver, one at 54 and one at 64 cents per ounce. How much must be taken of each sort to make the mixture worth 60 cents per ounce. (Formula 3.) $P = 60.$ $x = 54.$ $y = 64.$

$$a : b = (60 - 54) : (64 - 60) = 6 : 4, \text{ or,}$$

4 ounces at 54 cents; and 6 ounces at 64 cents.

Alligation of three ingredients, a, b, and c, with their prices or value per unit, z, y and x.

$$a' : c' = (P - x) : (z - P) \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 6,$$
$$a'' : b = (P - y) : (z - P) \text{ when } z > P > y > x, \quad \cdot \quad \cdot \quad 7,$$
$$b : c'' = (P - x) : (y - P) \text{ when } z > y > P > x, \quad \cdot \quad \cdot \quad 8,$$
$$a = a' + a'', \quad c = c' + c''.$$

Example 3. A Farmer will mix wheat at 94 cents per bushel, with barley at 72 cents, rye at 64 cents per bushel. How much of each sort must be taken to make the mixture worth 80 cents per bushel?

(Formula 6.) $z = 94$, $y = 72$, $x = 64$, and $P = 80$.

$$a' : c' = (80 - 64) : (94 - 80) = 16 : 14,$$
$$a' : b = (80 - 72) : (94 - 80) = 8 : 14.$$

The wheat $a = 16 + 8 = 24$ bushels at 94 cents per bushel.
" barley $b = 14$ " " 72 " "
" rye $c = 14$ " " 64 " "

Alligation of four ingredients a, b, c, and d, respective prices or value per unit; z, y, x, and w.

$$\left.\begin{array}{l} a : d = (P - w) : (z - P) \\ b : c = (P - y) : (x - P) \end{array}\right\} \text{when } z > y > P > x > w. \quad \left\{\begin{array}{l} 9, \\ 10, \end{array}\right.$$

$$\left.\begin{array}{l} a' : d = (P - w) : (z - P) \\ a'' : b = (P - y) : (z - P) \\ a''' : c = (P - x) : (z - P) \end{array}\right\} \text{when } z > P > y > x > w. \quad \cdot \quad \left\{\begin{array}{l} 11, \\ 12, \\ 13, \end{array}\right.$$
$$a = a' + a'' + a''',$$

$$\left.\begin{array}{l} a : d' = (P - w) : (z - P) \\ b : d'' = (P - w) : (y - P) \\ c : d''' = (P - w) : (x - P) \end{array}\right\} \text{when } z > y > x > P > w. \quad \cdot \quad \left\{\begin{array}{l} 14, \\ 15, \\ 16, \end{array}\right.$$
$$d = d' + d'' + d''',$$

In the same manner, formulæ can be set up for any number of ingredients.

INVOLUTION.

Involution is to multiply a number into itself a number of times; each product is called the *power* of the number, and the *dignity* of the power is marked by a small figure called *exponent*, on the right of the number; thus,

$$a \times a = a^2 \text{ the square of } a.$$
$$a \times a \times a = a^3 \text{ the cube of } a.$$
$$a \times a \times a \times a = a^4 \text{ the bisquare of } a.$$
$$a \times a \times a \times a \times a = a^5 \text{ the fifth power of } a.$$
&c., &c., &c.

$$3^2 = 3 \times 3 = 9.$$
$$2^3 = 2 \times 2 \times 2 = 8.$$
$$4^4 = 4 \times 4 \times 4 \times 4 = 256.$$
&c., &c., &c.

Binome is a factor or quantity which contains *two terms*; as $(a + b.)$

Binomial-Theorem is the rule which a binome follows, when it is raised to *any* power.

When a binome is to be multiplied by itself or any other binome, it is set up and performed like the common multiplication by numbers; thus,

Example 1.

$$\begin{array}{r} a+b \\ a+b \\ \hline +ab+b^2 \\ a^2+ab \quad \\ \hline a^2+2ab+b^2 \end{array} \Big\} \ (a+b)(a+b), \quad = (a+b)^2.$$

Example 2. Suppose $a+b = 358746$, and $a = 358000$, $b = 746$, then,

$$\begin{array}{rcr} (a+b)^2 & = & 358746^2, \\ +a^2 & = & 128164000000 \\ +2ab & = & 534136000 \\ +b^2 & = & 556516 \\ \hline & & 128698692516 = (a+b)^2. \end{array}$$

Ex. 3. $(a+b)^3 = a^3+3a^2b+3ab^2+b^3.$
Ex. 4. $(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4.$
Ex. 5. $(a-b)^7 = a^7-7a^6b+21a^5b^2-35a^4b^3+35a^3b^4-a^2b^5+7ab^7-b^7.$

Here you will discover the peculiarities of the *Binomial-Theorem*, which is thus expressed in words:

1st. The *exponent* of the first term a in the power, is equal to the exponent of the binome; and in every successive term, the exponent of a is decreased by 1, until the last term of the exponent of a is 0, and, therefore, disappears, because any quantity raised to no power is equal to 1, thus, $a^0 = 1$ and $a^1 = a$.

2d. In the first term of the power, the exponent of b is 0, and therefore b will first appear in the second term with the exponent 1, and in every successive term the exponent of b is increased by 1, until in the last term the exponent will be equal to the exponent of the binome.

3d. The coefficient of the second term in the power, is equal to the exponent of the binome, and the coefficient of any successive term is equal to the product of the coefficient and exponent of a in the foregoing term, divided by the number of terms before the sought coefficient.

4th. When the second term in the binome is negative, the first term in the power will be positive, the second negative, the third positive, the fourth negative, &c., &c. The odd terms are positive, and the even terms are negative.

5th. The number of terms in the power is *one more* than the exponent of the binome.

EVOLUTION.

Evolution is the reverse of Involution, or to find the number that has produced a given power. In this case the given power is called the *number*, and the number which has produced the given power is called the *root* of the *number*. The symbol $\sqrt{\ }$ is generally placed over the number of which the *root* is to be extracted. The dignity of the *root* is placed thus $\sqrt[3]{\ }$ of which the figure ^{3} is called the index of the root; for the square roots the index ^{2} is always omitted.

Example 1.	$\sqrt{9} = 3$	because $3^2 = 9$.
" 2.	$\sqrt[3]{64} = 4$	because $4^3 = 64$.
" 3.	$\sqrt[4]{531441} = 27$	" $27^4 = 531441$.
	&c.,	*&c.,*

In the accompanying Table are calculated the *Squares, Cubes, Square Roots* and *Cube Roots* of any number up to 1600. By means of this Table, there will be easy rules to find the *Square Root* and *Cube Root* of numbers exceeding 1600.

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
1	1	1	1·0000000	1·0000000	·100000000
2	4	8	1·4142136	1·2599210	·500000000
3	9	27	1·7320508	1·4422496	·333333333
4	16	64	2·0000000	1·5874011	·250000000
5	25	125	2·2360680	1·7099759	·200000000
6	36	216	2·4494897	1·8171206	·166666667
7	49	343	2·6457513	1·9129312	·142857143
8	64	512	2·8284271	2·0000000	·125000000
9	81	729	3·0000000	2·0800837	·111111111
10	100	1000	3·1622777	2·1544347	·100000000
11	121	1331	3·3166248	2·2239801	·090909091
12	144	1728	3·4641016	2·2894286	·083333333
13	169	2197	3·6055513	2·3513347	·076928077
14	196	2744	3·7416574	2·4101422	·071428571
15	225	3375	3·8729833	2·4662121	·066666667
16	256	4096	4·0000000	2·5198421	·062500000
17	289	4913	4·1231056	2·5712816	·058823529
18	324	5832	4·2426407	2·6207414	·055555556
19	361	6859	4·3588989	2·6684016	·052631579
20	400	8000	4·4721360	2·7144177	·050000000
21	441	9261	4·5825757	2·7589243	·047619048
22	484	10648	4·6904158	2·8020393	·045454545
23	529	12167	4·7958315	2·8438670	·043478261
24	576	13824	4·8989795	2·8844991	·041666667
25	625	15625	5·0000000	2·9240177	·040000000
26	676	17576	5·0990195	2·9624960	·038461538
27	729	19683	5·1961524	3·0000000	·037037037
28	784	21952	5·2915026	3·0365889	·035714286
29	841	24389	5·3851648	3·0723168	·034482759
30	900	27000	5·4772256	3·1072325	·033333333
31	961	29791	5·5677644	3·1413806	·032258065
32	1024	32768	5·6568542	3·1748021	·031250000
33	1089	35937	5·7445626	3·2075343	·030303030
34	1156	39304	5·8309519	3·2396118	·029411765
35	1225	42875	5·9160798	3·2710663	·028571429
36	1296	46656	6·0000000	3·3019272	·027777778
37	1369	50653	6·0827625	3·3322218	·027027027
38	1444	54872	6·1644140	3·3619754	·026315789
39	1521	59319	6·2449980	3·3912114	·025641026
40	1600	64000	6·3245553	3·4199519	·025000000
41	1681	68921	6·4031242	3·4482172	·024390244
42	1764	74088	6·4807407	3·4760266	·023809524
43	1849	79507	6·5574385	3·5033981	·023255814
44	1936	85184	6·6332496	3·5303483	·022727273
45	2025	91125	6·7082039	3·5568933	·022222222
46	2116	97336	6·7823300	3·5830479	·021739130
47	2209	103823	6·8556546	3·6088261	·021276600
48	2304	110592	6·9282032	3·6342411	·020833333
49	2401	117649	7·0000000	3·6593057	·020408163
50	2500	125000	7·0710678	3·6840314	·020000000
51	2601	132651	7·1414284	3·7084298	·019607843
52	2704	140608	7·2111026	3·7325111	·019230769

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
53	2809	148877	7·2801099	3·7562858	·018867925
54	2916	157464	7·3484692	3·7797631	·018518519
55	3025	166375	7·4161985	3·8029525	·018181818
56	3136	175616	7·4833148	3·8258624	·017857143
57	3249	185193	7·5498344	3·8485011	·017543860
58	3364	195112	7·6157731	3·8708766	·017241379
59	3481	205379	7·6811457	3·8929965	·016949153
60	3600	216000	7·7459667	3·9148676	·016666667
61	3721	226981	7·8102497	3·9304972	·016393443
62	3844	238328	7·8740079	3·9578915	·016129032
63	3969	250047	7·9372539	3·9790571	·015873016
64	4096	262144	8·0000000	4·0000000	·015625000
65	4225	274625	8·0622577	4·0207256	·015384615
66	4356	287496	8·1240384	4·0412401	·015151515
67	4489	300763	8·1853528	4·0615480	·014925373
68	4624	314432	8·2462113	4·0816551	·014705882
69	4761	328509	8·3066239	4·1015661	·014492754
70	4900	343000	8·3666003	4·1212853	·014285714
71	5041	357911	8·4261498	4·1408178	·014084517
72	5184	373248	8·4852814	4·1601676	·013888889
73	5329	389017	8·5440037	4·1793390	·013698630
74	5476	405224	8·6023253	4·1983364	·013513514
75	5625	421875	8·6602540	4·2171633	·013333333
76	5776	438976	8·7177979	4·2358236	·013157895
77	5929	456533	8·7749644	4·2543210	·012987013
78	6084	474552	8·8317609	4·2726586	·012820513
79	6241	493039	8·8881944	4·2908404	·012658228
80	6400	512000	8·9442719	4·3088695	·012500000
81	6561	531441	9·0000000	4·3267487	·012345679
82	6724	551368	9·0553851	4·3444815	·012195122
83	6889	571787	9·1104336	4·3620707	·012048193
84	7056	592704	9·1651514	4·3795191	·011904762
85	7225	614125	9·2195445	4·3968296	·011764706
86	7396	636056	9·2736185	4·4140049	·011627907
87	7569	658503	9·3273791	4·4310476	·011494253
88	7744	681472	9·3808315	4·4470692	·011363636
89	7921	704969	9·4339811	4·4647451	·011235955
90	8100	729000	9·4868330	4·4814047	·011111111
91	8281	753571	9·5393920	4·4979414	·010989011
92	8464	778688	9·5916630	4·5143574	·010869565
93	8649	804357	9·6436508	4·5306549	·010752688
94	8836	830584	9·6953597	4·5468359	·010638298
95	9025	857374	9·7467943	4·5629026	·010526316
96	9216	884736	9·7979590	4·5788570	·010416667
97	9409	912673	9·8488578	4·5947009	·010309278
98	9604	941192	9·8994949	4·6104363	·010204082
99	9801	970299	9·9498744	4·6260650	·010101010
100	10000	1000000	10·0000000	4·6415888	·010000000
101	10201	1030301	10·0498756	4·6570095	·009900990
102	10404	1061208	10·0995049	4·6723287	·009803922
103	10609	1092727	10·1488916	4·6875482	·009708738
104	10816	1124864	10·1980390	4·7026694	·009615385

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
105	11025	1157625	10·2469508	4·7176940	·009523810
106	11236	1191016	10·2956301	4·7326235	·009433962
107	11449	1225043	10·3440804	4·7474594	·009345794
108	11664	1259712	10·3923048	4·7622032	·009259259
109	11881	1295029	10·4403065	4·7768562	·009174312
110	12100	1331000	10·4880885	4·7914199	·009090909
111	12321	1367631	10·5356538	4·8058995	·009009009
112	12544	1404928	10·5830052	4·8202845	·008928571
113	12769	1442897	10·6301458	4·8345881	·008849558
114	12996	1481544	10·6770783	4·8488076	·008771930
115	13225	1520875	10·7238053	4·8629442	·008695652
116	13456	1560896	10·7703296	4·8769990	·008020690
117	13689	1601613	10·8166538	4·8909732	·008547009
118	13924	1643032	10·8627805	4·9048681	·008474576
119	14161	1685159	10·9087121	4·9186847	·008403361
120	14400	1728000	10·9544512	4·9324242	·008333333
121	14641	1771561	11·0000000	4·9460874	·008264463
122	14834	1815848	11·0453610	4·9596757	·008196721
123	15129	1860867	11·0905365	4·9731898	·008130081
124	15376	1906624	11·1355287	4·9866310	·008064516
125	15625	1953125	11·1803399	5·0000000	·008000000
126	15876	2000376	11·2249722	5·0132979	·007936508
127	16129	2048383	11·2694277	5·0265257	·007874016
128	16384	2097152	11·3137085	5·0396842	·007812500
129	16641	2146689	11·3578167	5·0527743	·007751938
130	16900	2197000	11·4017543	5·0657970	·007692308
131	17161	2248091	11·4455231	5·0787531	·007633588
132	17424	2299968	11·4891253	5·0916434	·007575758
133	17689	2352637	11·5325626	5·1044687	·007518797
134	17956	2406104	11·5758369	5·1172299	·007462687
135	18225	2460375	11·6189500	5·1299278	·007407407
136	18496	2515456	11·6619038	5·1425632	·007352941
137	18769	2571353	11·7046999	5·1551367	·007299270
138	19044	2628072	11·7473444	5·1676493	·007246377
139	19321	2685619	11·7898261	5·1801015	·007194245
140	19600	2744000	11·8321596	5·1924941	·007142857
141	19881	2803221	11·8743421	5·2048279	·007092199
142	20164	2863288	11·9163753	5·2171034	·007042254
143	20449	2924207	11·9582607	5·2293215	·006993007
144	20736	2985984	12·0000000	5·2414828	·006944444
145	21025	3048625	12·0415946	5·2535879	·006896552
146	21316	3112136	12·0830460	5·2656374	·006849315
147	21609	3176523	12·1243557	5·2776321	·006802721
148	21904	3241792	12·1655251	5·2895725	·006756757
149	22201	3307949	12·2065556	5·3014592	·006711409
150	22500	3375000	12·2474487	5·3132928	·006666667
151	22801	3442951	12·2882057	5·3250740	·006622517
152	23104	3511008	12·3288280	5·3368033	·006578947
153	23409	3581577	12·3693169	5·3484812	·006535948
154	23716	3652264	12·4096736	5·3601084	·006493506
155	24025	3723875	12·4498996	5·3716854	·006451613
156	24336	3796416	12·4899960	5·3832126	·006410256

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
157	24649	3869893	12·5299641	5·3946907	·006369427
158	24964	3944312	12·5698051	5·4061202	·006329114
159	25281	4019679	12·6095202	5·4175015	·006289308
160	25600	4096000	12·6491106	5·4288352	·006250000
161	25921	4173281	12·6885775	5·4401218	·006211180
162	26244	4251528	12·7279221	5·4513618	·006172840
163	26569	4330747	12·7671453	5·4625556	·006134969
164	26896	4410944	12·8062485	5·4737037	·006097561
165	27225	4492125	12·8452326	5·4848066	·006060606
166	27556	4574296	12·8840987	5·4958647	·006024096
167	27889	4657463	12·9228480	5·5068784	·005988024
168	28224	4741632	12·9614814	5·5178484	·005952381
169	28561	4826809	13·0000000	5·5287748	·005917160
170	28900	4913000	13·0384048	5·5396583	·005882353
171	29241	5000211	13·0766968	5·5504991	·005847953
172	29584	5088448	13·1148770	5·5612978	·005813953
173	29929	5177717	13·1529464	5·5720546	·005780347
174	30276	5268024	13·1909060	5·5827702	·005747126
175	30625	5359375	13·2287566	5·5934447	·005714286
176	30976	5451776	13·2664992	5·6040787	·005681818
177	31329	5545233	13·3041347	5·6146724	·005649718
178	31684	5639752	13·3416641	5·6252263	·005617978
179	32041	5735339	13·3790882	5·6357408	·005586592
180	32400	5832000	13·4164079	5·6462162	·005555556
181	32761	5929741	13·4536240	5·6566528	·005524862
182	33124	6028568	13·4907376	5·6670511	·005494505
183	33489	6128487	13·5277493	5·6774114	·005464481
184	33856	6229504	13·5646600	5·6877340	·005434783
185	34225	6331625	13·6014705	5·6980192	·005405405
186	34596	6434856	13·6381817	5·7082675	·005376344
187	34969	6539203	13·6747943	5·7184791	·005347594
188	35344	6644672	13·7113092	5·7286543	·005319149
189	35721	6751269	13·7477271	5·7387936	·005291005
190	36100	6859000	13·7840488	5·7488971	·005263158
191	36481	6967871	13·8202750	5·7589652	·005235602
192	36864	7077888	13·8564065	5·7689982	·005208333
193	37249	7189517	13·8924400	5·7789966	·005181347
194	37636	7301384	13·9283883	5·7889604	·005154639
195	38025	7414875	13·9642400	5·7988900	·005128205
196	38416	7529536	14·0000000	5·8087857	·005102041
197	38809	7645373	14·0356688	5·8186479	·005076142
198	39204	7762392	14·0712473	5·8284867	·005050505
199	39601	7880599	14·1067360	5·8382725	·005025126
200	40000	8000000	14·1421356	5·8480355	·005000000
201	40401	8120601	14·1774469	5·8577660	·004975124
202	40804	8242408	14·2126704	5·8674673	·004950495
203	41209	8365427	14·2478068	5·8771307	·004926108
204	41616	8489664	14·2828569	5·8867653	·004901961
205	42025	8615125	14·3178211	5·8963685	·004878049
206	42436	8741816	14·3527001	5·9059406	·004854369
207	42849	8869743	14·3874946	5·9154817	·004830918
208	43264	8998912	14·4222051	5·9249921	·004807692

Number.	Squares	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
209	43681	9129329	14·4568323	5·9344721	·004784689
210	44100	9261000	14·4913767	5·9439220	·004761905
211	44521	9393931	14·5258390	5·9533418	·004739336
212	44944	9528128	14·5602198	5·9627320	·004716981
213	45369	9663597	14·5945195	5·9720926	·004694836
214	45796	9800344	14·6287388	5·9814240	·004672897
215	46225	9938375	14·6628783	5·9907264	·004651163
216	46656	10077696	14·6969385	6·0000000	·004629630
217	47089	10218313	14·7309199	6·0092450	·004608295
218	47524	10360232	14·7648231	6·0184617	·004587156
219	47961	10503459	14·7986486	6·0276502	·004566210
220	48400	10648000	14·8323970	6·0368107	·004545455
221	48841	10793861	14·8660687	6·0459435	·004524887
222	49284	10941048	14·8996644	6·0550489	·004504505
223	49729	11089567	14·9331845	6·0641270	·004484305
224	50176	11239424	14·9666295	6·0731779	·004464286
225	50625	11390625	15·0000000	6·0824020	·004444444
226	51076	11543176	15·0332964	6·0991994	·004424779
227	51529	11697083	15·0665192	6·1001702	·004405286
228	51984	11852352	15·0996689	6·1091147	·004885965
229	52441	12008989	15·1327460	6·1180332	·004366812
230	52900	12167000	15·1657509	6·1269257	·004347826
231	53361	12326391	15·1986842	6·1357924	·004329004
232	53824	12487168	15·2315462	6·1446337	·004310345
233	54289	12649337	15·2643375	6·1534495	·004291845
234	54756	12812904	15·2970585	6·1622401	·004273504
235	55225	12977875	15·3297097	6·1710058	·004255319
236	55696	13144256	15·3622915	6·1797466	·004237288
237	56169	13312053	15·3948043	6·1884628	·004219409
238	56644	13481272	15·4272486	6·1971544	·004201681
239	57121	13651919	15·4596248	6·2058218	·004184100
240	57600	13824000	15·4919334	6·2144650	·004166667
241	58081	13997521	15·5241747	6·2230843	·004149378
242	58564	14172488	15·5563492	6·2316797	·004132231
243	59049	14348907	15·5884573	6·2402515	·004115226
244	59536	14526784	15·6204994	6·2487998	·004098361
245	60025	14706125	15·6524758	6·2573248	·004081633
246	60516	14886936	15·6843871	6·2658266	·004065041
247	61009	15069223	15·7162336	6·2743054	·004048583
248	61504	15252992	15·7480157	6·2827613	·004032258
249	62001	15438249	15·7797338	6·2911946	·004016064
250	62500	15625000	15·8113883	6·2996053	·004000000
251	63001	15813251	15·8429795	6·3079935	·003984064
252	63504	16003008	15·8745079	6·3163596	·003968254
253	64009	16194277	15·9059737	6·3247035	·003952569
254	64516	16387064	15·9373775	6·3330256	·003937008
255	65025	16581375	15·9687194	6·3413257	·003921569
256	65536	16777216	16·0000000	6·3496042	·003906250
257	66049	16974593	16·0312195	6·3578611	·003891051
258	66564	17173512	16·0623784	6·3660968	·003875969
259	67081	17373979	16·0934769	6·3743111	·003861004
260	67600	17576000	16·1245155	6·3825043	·003846154

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
261	68121	17779581	16·1554944	6·3906765	·003831418
262	68644	17984728	16·1864141	6·3988279	·003816794
263	69169	18191447	16·2172747	6·4069585	·003802281
264	69696	18399744	16·2480768	6·4150687	·003787879
265	70225	18609625	16·2788206	6·4231583	·003773585
266	70756	18821096	16·3095064	6·4312276	·003759398
267	71289	19034163	16·3401346	6·4392767	·003745318
268	71824	19248832	16·3707055	6·4473057	·003731343
269	72361	19465109	16·4012195	6·4553148	·003717472
270	72900	19683000	16.4316767	6·4633041	·003703704
271	73441	19902511	16·4620776	6·4712736	·003690037
272	73984	20123643	16·4924225	6·4792236	·003676471
273	74529	20346417	16·5227116	6·4871541	·003663004
274	75076	20570824	16·5529454	6·4950653	·003649635
275	75625	20796875	16·5831240	6·5029572	·003636364
276	76176	21024576	16·6132477	6·5108300	·003623188
277	76729	21253933	16·6433170	6·5186839	·003610108
278	77284	21484952	16·6783320	6·5265189	·003597122
279	77841	21717639	16·7032931	6·5343351	·003584229
280	78400	21952000	16·7332005	6·5421326	·003571429
281	78961	22188041	16·7630546	6·5499116	·003558719
282	79524	22425768	16·7928556	6·5576722	·003546099
283	80089	22665187	16·8226038	6·5654144	·003533569
284	80656	22906304	16·8522995	6·5731385	·003522127
285	81225	23149125	16·8819430	6·5808443	·003508772
286	81796	23393656	16·9115345	6·5885323	·003496503
287	82369	23639903	16·9410743	6·5962023	·003484321
288	82944	23887872	16·9705627	6·6038545	·003472222
289	83521	24137569	17·0000000	6·6114890	·003460208
290	84100	24389000	17·0293864	6·6191060	·003448276
291	84681	24642171	17·0587221	6·6267054	·003436426
292	85264	24897088	17·0880075	6·6342874	·003424658
293	85849	25153757	17·1172428	6·6418522	·003412969
294	86436	25412184	17·1464282	6·6493998	·003401361
295	87025	25672375	17·1755640	6·6569302	·003389831
296	87616	25934836	17·2046505	6·6644437	·003378378
297	88209	26198073	17·2336879	6·6719403	·003367003
298	88804	26463592	17·2626765	6.6794200	·003355705
299	89401	26730899	17·2916165	6.6868831	·003344482
300	90000	27000000	17·3205081	6.6943295	·003333333
301	90601	27270901	17·3493516	6.7017593	·003322259
302	91204	27543608	17·3781472	6·7091729	·003311258
303	91809	27818127	17·4068952	6·7165700	·003301330
304	92416	28094464	17·4355958	6·7239508	·003289474
305	93025	28372625	17·4642492	6·7313155	·003278689
306	93636	28652616	17·4928557	6·7386641	·003267974
307	94249	28934443	17·5214155	6·7459967	·003257329
308	94864	29218112	17·5499288	6·7533134	·003246753
309	95481	29503609	17·5783958	6·7606143	·003236246
310	96100	29791000	17·6068169	6·7678995	·003225806
311	96721	30080231	17·6351921	6·7751690	·003215434
312	97344	30371328	17·6635217	6·7824229	·003205128

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
313	97969	30664297	17·6918060	6·7896613	·003194888
314	98596	30959144	17·7200451	6·7968844	·003184713
315	99225	31255875	17·7482393	6·8040921	·003174603
316	99856	31554496	17·7763888	6·8112847	·003164557
317	100489	31855013	17·8044938	6·8184620	.003154574
318	101124	32157432	17·8325545	6·8256242	·003144654
319	101761	32461759	17·8605711	6·8327714	·003134796
320	102400	32768000	17·8885438	6·8399037	·003125000
321	103041	33076161	17·9164729	6·8470213	·003115265
322	103684	33386248	17·9443584	6·8541240	·003105590
323	104329	33698267	17·9722008	6·8612120	·003095975
324	104976	34012224	18·0000000	6·8682855	·003086420
325	105625	34328125	18·0277564	6·8753433	·003076923
326	106276	34645976	18·0554701	6·8823888	·003067485
327	106929	34965783	18·0831413	6·8894188	·003048104
328	107584	35287552	18·1107703	6·8964345	·003048780
329	108241	35611289	18·1383571	6·9034359	·003039514
330	108900	35937000	18·1659021	6·9104232	·003030303
331	109561	36264691	18·1934054	6·9173964	·003021148
332	110224	36594368	18·2208672	6·9243556	·003012048
333	110889	36926037	18·2482876	6·9313088	·003003003
334	111556	37259704	18·2756669	6·9382321	·002994012
335	112225	37595375	18·3030052	6·9451496	·002985075
336	112896	37933056	18·3303028	6·9520533	·002976190
337	113569	38272753	18·3575598	6·9589434	·002967359
338	114244	38614472	18·3847763	6·9658198	·002958580
339	114921	38958219	18·4119526	6·9726826	·002949853
340	115600	39304000	18·4390889	6·9795321	·002941176
341	116281	39651821	18·4661853	6·9863681	·002932551
342	116964	40001688	18·4932420	6·9931906	·002923977
343	117649	40353607	18·5202592	7·0000000	·002915452
344	118336	40707584	18·5472370	7·0067962	·002906977
345	119025	41063625	18·5741756	7·0135791	·002898551
346	119716	41421736	18·6010752	7·0203490	·002890173
347	120409	41781923	18·6279360	7·0271058	·002881844
348	121104	42144192	18·6547581	7·0338497	·002873563
349	121801	42508549	18·6815417	7·0405860	·002865330
350	122500	42875000	18·7082869	7·0472987	·002857143
351	123201	43243551	18·7349940	7·0540041	·002849003
352	123904	43614208	18·7616630	7·0606967	·002840909
353	124609	43986977	18·7882942	7·0673767	·002832861
354	125316	44361864	18·8148877	7·0740440	·002824859
355	126025	44738875	18·8414437	7·0806988	·002816901
356	126736	45118016	18·8679623	7·0873411	·002808989
357	127449	45499293	18·8944436	7·0939709	·002801120
358	128164	45882712	18·9208879	7·1005885	·002793296
359	128881	46268279	18·9472953	7·1071937	·002785515
360	129600	46656000	18·9736660	7·1137866	·002777778
361	130321	47045831	19·0000000	7·1203674	·002770083
362	131044	47437928	19·0262976	7·1269360	·002762431
363	131769	47832147	19·0525589	7·1334925	·002754821
364	132496	48228544	19·0787840	7·1400370	·002747253

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
365	133225	48627125	19·1049732	7·1465695	·002739726
366	133956	49027896	19·1311265	7·1530901	·002732240
367	134689	49430863	19·1572441	7·1595988	·002724796
368	135424	49836032	19·1833261	7·1660957	·002717391
369	136161	50243409	19·2093727	7·1725809	·002710027
370	136900	50653000	19·2353841	7·1790544	·002702703
371	137641	51064811	19·2613603	7·1855162	·002695418
372	138384	51478848	19·2873015	7·1919663	·002688172
373	139129	51895117	19·3132079	7·1984050	·002680965
374	139876	52313624	19·3390796	7·2048322	·002673797
375	140625	52734375	19·3649167	7·2112479	·002666667
376	141376	53157376	19·3907194	7·2176522	·002659574
377	142129	53582633	19·4164878	7·2240450	·002652520
378	142884	54010152	19·4422221	7·2304268	·002645503
379	143641	54439939	19·4679223	7·2367972	·002638521
380	144400	54872000	19·4935887	7·2431565	·002631579
381	145161	55306341	19·5192213	7·2495045	·002624672
382	145924	55742968	19·5448203	7·2558415	·002617801
383	146689	56181887	19·5703858	7·2621675	·002610966
384	147456	56623104	19·5959179	7·2684824	·002604167
385	148225	57066625	19·6214169	7·2747864	·002597403
386	148996	57512456	19·6468827	7·2810794	·002590674
387	149769	57960603	19·6723156	7·2873617	·002583979
388	150544	58411072	19·6977156	7·2936330	·002577320
389	151321	58863869	19·7230829	7·2998936	·002570694
390	152100	59319000	19·7484177	7·3061436	·002564103
391	152881	59776471	19·7737199	7·3123828	·002557545
392	153664	60236288	19·7989899	7·3186114	·002551020
393	154449	60698457	19·8242276	7·3248295	·002544529
394	155236	61162984	19·8494332	7·3310369	·002538071
395	156025	61629875	19·8746069	7·3372339	·002531646
396	156816	62099136	19·8997487	7·3434205	·002525253
397	157609	62570773	19·9248588	7·3495966	·002518892
398	158404	63044792	19·9499373	7·3557624	·002512563
399	159201	63521199	19·9749844	7·3619178	·002506266
400	160000	64000000	20·0000000	7·3680630	·002500000
401	160801	64481201	20·0249844	7·3741979	·002493766
402	161604	64964808	20·0499377	7·3803227	·002487562
403	162409	65450827	20·0748599	7·3864373	·002481390
404	163216	65939264	20·0997512	7·3925418	·002475248
405	164025	66430125	20·1246118	7·3986363	·002469136
406	164836	66923416	20·1494417	7·4047206	·002463054
407	165649	67419143	20·1742410	7·4107950	·002457002
408	166464	67917312	20·1990099	7·4168595	·002450980
409	167281	68417929	20·2237484	7·4229142	·002444988
410	168100	68921000	20·2484567	7·4289589	·002439024
411	168921	69426531	20·2731349	7·4349938	·002433090
412	169744	69934528	20·2977831	7·4410189	·002427184
413	170569	70444997	20·3224014	7·4470343	·002421308
414	171396	70957944	20·3469899	7·4530399	·002415459
415	172225	71473375	20·3715488	7·4590359	·002409639
416	173056	71991296	20·3960781	7·4650223	·002406846

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
417	173889	72511713	20·4205779	7·4709991	·002398082
418	174724	73034632	20·4450483	7·4769664	·002392344
419	175561	73560059	20·4694895	7·4829242	·002386635
420	176400	74088000	20·4939015	7·4888724	·002380952
421	177241	74618461	20·5182845	7·4948113	·002375297
422	178084	75151448	20·5426386	7·5007406	·002369668
423	178929	75686967	20·5669638	7·5066607	·002364066
424	179776	76225024	20·5912603	7·5125715	·002358491
425	180625	76765625	20·6155281	7·5184730	·002352941
426	181476	77308776	20·6397674	7·5243652	·002347418
427	182329	77854483	20·6639783	7·5302482	·002341920
428	183184	78402752	20·6881609	7·5361221	·002336449
429	184041	78953589	20·7123152	7·5419867	·002331002
430	184900	79507000	20·7364414	7·5478423	·002325581
431	185761	80062991	20·7605395	7·5536888	·002320186
432	186624	80621568	20·7846097	7·5595263	·002314815
433	187489	81182737	20·8086520	7·5653548	·002309469
434	188356	81746504	20·8326667	7·5711743	·002304147
435	189225	82312875	20·8566536	7·5769849	·002298851
436	190096	82881856	20·8806130	7·5827865	·002293578
437	190969	83453453	20·9045450	7·5885793	·002288330
438	191844	84027672	20·9284495	7·5943633	·002283105
439	192721	84604519	20·9523268	7·6001385	·002277904
440	193600	85184000	20·9761770	7·6059049	·002272727
441	194481	85766121	21·0000000	7·6116626	·002267574
442	195364	86350888	21·0237960	7·6174116	·002262443
443	196249	86938307	21·0475652	7·6231519	·002257336
444	197136	87528384	21·0713075	7·6288837	·002252252
445	198025	88121125	21·0950231	7·6346067	·002247191
446	198916	88716536	21·1187121	7·6403213	·002242152
447	199809	89314623	21·1423745	7·6460272	·002237136
448	200704	89915392	21·1660105	7·6517247	·002232143
449	201601	90518849	21·1896201	7·6574138	·002227171
450	202500	91125000	21·2132034	7·6630943	·002222222
451	203401	91733851	21·2367606	7·6687665	·002217295
452	204304	92345408	21·2602916	7·6744303	·002212389
453	205209	92959677	21·2837967	7·6800857	·002207506
454	206116	93576664	21·3072758	7·6857328	·002202643
455	207025	94196375	21·3307290	7·6913717	·002197802
456	207936	94818816	21·3541565	7·6970023	·002192982
457	208849	95443993	21·3775583	7·7026246	·002188184
458	209764	96071912	21·4009346	7·7082388	·002183406
459	210681	96702579	21·4242853	7·7188448	·002178649
460	211600	97336000	21·4476106	7·7194426	·002173913
461	212521	97972181	21·4709106	7·7250325	·002169197
462	213444	98611128	21·4941853	7·7306141	·002164502
463	214369	99252847	21·5174348	7·7361877	·002159827
464	215296	99897344	21·5406592	7·7417532	·002155172
465	216225	100544625	21·5638587	7·7473109	·002150538
466	217156	101194696	21·5870331	7·7528606	·002145923
467	218089	101847563	21·6101828	7·7584023	·002141328
468	219024	102503232	21·6333077	7·7639361	·002136752

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
469	219961	103161709	21·6564078	7·7694620	·002132196
470	220900	103823000	21·6794834	7·7749801	·002127660
471	221841	104487111	21·7025344	7·7804904	·002123142
472	222784	105154048	21·7255610	7·7859928	·002118644
473	223729	105828817	21·7485632	7·7914875	·002114165
474	224676	106496424	21·7715411	7·7969745	·002109705
475	225625	107171875	21·7944947	7·8024538	·002105263
476	226576	107850176	21·8174242	7·8079254	·002100840
477	227529	108531333	21·8403297	7·8133892	·002096486
478	228484	109215352	21·8632111	7·8188456	·002092050
479	229441	109902239	21·8860686	7·8242942	·002087683
480	230400	110592000	21·9089023	7·8297353	·002083333
481	231361	111284641	21·9317122	7·8351688	·002079002
482	232324	111980168	21·9544984	7·8405949	·002074689
483	233289	112678587	21·9772610	7·8460134	·002070393
484	234256	113379904	22·0000000	7·8514244	·002066116
485	235225	114084125	22·0227155	7·8568281	·002061856
486	236196	114791256	22·0454077	7·8622242	·002057613
487	237169	115501303	22·0680765	7·8676130	·002053388
488	238144	116214272	22·0907220	7·8729944	·002049180
489	239121	116930169	22·1133444	7·8783684	·002044990
490	240100	117649000	22·1359436	7·8837352	·002040816
491	241081	118370771	22·1585198	7·8890946	·002036660
492	242064	119095488	22·1810730	7·8944468	·002032520
493	243049	119823157	22·2036033	7·8997917	·002028398
494	244036	120553784	22·2261108	7·9051294	·002024291
495	245025	121287375	22·2485955	7·9104599	·002020202
496	246016	122023936	22·2710575	7·9157832	·002016129
497	247009	122763473	22·2934968	7·9210994	·002012072
498	248004	123505992	22·3159136	7·9264085	·002008032
499	249001	124251499	22·3383079	7·9317104	·002004008
500	250000	125000000	22·3606798	7·9370053	·002000000
501	251001	125751501	22·3830293	7·9422931	·001996008
502	252004	126506008	22·4053565	7·9475739	·001992032
503	253009	127263527	22·4276615	7·9528477	·001988072
504	254016	128024064	22·4499443	7·9581144	·001984127
505	255025	128787625	22·4722051	7·9633743	·001980198
506	256036	129554216	22·4944438	7·9686271	·001976285
507	257049	130323843	22·5166605	7·9738731	·001972387
508	258064	131096512	22·5388553	7·9791122	·001968504
509	259081	131872229	22·5610283	7·9843444	·001964637
510	260100	132651000	22·5831796	7·9895697	·001960784
511	261121	133432831	22·6053091	7·9947883	·001956947
512	262144	134217728	22·6274170	8·0000000	·001953125
513	263169	135005697	22·6495033	8·0052049	·001949318
514	264196	135796744	22·6715681	8·0104032	·001945525
515	265225	136590875	22·6936114	8·0155946	·001941748
516	266256	137388096	22·7156334	8·0207794	·001937984
517	267289	138188413	22·7376341	8·0259574	·001934236
518	268324	138991832	22·7596134	8·0311287	·001930502
519	269361	139798359	22·7815715	8·0362935	·001926782
520	270400	140608000	22·8035085	8·0414515	·001923077

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
521	271441	141420761	22·8254244	8·0466030	·001919386
522	272484	142236648	22·8473193	8·0517479	·001915709
523	273529	143055667	22·8691933	8·0568862	·001912046
524	274576	143877824	22·8910463	8·0620180	·001908397
525	275625	144703125	22·9128785	8·0671432	·001904762
526	276676	145531576	22·9346899	8·0722620	·001901141
527	277729	146363183	22·9564806	8·0773743	·001897533
528	278784	147197952	22·9782506	8·0824800	·001893939
529	279841	148035889	23·0000000	8·0875794	·001890359
530	280900	148877001	23·0217289	8·0926723	·001886792
531	281961	149721291	23·0434372	8·0977589	·001883239
532	283024	150568768	23·0651252	8·1028390	·001879699
533	284089	151419437	23·0867928	8·1079128	·001876173
534	285156	152273304	23·1084400	8·1129803	·001872659
535	286225	153130375	23·1300670	8·1180414	·001869159
536	287296	153990656	23·1516738	8·1230962	·001865672
537	288369	154854153	23·1732605	8·1281447	·001862197
538	289444	155720872	23·1948270	8·1331870	·001858736
539	290521	156590819	23·2163735	8·1382230	·001855288
540	291600	157464000	23·2379001	8·1432529	·001851852
541	292681	158340421	23·2594067	8·1482765	·001848429
542	293764	159220088	23·2808935	8·1532939	·001845018
543	294849	160103007	23·3023604	8·1583051	·001841621
544	295936	160989184	23·3238076	8·1633102	·001838235
545	297025	161878625	23·3452351	8·1683092	·001834862
546	298116	162771336	23·3666429	8·1733020	·001831502
547	299209	163667323	23·3880311	8·1782888	·001828154
548	300304	164566592	23·4093998	8·1832695	·001824818
549	301401	165469149	23·4307490	8·1882441	·001821494
550	302500	166375000	23·4520788	8·1932127	·001818182
551	303601	167284151	23·4733892	8·1981753	·001814882
552	304704	168196608	23·4946802	8·2031319	·001811594
553	305809	169112377	23·5159520	8·2080825	·001808318
554	306916	170031464	23·5372046	8·2130271	·001805054
555	308025	170953875	23·5584380	8·2179657	·001801802
556	309136	171879616	23·5796522	8·2228985	·001798561
557	310249	172808693	23·6008474	8·2278254	·001795332
558	311364	173741112	23·6220236	8·2327463	·001792115
559	312481	174676879	23·6431808	8·2376614	·001788909
560	313600	175616000	23·6643191	8·2425706	·001785714
561	314721	176558481	23·6854386	8·2474740	·001782531
562	315844	177504328	23·7065392	8·2523715	·001779359
563	316969	178453547	23·7276210	8·2572635	·001776199
564	318096	179406144	23·7486842	8·2621492	·001773050
565	319225	180362125	23·7697286	8·2670294	·001769912
566	320356	181321496	23·7907545	8·2719039	·001766784
567	321489	182284263	23·8117618	8·2767726	·001763668
568	322624	183250432	23·8327506	8·2816255	·001760563
569	323761	184220009	23·8537209	8·2864928	·001757469
570	324900	185193000	23·8746728	8·2913444	·001754386
571	326041	186169411	23·8956063	8·2961903	·001751313
572	327184	187149248	23·9165215	8·3010304	·001748252

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
573	328329	188132517	23·9374184	8·3058651	·001745201
574	329476	189119224	23·9582971	8·3106941	·001742160
575	330625	190109375	23·9791576	8·3155175	·001739130
576	331776	191102976	24·0000000	8·3203353	·001736111
577	332927	192100033	24·0208243	8·3251475	·001733102
578	334084	193100552	24·0416306	8·3299542	·001730104
·579	335241	194104539	24·0624188	8·3347553	·001727116
580	336400	195112000	24·0831891	8·3395509	·001724138
581	337561	196122941	24·1039416	8·3443410	·001721170
582	338724	197137368	24·1246762	8·3491256	·001718213
583	339889	198155287	24·1453929	8·3539047	·001715266
584	341056	199176704	24·1660919	8·3586784	·001712329
585	342225	200201625	24·1867732	8·3634466	·001709402
586	343396	201230056	24·2074369	8·3682095	·001706485
587	344569	202262003	24·2280829	8·3729668	·001703578
588	345744	203297472	24·2487113	8·3777188	·001700680
589	346921	204336469	24·2693222	8·3824653	·001697793
590	348100	205379000	24·2899156	8·3872065	·001694915
591	349281	206425071	24·3104996	8·3919428	·001692047
592	350464	207474688	24·3310501	8·3966729	·001689189
593	351649	208527857	24·3515913	8·4013981	·001686341
594	352836	209584584	24·3721152	8·4061180	·001683502
595	354025	210644875	24·3926218	8·4108326	·001680672
596	355216	211708736	24·4131112	8·4155419	·001677852
597	356409	212776173	24·4335834	8·4202460	·001675042
598	357604	213847192	24·4540385	8·4249448	·001672241
599	358801	214921799	24·4744765	8·4296383	·001669449
600	360000	216000000	24·4948974	8·4343267	·001666667
601	361201	217081801	24·5153013	8·4390098	·001663894
602	362404	218167208	24·5356883	8·4436877	·001661130
603	363609	219256227	24·5560583	8·4483605	·001658375
604	364816	220348864	24·5764115	8·4530281	·001655629
605	366025	221445125	24·5967478	8·4576906	·001652893
606	367236	222545016	24·6170673	8·4623479	·001650165
607	368449	223648543	24·6373700	8·4670001	·001647446
608	369664	224755712	24·6576560	8·4716471	·001644737
609	370881	225866529	24·6779254	8·4762892	·001642036
610	372100	226981000	24·6981781	8·4809261	·001639344
611	373321	228099131	24·7184142	8·4855579	·001636661
612	374544	229220928	24·7386338	8·4901848	·001633987
613	375769	230346397	24·7588368	8·4948065	·001631321
614	376996	231475544	24·7790234	8·4994233	·001628664
615	378225	232608375	24·7991935	8·5040350	·001626016
616	379456	233744896	24·8193473	8·5086417	·001623377
617	380689	234885113	24·8394847	8·5132435	·001620746
618	381924	236029032	24·8596058	8·5178403	·001618123
619	383161	237176659	24·8797106	8·5224331	·001615509
620	384400	238328000	24·8997992	8·5270189	·001612903
621	385641	239483061	24·9198716	8·5316009	·001610306
622	386884	240641848	24·9399278	8·5361780	·001607717
623	388129	241804367	24·9599679	8·5407501	·001605136
624	389376	242970624	24·9799920	8·5453173	001602564

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
625	390625	244140625	25·0000000	8·5498797	·001600000
626	391876	245134376	25·0199920	8·5544372	·001597444
627	393129	246491883	25·0399681	8·5589899	·001594896
628	394384	247673152	25·0599282	8·5635377	·001592357
629	395641	248858189	25·0798724	8·5680807	·001589825
630	396900	250047000	25·0998008	8·5726189	·001587302
631	398161	251239591	25·1197134	8·5771523	·001584786
632	399424	252435968	25·1396102	8·5816809	·001582278
633	400689	253636137	25·1594913	8·5862247	·001579779
634	401956	254840104	25·1793566	8·5907238	·001577287
635	403225	256047875	25·1992063	8·5952380	·001574803
636	404496	257259456	25·2190404	8·5997476	·001572327
637	405769	258474853	25·2388589	8·6042525	·001569859
638	407044	259694072	25·2586619	8·6087526	·001567398
639	408321	260917119	25·2784493	8·6132480	·001564945
640	409600	262144000	25·2982213	8·6177388	·001562500
641	410881	263374721	25·3179778	8·6222248	·001560062
642	412164	264609288	25·3377189	8·6267063	·001557632
643	413449	265847707	25·3574447	8·6311830	·001555210
644	414736	267089984	25·3771551	8·6356551	·001552795
645	416125	268336125	25·3968502	8·6401226	·001550388
646	417316	269585136	25·4165302	8·6445855	·001547988
647	418609	270840023	25·4361947	8·6490437	·001545595
648	419904	272097792	25·4558441	8·6534974	·001543210
649	421201	273359449	25·4754784	8·6579465	·001540832
650	422500	274625000	25·4950976	8·6623911	·001538462
651	423801	275894451	25·5147013	8·6668310	·001536098
652	425104	277167808	25·5342907	8·6712665	·001533742
653	426409	278445077	25·5538647	8·6756974	·001531394
654	427716	279726264	25·5734237	8·6801237	·001529052
655	429025	281011375	25·5929678	8·6845456	·001526718
656	430336	282300416	25·6124969	8·6889630	·001524390
657	431639	283593393	25·6320112	8·6933759	·001522070
658	432964	284890312	25·6515107	8·6977843	·001519751
659	434281	286191179	25·6709953	8·7021882	·001517451
660	435600	287496000	25·6904652	8·7065877	·001515152
661	436921	288804781	25.7099203	8·7109827	·001512859
662	438244	290117528	25·7293607	8·7153734	·001510574
663	439569	291434247	25·7487864	8·7197596	·001508296
664	440896	292754944	25·7681975	8·7241414	·001506024
665	442225	294079625	25·7875939	8·7285187	·001503759
666	443556	295408296	25·8069758	8·7328918	·001501502
667	444899	296740963	25·8263431	8·7372604	·001499250
668	446224	298077632	25·8456960	8·7416246	·001497006
669	447561	299418309	25·8650343	8·7459846	·001494768
670	448900	300763000	25·8843582	8·7503401	·001492537
671	450241	302111711	25·9036677	8·7546913	·001490313
672	451584	303464448	25·9229628	8·7590383	·001488095
673	452929	304821217	25·9422435	8·7633809	·001485884
674	454276	306182024	25·9615100	8·7677192	·001483680
675	455625	307546875	25·9807621	8·7720532	·001481481
676	456976	308915776	26·0000000	8·7763830	·001479290

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
677	458329	310288733	26·0192237	8·7807084	·001477105
678	459684	311665752	26·0384331	8·7850296	·001474926
679	461041	313046839	26·0576284	8·7893466	·001472754
680	462400	314432000	26·0768096	8·7936593	·001470588
681	463761	315821241	26·0959767	8·7979679	·001468429
682	465124	317214568	26·1151297	8·8022721	·001466276
683	466489	318611987	26·1342687	8·8065722	·001464129
684	467856	320013504	26·1533937	8·8108681	·001461988
685	469225	321419125	26·1725047	8·8151598	·001459854
686	470596	322828856	26·1916017	8·8194474	·001457726
687	471969	324242703	26·2106848	8·8237307	·001455604
688	473344	325660672	26·2297541	8·8280099	·001453488
689	474721	327082769	26·2488095	8·8322850	·001451379
690	476100	328509000	26·2678511	8·8365559	·001449275
691	477481	329939371	26·2868789	8·8408227	·001447178
692	478864	331373888	26·3058929	8·8450854	·001445087
693	480249	332812557	26·3248932	8·8493440	·001443001
694	481636	334255384	26·3438797	8·8535985	·001440922
695	483025	335702375	26·3628527	8·8578489	·001438849
696	484416	337153536	26·3818119	8·8620952	·001436782
697	485809	338608873	26·4007576	8·8663375	·001434720
698	487204	340068392	26·4196896	8·8705757	·001432665
699	488601	341532099	26·4386081	8·8748099	·001430615
700	490000	343000000	26·4575131	8·8790400	·001428571
701	491401	344472101	26·4764046	8·8832661	·001426534
702	492804	345948408	26·4952826	8·8874882	·001424501
703	494209	347428927	26·5141472	8·8917063	·001422475
704	495616	348913664	26·5329983	8·8959204	·001420455
705	497025	350402625	26·5518361	8·9001304	·001418440
706	498436	351895816	26·5706605	8·9043366	·001416431
707	499849	353393243	26·5894716	8·9085387	·001414427
708	501264	354894912	26·6082694	8·9127369	·001412429
709	502681	356400829	26·6270539	8·9169311	·001410437
710	504100	357911000	26·6458252	8·9211214	·001408451
711	505521	359425431	26·6645833	8·9253078	·001406470
712	506944	360944128	26·6833281	8·9294902	·001404494
713	508369	362467097	26·7020598	8·9336687	·001402525
714	509796	363994344	26·7207784	8·9378433	·001400560
715	511225	365525875	26·7394839	8·9420140	·001398601
716	512656	367061696	26·7581763	8·9461809	·001396648
717	514089	368601813	26·7768557	8·9503438	·001394700
718	515524	370146232	26·7955220	8·9545029	·001392758
719	516961	371694959	26·8141754	8·9586581	·001390821
720	518400	373248000	26·8328157	8·9628095	·001388889
721	519841	374805361	26·8514432	8·9669570	·001386963
722	521284	376367048	26·8700577	8·9711007	·001385042
723	522729	377933067	26·8886593	8·9752406	·001383126
724	524176	379503424	26·9072481	8·9793766	·001381215
725	525625	381078125	26·9258240	8·9835089	·001379310
726	527076	382657176	26·9443872	8·9876373	·001377410
727	528529	384240583	26·9629375	8·9917620	·001375516
728	529984	385828352	26·9814751	8·9958899	·001373626

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
729	531441	387420489	27·0000000	9·0000000	·001371742
730	532900	389017000	27·0185122	9·0041134	·001369863
731	534361	390617891	27·0370117	9·0082229	·001367989
732	535824	392223168	27·0554985	9·0123288	·001366120
733	537289	393832837	27·0739727	9·0164309	·001364256
734	538756	395446904	27·0924344	9·0205293	·001362398
735	540225	397065375	27·1108834	9·0246239	·001360544
736	541696	398688256	27·1293199	9·0287149	·001358696
737	543169	400315553	27·1477149	9·0328021	·001356852
738	544644	401947272	27·1661554	9·0368857	·001355014
739	546121	403583419	27·1845544	9·0409655	·001353180
740	547600	405224000	27·2029140	9·0450419	·001351351
741	549801	406869021	27·2213152	9·0491142	·001349528
742	550564	408518488	27·2396769	9·0531831	·001347709
743	552049	410172407	27·2580263	9·0572482	·001345895
744	553536	411830784	27·2763634	9·0613098	·001344086
745	555025	413493625	27·2946881	9·0653677	·001342282
746	556516	415160936	27·3130006	9·0694220	·001340483
747	558009	416832723	27·3313007	9·0734726	·001338688
748	559504	418508992	27·3495887	9·0775197	·001336898
749	561001	420189749	27·3678644	9·0815631	·001335113
750	562500	421875000	27·3861279	9·0856030	·001333333
751	564001	423564751	27·4043792	9·0896352	·001331558
752	565504	425259008	27·4226184	9·0936719	·001329787
753	567009	426957777	27·4408455	9·0977010	·001328021
754	568516	428661064	27·4590604	9·1017265	·001326260
755	570025	430368875	27·4772633	9·1057485	·001324503
756	571536	432081216	27·4954542	9·1097669	·001322751
757	573049	433798093	27·5136330	9·1137818	·001321004
758	574564	435519512	27·5317998	9·1177931	·001319261
759	576081	437245479	27·5499546	9·1218010	·001317523
760	577600	438976000	27·5680975	9·1258053	·001315789
761	579121	440711081	27·5862284	9·1298061	·001314060
762	580644	442450728	27·6043475	9·1338034	·001312336
763	582169	444194947	27·6224546	9·1377971	·001310616
764	583696	445943744	27·6405499	9·1417874	·001308901
765	585225	447697125	27·6586334	9·1457742	·001307190
766	586756	449455096	27·6767050	9·1497576	·001305483
767	588289	451217663	27·6947648	9·1537375	·001303781
768	589824	452984832	27·7128129	9·1577139	·001302083
769	591361	454756609	27·7308492	9·1616869	·001300390
770	592900	456533000	27·7488739	9·1656565	·001298701
771	594441	458314011	27·7668868	9·1696225	·001297017
772	595984	460099648	27·7848880	9·1735852	·001295337
773	597529	461889917	27·8028775	9·1775445	·001293661
774	599076	463684824	27·8208555	9·1815003	·001291990
775	600625	465484375	27·8388218	9·1854527	·001290323
776	602176	467288576	27·8567766	9·1894018	·001288660
777	603729	469097433	27·8747197	9·1933474	·001287001
778	605284	470910952	27·8926514	9·1972897	·001285347
779	606841	472729139	27·9105715	9·2012286	·001283697
780	608400	474552000	27·9284801	9·2051641	·001282051

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
781	609961	476379541	27·9463772	9·2090962	·001280410
782	611524	478211768	27·9642629	9·2130250	·001278772
783	613089	480048687	27·9821372	9·2169505	·001277139
784	614656	481890304	28·0000000	9·2208726	·001275510
785	616225	483736625	28·0178515	9·2247914	·001273885
786	617796	485587656	28·0356915	9·2287068	·001272265
787	619369	487443403	28·0535203	9·2326189	·001270648
788	620944	489303872	28·0713377	9·2365277	·001269036
789	622521	491169069	28·0891438	9·2404333	·001267427
790	624100	493039000	28·1069386	9·2443355	·001265823
791	625681	494913671	28·1247222	9·2482344	·001264223
792	627624	496793088	28·1424946	9·2521300	·001262626
793	628849	498677257	28·1602557	9·2560224	·001261034
794	630436	500566184	28·1780056	9·2599114	·001259446
795	632025	502459875	28·1957444	9·2637973	·001257862
796	633616	504358336	28·2134720	9·2676798	·001256281
797	635209	506261573	28·2311884	9·2715592	·001254705
798	636804	508169592	28·2488938	9·2754352	·001253133
799	638401	510082399	28·2665881	9·2793081	·001251364
800	640000	512000000	28·2842712	9·2831777	·001250000
801	641601	513922401	28·3019434	9·2870444	·001248439
802	643204	515849608	28·3196045	9·2909072	·001246883
803	644809	517781627	28·3372546	9·2947671	·001245330
804	646416	519718464	28·3548938	9·2986239	·001243781
805	648025	521660125	28·3725219	9·3024775	·001242236
806	649636	523606616	28·3901391	9·3063278	·001240695
807	651249	525557943	28·4077454	9·3101750	·001239157
808	652864	527514112	28·4253408	9·3140190	·001237624
809	654481	529475129	28·4429253	9·3178599	·001236094
810	656100	531441000	28·4604989	9·3216975	·001234568
811	657721	533411731	28·4780617	9·3255320	·001233046
812	659344	535387328	28·4956137	9·3293634	·001231527
813	660969	537367797	28·5131549	9·3331916	·001230012
814	662596	539353144	28·5306852	9·3370167	·001228501
815	664225	541343375	28·5482048	9·3408386	·001226994
816	665856	543338496	28·5657137	9·3446575	·001225499
817	667489	545338513	28·5832119	9·3484731	·001223990
818	669124	547343432	28·6006993	9·3522857	·001222494
819	670761	549353259	28·6181760	9·3560952	·001221001
820	672400	551368000	28·6356421	9·3599016	·001219512
821	674041	553387661	28·6530976	9·3637049	·001218027
822	675684	555412248	28·6705424	9·3675051	·001216545
823	677329	557441767	28·6879716	9·3713022	·001215067
824	678976	559476224	28·7054002	9·3750963	·001213592
825	680625	561515625	28·7228132	9·3788873	·001212121
826	682276	563559976	28·7402157	9·3826752	·001210654
827	683929	565609283	28·7576077	9·3864600	·001209190
828	685584	567663552	28·7749891	9·3902419	·001207729
829	687241	569722789	28·7923601	9·3940206	·001206273
830	688900	571787000	28·8097206	9·3977964	·001204819
831	690561	573856191	28·8270706	9·4015691	·001203369
832	692224	575930368	28·8444102	9·4053387	·001201923

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
833	693889	578009537	28·8617394	9·4091054	·001200480
834	695556	580093704	28·8790582	9·4128690	·001199041
835	697225	582182875	28·8963666	9·4166297	·001197605
836	698896	584277056	28·9136646	9·4203873	·001196172
837	700569	586376253	28·9309523	9·4241420	·001194743
838	702244	588480472	28·9482297	9·4278936	·001193317
839	703921	590589719	28·9654967	9·4316423	·001191895
840	705600	592704000	28·9827535	9·4353800	·001190476
841	707281	594823321	29·0000000	9·4391307	·001189061
842	708964	596947688	29·0172363	9·4428704	·001187648
843	710649	599077107	29·0344623	9·4466072	·001186240
844	712336	601211584	29·0516781	9·4503410	·001184834
845	714025	603351125	29·0688837	9·4540719	·001183432
846	715716	605495736	29·0860791	9·4577999	·001182033
847	717409	607645423	29·1032644	9·4615249	·001180638
848	719104	609800192	29·1204396	9·4652470	·001179245
849	720801	611960049	29·1376046	9·4689661	·001177856
850	722500	614125000	29·1547595	9·4726824	·001176471
851	724201	616295051	29·1719043	9·4763957	·001175088
852	725904	618470208	29·1890390	9·4801061	·001173709
853	727609	620650477	29·2061637	9·4838136	·001172333
854	729316	622835864	29·2232784	9·4875182	·001170960
855	731025	625026375	29·2403830	9·4912200	·001169591
856	732736	627222016	29·2574777	9·4949188	·001168224
857	734449	629422793	29·2745623	9·4986147	·001166861
858	736164	631628712	29·2916370	9·5023078	·001165501
859	737881	633839779	29·3087018	9·5059980	·001164144
860	739600	636056000	29·3257566	9·5096854	·001162791
861	741321	638277381	29·3428015	9·5133699	·001161440
862	743044	640503928	29·3598365	9·5170515	·001160093
863	744769	642735647	29·3768616	9·5207303	·001158749
864	746496	644972544	29·3938769	9·5244063	·001157407
865	748225	647214625	29·4108823	9·5280794	·001156069
866	749956	649461896	29·4278779	9·5317497	·001154734
867	751689	651714363	29·4448637	9·5354172	·001153403
868	753424	653972032	29·4618397	9·5390818	·001152074
869	755161	656234909	29·4788059	9·5427437	·001150748
870	756900	658503000	29·4957624	9·5464027	·001149425
871	758641	660776311	29·5127091	9·5500589	·001148106
872	760384	663054848	29·5296461	9·5537123	·001146789
873	762129	665338617	29·5465734	9·5573630	·001145475
874	763876	667627624	29·5634910	9·5610108	·001144165
875	765625	669921875	29·5803989	9·5646559	·001142857
876	767376	672221376	29·5972972	9·5682782	·001141553
877	769129	674526133	29·6141858	9·5719377	·001140251
878	770884	676836152	29·6310648	9·5755745	·001138952
879	772641	679151439	29·6479342	9·5792085	·001137656
880	774400	681472000	29·6647939	9·5828397	·001136364
881	776161	683797841	29·6816442	9·5864682	·001135074
882	777924	686128968	29·6984848	9·5900937	·001133787
883	779689	688465387	29·7153159	9·5937169	·001132503
884	781456	690807104	29·7321375	9·5973373	·001131222

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
885	783225	693154125	29·7489496	9·6009548	·001129944
886	784996	695506456	29·7657521	9·6045696	·001128668
887	786769	697864103	29·7825452	9·6081817	·001127396
888	788544	700227072	29·7993289	9·6117911	·001126126
889	790321	702595369	29·8161030	9·6153977	·001124859
890	792100	704969000	29·8328678	9·6190017	·001123596
891	793881	707347971	29·8496231	9·6226030	·001122334
892	795664	707932288	29·8663690	9·6262016	·001121076
893	797449	712121957	29·8831056	9·6297975	·001119821
894	799236	714516984	29·8998328	9·6333907	·001118568
895	801025	716917375	29·9165506	9·6369812	·001117818
896	802816	719323136	29·9332591	9·6405690	·001116071
897	804609	721734273	29·9499583	9·6441542	·001114827
898	806404	724150792	29·9666481	9·6477367	·001113586
899	808201	726572699	29·9833287	9·6513166	·001112347
900	810000	729000000	30·0000000	9·6548938	·001111111
901	811801	731432701	30·0166621	9·6584684	·001109878
902	813604	733870808	30·0333148	9·6620403	·001108647
903	815409	736314327	30·0499584	9·6656096	·001107420
904	817216	738763264	30·0665928	9·6691762	·001106195
905	819025	741217625	30·0832179	9·6727403	·001104972
906	820836	743677416	30·0998339	9·6763017	·001103753
907	822649	746142643	30·1164407	9·6798604	·001102536
908	824464	748613312	20·1330383	9·6834166	·001101322
909	826281	751089429	30·1496269	9·6869701	·001100110
910	828100	753571000	30·1662063	9·6905211	·001098901
911	829921	756058031	30·1827765	9·6940694	·001097695
912	831744	758550825	30·1993377	9·6976151	·001096491
913	833569	761048497	30·2158899	9·7011583	·001095290
914	835396	763551944	30·2324329	9·7046989	·001094092
915	837225	766060875	30·2489669	9·7082369	·001092896
916	839056	768575296	30·2654919	9·7117723	·001091703
917	840889	771095213	30·2820079	9·7153051	·001090513
918	842724	773620632	30·2985148	9·7188354	·001089325
919	844561	776151559	30·3150128	9·7223631	·001088139
920	846400	778688000	30·3315018	9·7258883	·001086957
921	848241	781229961	30·3479818	9.7294109	·001085776
922	850084	783777448	30·3644529	9·7329309	·001084599
923	851929	786330467	30·3809151	9·7364484	·001083423
924	853776	788889024	30·3973683	9·7399634	·001082251
925	855625	791453125	30·4138127	9·7434758	·001081081
926	857476	794022776	30·4302481	9·7469857	·001079914
927	859329	796597983	30·4466747	9·7504930	·001078749
928	861184	799178752	30·4630924	9·7539979	·001077586
929	863041	801765089	30·4795013	9·7575002	·001076426
930	864900	804357000	30·4959014	9·7610001	·001075269
931	866761	806954491	30·5122926	9·7644974	·001074114
932	868624	809557568	30·5286750	9·7679922	·001072961
933	870489	812166237	30·5450487	9·7714845	·001071811
934	872356	814780504	30·5614136	9·7749743	·001070664
935	874225	817400375	30·5777697	9·7784616	·001069519
936	876096	820025856	30·5941171	9·7829466	·001068376

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
937	877969	822656953	30·6104557	9·7854288	·001067236
938	879844	825293672	30·6267857	9·7889087	·001066098
939	881721	827936019	30·6431069	9·7923861	·001064963
940	883600	830584000	30·6594194	9·7958611	·001063830
941	885481	833237621	30·6757233	9·7993336	·001062699
942	887364	835896888	30·6920185	9·8028036	·001061571
943	889249	838561807	30·7083051	9·8062711	·001060445
944	891136	841232384	30·7245830	9·8097362	·001059322
945	893025	843908625	30·7408523	9·8131989	·001058201
946	894916	846590536	30·7571130	9·8166591	·001057082
947	896808	849278123	30·7733651	9·8201169	·001055966
948	898704	851971392	30·7896086	9·8235723	·001054852
949	900601	854670349	30·8058436	9·8270252	·001053741
950	902500	857375000	30·8220700	9·8304757	·001052632
951	904401	860085351	30·8382879	9·8339238	·001051525
952	906304	862801408	30·8544972	9·8373695	·001050420
953	908209	865523177	30·8706981	9·8408127	·001049318
954	910116	868250664	30·8868904	9·8442536	·001048218
955	912025	870983875	30·9030743	9·8476920	·001047120
956	913936	873722816	30·9192477	9·8511280	·001046025
957	915849	876467493	30·9354166	9·8545617	·001044932
958	917764	879217912	30·9515751	9·8579929	·001043841
959	919681	881974079	30·9677251	9·8614218	·001042753
960	921600	884736000	30·9838668	9·8648483	·001041667
961	923521	887503681	31·0000000	9·8682724	·001040583
962	925444	890277128	31·0161248	9·8716941	·001039501
963	927369	893056347	31·0322413	9·8751135	·001038422
964	929296	895841344	31·0483494	9·8785305	·001037344
965	931225	898632125	31·0644491	9·8819451	·001036269
966	933156	901428696	31·0805405	9·8853574	·001035197
967	935089	904231063	31·0966236	9·8887673	·001034126
968	937024	907039232	31·1126984	9·8921749	·001033058
969	938961	909853209	31·1287648	9·8955801	·001031992
970	940900	912673000	31·1448230	9·8989830	·001030928
971	942841	915498611	31·1608729	9·9023835	·001029866
972	944784	918330048	31·1769145	9·9057817	·001028807
973	946729	921167317	31·1929479	9·9091776	·001027749
974	948676	924010424	31·2089731	9·9125712	·001026694
975	950625	926859375	31·2249900	9·9159624	·001025641
976	952576	929714176	31·2409987	9·9193513	·001024590
977	954529	932574833	31·2569992	9·9227379	·001023541
978	956484	935441352	31·2729915	9·9261222	·001022495
979	958441	938313739	31·2889757	9·9295042	·001021450
980	960400	941192000	31·3049517	9·9328839	·001020408
981	962361	944076141	31·3209195	9·9362613	·001019168
982	964324	946966168	31·3368792	9·9396363	·001018330
993	966289	949862087	31·3528308	9·9430092	·001017294
984	968256	952763904	31·3687743	9·9463797	·001016260
985	970225	955671625	31·3847097	9·9497479	·001015228
986	972196	958585256	31·4006369	9·9531138	·001014199
987	974169	961504803	31·4165561	9·9564775	·001013171
988	976144	964430272	31·4324673	9·9598389	·001012146

Number.	Squares.	Cubes.	$\sqrt{\text{Roots.}}$	$\sqrt[3]{\text{Roots.}}$	Reciprocals.
989	978121	967361669	31·4483704	9·9631981	·001011122
990	980100	970299000	31·4642654	9·9665549	·001010101
991	982081	973242271	31·4801525	9·9699055	·001009082
992	984064	976191488	31·4960315	9·9732619	·001008065
993	986049	979146657	31·5119025	9·9766120	·001007049
994	988036	982107784	31·5277655	9·9799599	·001006036
995	990025	985074875	31·5436206	9·9833055	·001005025
996	992016	988047936	31·5594677	9·9866488	·001004016
997	994009	991026973	31·5753068	9·9899900	·001003009
998	996004	994011992	31·5911380	9·9933289	·001002004
999	998001	997002999	31·6069613	9·9966656	·001001001
1000	1000000	1000000000	31·6227766	10·0000000	·001000000
1001	1000201	1003003001	31·6385840	10·0033222	·0009990010
1002	1004004	1006012008	31·6543866	10·0066622	·0009980040
1003	1006009	1009027027	31·6701752	10·0099899	·0009970090
1004	1008016	1012048064	31·6859590	10·0133155	·0009960159
1005	1010025	1015075125	31·7017349	10·0166389	·0009950249
1006	1010036	1018108216	31·7175030	10·0199601	·0009940358
1007	1014049	1021147343	31·7332633	10·0232791	·0009930487
1008	1016064	1024192512	31·7490157	10·0265958	·0009920635
1009	1018081	1027243729	31·7647603	10·0299104	·0009910803
1010	1020100	1030301000	31·7804972	10·0332228	·0009900990
1011	1020121	1033364331	31·7962262	10·0365330	·0009891197
1012	1024144	1036433728	31·8119474	10·0398410	·0009881423
1013	1026169	1039509197	31·8276609	10·0431469	·0009871668
1014	1028196	1042590744	31·8433666	10·0464506	·0009861933
1015	1030225	1045678375	31·8590646	10·0497521	·0009852217
1016	1032256	1048772096	31·8747549	10·0530514	·0009842520
1017	1034289	1051871913	31·8904374	10·0563485	·0009832842
1018	1036324	1054977832	31·9061123	10·0596435	·0009823183
1019	1038361	1058089859	31·9217794	10·0629364	·0009813543
1020	1040400	1061208000	31·9374388	10·0662271	·0009803922
1021	1042441	1064332261	31·9530906	10·0695156	·0009794319
1022	1044484	1067462648	31·9687347	10·0728020	·0009784736
1023	1046529	1070599167	31·9843712	10·0760863	·0009775171
1024	1048576	1073741824	32·0000000	10·0793684	·0009765625
1025	1050625	1076890625	32·0156212	10·0826484	·0009756098
1026	1052676	1080045576	32·0312348	10·0859262	·0009746589
1027	1054729	1083206683	32·0468407	10·0892019	·0009737098
1028	1056784	1086373952	32·0624391	10·0924755	·0009727626
1029	1058841	1089547389	32·0780298	10·0957469	·0009718173
1030	1060900	1092727000	32·0936131	10·0990163	·0009708738
1031	1062961	1095912791	32·1091887	10·1022835	·0009699321
1032	1065024	1099104768	32·1247568	10·1055487	·0009689922
1033	1067089	1102302937	32·1403173	10·1088117	·0009680542
1034	1069156	1105507304	32·1558704	10·1120726	·0009671180
1035	1071225	1108717875	32·1714159	10·1153314	·0009661836
1036	1073296	1111934656	32·1869539	10·1185882	·0009652510
1037	1075369	1115157653	32·2024844	10·1218428	·0009643202
1038	1077444	1118386872	32·2180074	10·1250953	·0009633911
1039	1079521	1121622319	32·2335229	10·1283457	·0009624639
1040	1081600	1124864000	32·2490310	10·1315941	·0009615385

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
1041	1083681	1128111921	32·2645316	10·1348403	·0009606148
1042	1085764	1131366088	32·2800248	10·1380845	·0009596929
1043	1087849	1134626507	32·2955105	10·1413266	·0009587738
1044	1089936	1137893184	32·3109888	10·1445667	·0009578544
1045	1092025	1141166125	32·3264598	10·1478047	·0009569378
1046	1094116	1144445336	32·3419233	10·1510406	·0009560229
1047	1096209	1147730823	32·3573794	10·1542744	·0009551098
1048	1098304	1151022592	32·3728281	10·1575062	·0009541985
1049	1100401	1154320649	32·3882695	10·1607359	·0009532888
1050	1102500	1157625000	32·4037035	10·1639636	·0009523810
1051	1104601	1160935651	32·4191301	10·1671893	·0009514748
1052	1106704	1164252608	32·4345495	10·1704129	·0009505703
1053	1108809	1167575877	32·4499615	10·1736344	·0009496676
1054	1110916	1170905464	32·4653662	10·1768539	·9009487666
1055	1113125	1174241375	32·4807635	10·1800714	·0009478673
1056	1115136	1177583616	32·4961536	10·1832868	·0009469697
1057	1117249	1180932193	32·5115364	10·1865002	·0009460738
1058	1119364	1184287112	32·5269119	10·1897116	·0009451796
1059	1121481	1187648379	32·5422802	10·1929209	·0009442871
1060	1123600	1191016000	32·5576412	10·1961283	·0009433962
1061	1125721	1194389981	32·5729949	10·1993336	·0009425071
1062	1127844	1197770328	32·5883415	10·2025369	·0009416196
1063	1129969	1201157047	32·6035807	10·2057382	·0009407338
1064	1132096	1204550144	32·6190129	10·2089375	·0009398496
1065	1134225	1207949625	32·6343377	10·2121347	·0009389671
1066	1136356	1211355496	32·6496554	10·2153300	·0009380863
1067	1138489	1214767763	32·6649659	10·2185233	·0009372071
1068	1140624	1218186432	32·6802693	10·2217146	·0009363296
1069	1142761	1221611509	32·6955654	10·2249039	·0009354537
1070	1144900	1225043000	32·7108544	10·2280912	·0009345794
1071	1147041	1228480911	32·7261363	10·2312766	·0009337068
1072	1149184	1231925248	32·7414111	10·2344599	·0009328358
1073	1151329	1235376017	32·7566787	10·2376413	·0009319664
1074	1153476	1238833224	32·7719392	10·2408207	·0009310987
1075	1155625	1242296875	32·7871926	10·2439981	·0009302326
1076	1157776	1245766976	32·8024398	10·2471735	·0009293680
1077	1159929	1249243533	32·8176782	10·2503470	·0009285051
1078	1162084	1252726552	32·8329103	10·2535186	·0009276438
1079	1164241	1256216039	32·8481354	10·2566881	·0009267841
1080	1166400	1259712000	32·8633535	10·2598557	·0009259259
1081	1168561	1263214441	32·8785644	10·2630213	·0009250694
1082	1170724	1266723368	32·8937684	10·2661850	·0009242144
1083	1172889	1270238787	32·9089653	10·2693467	·0009233610
1084	1175056	1273760704	32·9241553	10·2725065	·0009225092
1085	1177225	1277289125	32·9393382	10·2756644	·0009216590
1086	1179396	1280824056	32·9545141	10·2788203	·0009208103
1087	1181569	1284365503	32·9696830	10·2819743	·0009199632
1088	1183744	1287913472	32·9848450	10·2851264	·0009191176
1089	1185921	1291467969	33·0000000	10·2882765	·0009182736
1090	1188100	1295029000	33·0151480	10·2914247	·0009174312
1091	1190281	1298596571	33·0302891	10·2945709	·0009165903
1092	1192464	1302170688	33·0454233	10·2977153	·0009157509

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
1093	1194649	1305751357	33·0605505	10·3008577	·0009149131
1094	1196836	1309338584	33·0756708	10·3039982	·0009140768
1095	1199025	1312932375	33·0907842	10·3071368	·0009132420
1096	1201216	1316532736	33·1058907	10·3102735	·0009124008
1097	1203409	1320139673	33·1209903	10·3134083	·0009115770
1098	1205604	1323753192	33·1360830	10·3165411	·0009107468
1099	1207801	1327373299	33·1511689	10·3196721	·0009099181
1100	1210000	1331000000	33·1662479	10·3228012	·0009090909
1101	1212201	1334633301	33·1813200	10·3259284	·0009082652
1102	1214404	1338273208	33·1963853	10·3290537	·0009074410
1103	1216609	1341919727	33·2114438	10·3321770	·0009066183
1104	1218816	1345572864	33·2266955	10·3352985	·0009057971
1105	1221025	1349232625	33·2415403	10·3384181	·0009049774
1106	1223236	1352899016	33·2565783	10·3415358	·0009041591
1107	1225449	1356572043	33·2716095	10·3446517	·0009033424
1108	1227664	1360251712	33·2866339	10·3477657	·0009025271
1109	1229881	1363938029	33·3016516	10·3508778	·0009017133
1110	1232100	1367631000	33·3166625	10·3539880	·0009009009
1111	1234321	1371330631	33·3316666	10·3570964	·0009000900
1112	1236544	1375036928	33·3466640	10·3602029	·0008992806
1113	1238769	1378749897	33·3616546	10·3633076	·0008984726
1114	1240996	1382469544	33·3766385	10·3664103	·0008976661
1115	1243225	1386195875	33·3916157	10·3695113	·0008968610
1116	1245456	1389928896	33·4065862	10·3726103	·0008960753
1117	1247689	1393668613	33·4215499	10·3757076	·0008952551
1118	1249924	1397415032	33·4365070	10·3788030	·0008944544
1119	1252161	1401168159	33·4514573	10·3818965	·0008936550
1120	1254400	1404928000	33·4664011	10·3849882	·0008928571
1121	1256641	1408694561	33·4813381	10·3880781	·0008960607
1122	1258884	1412467848	33·4962684	10·3911661	·0008912656
1123	1261129	1416247867	33·5111921	10·3942527	·0008904720
1124	1263376	1420034624	33·5261092	10·3973366	·0008896797
1125	1265625	1423828125	33·5410196	10·4004192	·0008888889
1126	1267876	1427628376	33·5559234	10·4034999	·0008880995
1127	1270129	1431435383	33·5708206	10·4065787	·0008873114
1128	1272384	1435249152	33·5857112	10·4096557	·0008865248
1129	1274641	1439069689	33·6005952	10·4127310	·0008857396
1130	1276900	1442897000	33·6154726	10·4158044	·0008849558
1131	1279161	1446731091	33·6303434	10·4188760	·0008841733
1132	1281424	1450571968	33·6452077	10·4219458	·0008833922
1133	1283689	1454419637	33·6600653	10·4250138	·0008826125
1134	1285956	1458274104	33·6749165	10·4280800	·0008818342
1135	1288225	1462135375	33·6897610	10·4311443	·0008810573
1136	1290496	1466003456	83·7045991	10·4342069	·0008802817
1137	1292769	1469878353	33·7174306	10·4372677	·0008795075
1138	1295044	1473760072	33·7340556	10·4403677	·0008787346
1139	1297321	1477648619	33·7490741	10·4433839	·0008779631
1140	1299600	1481544000	33·7638860	10·4464393	·0008771930
1141	1301881	1485446221	33·7786915	10·4494929	·0008764242
1142	1304164	1489355288	33·7934905	10·4525448	·0008756567
1143	1306449	1493271207	33·8082830	10·4555948	·0008748906
1144	1308736	1497193984	33·8230691	10·4586431	·0008741259

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
1145	1311025	1501123625	33·8378486	10·4616896	·0008733624
1146	1313316	1505060136	33·8526218	10·4647343	·0008726003
1147	1315609	1509003523	33·8673884	10·4677773	·0008718396
1148	1317904	1512953792	33·8821487	10·4708158	·0008710801
1149	1320201	1516910949	33·8969025	10·4738579	·0008703220
1150	1322500	1520875000	33·9116499	10·4768955	·0008695652
1151	1324801	1524845951	33·9263909	10·4799314	·0008688097
1152	1327104	1528823808	33·9411255	10·4829656	·0008680556
1153	1329409	1532808577	33·9558537	10·4859980	·0008673027
1154	1331716	1536800264	33·9705755	10·4890286	·0008665511
1155	1334025	1540798875	33·9852910	10·4920575	·0008658009
1156	1336336	1544804416	34·0000000	10·4950847	·0008650519
1157	1338649	1548816893	34·0147027	10·4981101	·0008643042
1158	1340964	1552836312	34·0293990	10·5011337	·0008635579
1159	1343281	1556862679	34·0440890	10·5041556	·0008628128
1160	1345600	1560896000	34·0587727	10·5071757	·0008620690
1161	1347921	1564936281	34·0734501	10·5101942	·0008613244
1162	1350244	1568983528	34·0881211	10·5132109	·0008605852
1163	1352569	1573037749	34·0127858	10·5162259	·0008598452
1164	1354896	1577098944	34·1174442	10·5192391	·0008591065
1165	1357225	1581167125	34·1320963	10·5222506	·0008583691
1166	1359556	1585242296	34·1467422	10·5252604	·0008576329
1167	1361889	1589324463	34·1613817	10·5282685	·0008568980
1168	1364224	1593413632	34·1760150	10·5312749	·0008561644
1169	1366561	1597509809	34·1906420	10·5342795	·0008554320
1170	1368900	1601613000	34·2052627	10·5372825	·0008547009
1171	1371241	1605723211	34·2198773	10·5402837	·0008539710
1172	1373584	1609840448	34·2344855	10·5432832	·0008532423
1173	1375929	1613964717	34·2490875	10·5462810	·0008525149
1174	1378276	1618096024	34·2636834	10·5492771	·0008517888
1175	1380625	1622234375	34·2782730	10·5522715	·0008510638
1176	1382976	1626379776	34·2928564	10·5552642	·0008503401
1177	1385329	1630532233	34·3074336	10·5582552	·0008496177
1178	1387684	1634691752	34·3220046	10·5612445	·0008488964
1179	1390041	1638858339	34·3365694	10·5642322	·0008481764
1180	1392400	1643032000	34·3511281	10·5672181	·0008471576
1181	1394761	1647212741	34·3656805	10·5702024	·0008467401
1182	1397124	1651400568	34·3802268	10·5731849	·0008460237
1183	1399489	1655595487	34·3947670	10·5761658	·0008453085
1184	1401856	1659797504	34·4093011	10·5791449	·0008445946
1185	1404225	1664006625	34·4238289	10·5821225	·0008438819
1186	1406596	1668222856	34·4383507	10·5850983	·0008431703
1187	1408969	1672446203	34·4528663	10·5880725	·0008424600
1188	1411344	1676676672	34·4673759	10·5910450	·0008417508
1189	1413721	1680914629	34·4818793	10·5940158	·0008410429
1190	1416100	1685159000	34·4963766	10·5969850	·0008403361
1191	1418481	1689410871	34·5108678	10·5999525	·0008396306
1192	1420864	1693669888	34·5253530	10·6029184	·0008389262
1193	1423249	1697936057	34·5398321	10·6058826	·0008382320
1194	1425636	1702209384	34·5543051	10·6088451	·0008375209
1195	1428025	1706489875	34·5687720	10·6118060	·0008368201
1196	1430416	1710777536	34·5832329	10·6147652	·0008361204

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
1197	1432809	1715072373	34·5976879	10·6177228	·0008354219
1198	1435204	1719374392	34·6121366	10·6206788	·0008347245
1199	1437601	1723683599	34·6265794	10·6236331	·0008340284
1200	1440000	1728000000	34·6410162	10·6265857	·0008333333
1201	1442401	1732323601	34·6554469	10·6295367	·0008326395
1202	1444804	1736654408	34·6698716	10·6324860	·0008319468
1203	1447209	1740992427	34·6842904	10·6354338	·0008312552
1204	1449616	1745337664	34·6987031	10·6383799	·0008305648
1205	1452025	1749690125	34·7131099	10·6413244	·0008298755
1206	1454436	1754049816	34·7275107	10·6442672	·0008291874
1207	1456849	1758416743	34·7419055	10·6472085	·0008285004
1208	1459264	1762790912	34·7562944	10·6501480	·0008278146
1209	1461681	1767172329	34·7706773	10·6530860	·0008271299
1210	1464100	1771561000	34·7850543	10·6560223	·0008264463
1211	1466521	1775956931	34·7994253	10·6589570	·0008257638
1212	1468944	1780360128	34·8137904	10·6618902	·0008250825
1213	1471369	1784770597	34·8281495	10·6648217	·0008244023
1214	1473796	1789188344	34·8425028	10·6677516	·0008237232
1215	1476225	1793613375	34·8568501	10·6706799	·0008230453
1216	1478656	1798045696	34·8711915	10·6736066	·0008223684
1217	1481089	1802485313	34·8855271	10·6765317	·0008216927
1218	1483524	1806932232	34·8998567	10·6794552	·0008210181
1219	1485961	1811386459	34·9141805	10·6823771	·0008203445
1220	1488400	1815848000	34·9284984	10·6852973	·0008196721
1221	1490841	1820316861	34·9428104	10·6882160	·0008190008
1222	1493284	1824793048	34·9571166	10·6911331	·0008183306
1223	1495729	1829276567	34·9714169	10·6940486	·0008176615
1224	1498176	1833764247	34·9857114	10·6969625	·0008169935
1225	1500625	1838265625	35·0000000	10·6998748	·0008163265
1226	1503276	1842771176	35·0142828	10·7027855	·0008156607
1227	1505529	1847284083	35·0285598	10·7056947	·0008149959
1228	1507984	1851804352	35·0428309	10·7086023	·0008143322
1229	1510441	1856331989	35·0570963	10·7115083	·0008136696
1230	1512900	1860867000	35·0713558	10·7144127	·0008130081
1231	1515361	1865409391	35·0856096	10·7173155	·0008123477
1232	1517824	1869959168	35·0998575	10·7202168	·0008116883
1233	1520289	1874516337	35·1140997	10·7231165	·0008110300
1234	1522756	1879080904	35·1283361	10·7260146	·0008103728
1235	1525225	1883652875	35·1425568	10·7289112	·0008097166
1236	1527696	1888232256	35·1567917	10·7318062	·0008090615
1237	1530169	1892819053	35·1710108	10·7346997	·0008084074
1238	1532644	1897413272	35·1852242	10·7375916	·0008077544
1239	1535121	1902014919	35·1994318	10·7404819	·0008071025
1240	1537600	1906624000	35·2136337	10·7433707	·0008064516
1241	1540081	1911240521	35·2278299	10·7462579	·0008058018
1242	1542564	1915864488	35·2420204	10·7491436	·0008051530
1243	1545049	1920495907	35·2562051	10·7520277	·0008045052
1244	1547536	1925134784	35·2703842	10·7549103	·0008038585
1245	1550025	1929781125	35·2845575	10·7577913	·0008032129
1246	1552521	1934434936	35·2987252	10·7606708	·0008025682
1247	1555009	1939096223	35·3128872	10·7635488	·0008019246
1248	1557504	1943764992	35·3270435	10·7664252	·0008012821

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
1249	1560001	1948441249	35·3411941	10·7693001	·0008006405
1250	1562500	1953125000	35·3553391	10·7721735	·0008000000
1251	1565001	1957816251	35·3694784	10·7750453	·0007993605
1252	1567504	1962515008	35·3836120	10·7779156	·0007987220
1253	1570009	1967221277	35·3977400	10·7807843	·0007980846
1254	1572516	1971935064	35·4118624	10·7836516	·0007974482
1255	1575025	1976656375	35·4259792	10·7865173	·0007968127
1256	1577536	1981385216	35·4400903	10·7893815	·0007961783
1257	1580049	1986121593	35·4541958	10·7922441	·0007955449
1258	1582564	1990865512	35·4682957	10·7951053	·0007949126
1259	1585081	1995616979	35·4823900	10·7979649	·0007942812
1260	1587600	2000376000	35·4964787	10·8008230	·0007936508
1261	1590121	2005142581	35·5105618	10·8036797	·0007930214
1262	1592644	2009916728	35·5246393	10·8065348	·0007923930
1263	1595166	2014698447	35·5387113	10·8093884	·0007917656
1264	1597696	2019487744	35·5527777	10·8122404	·0007911392
1265	1600225	2024284625	35·5668385	10·8150909	·0007905138
1266	1602756	2029089096	35·5808937	10·8179400	·0007898894
1267	1605289	2033901163	35·5949434	10·8207876	·0007892660
1268	1607824	2038720832	35·6089876	10·8236336	·0007886435
1269	1610361	2043548109	35·6230262	10·8264782	·0007880221
1270	1612900	2048383000	35·6370593	10·8293213	·0007874016
1271	1615441	2053225511	35·6510869	10·8321629	·0007867821
1272	1617984	2058075648	35·6651090	10·8350030	·0007861635
1273	1620529	2062933417	35·6791255	10·8378416	·0007855460
1274	1623076	2067798824	35·6931366	10·8406788	·0007849294
1275	1625625	2072671875	35·7071421	10·8435144	·0007843137
1276	1628176	2077552576	35·7211422	10·8463485	·0007836991
1277	1630729	2082440933	35·7351367	10·8491812	·0007830854
1278	1633284	2087336952	35·7491258	10·8520125	·0007824726
1279	1635841	2092240639	35·7631095	10·8548422	·0007818608
1280	1638400	2097152000	35·7770876	10·8576704	·0007812500
1281	1640961	2102071841	35·7910603	10·8604972	·0007806401
1282	1643524	2106997768	35·8050276	10·8633225	·0007800312
1283	1646089	2111932187	35·8189894	10·8661454	·0007794232
1284	1648656	2116874304	35·8329457	10·8689687	·0007788162
1285	1651225	2121824125	35·8468966	10·8717897	·0007782101
1286	1653796	2126781656	35·8608421	10·8746091	·0007776050
1287	1656369	2131746903	35·8747822	10·8774271	·0007770008
1288	1658944	2136719872	35·8887169	10·8802436	·0007763975
1289	1661521	2141700569	35·9026461	10·8830587	·0007757952
1290	1664100	2146689000	35·9165699	10·8858723	·0007751938
1291	1666681	2151685171	35·9304884	10·8886845	·0007745933
1292	1669264	2156689088	35·9444015	10·8914952	·0007739938
1293	1671849	2161700757	35·9583092	10·8943044	·0007733952
1294	1674436	2166720184	35·9722115	10·8971123	·0007727975
1295	1677025	2171747375	35·9861084	10·8999186	·0007722008
1296	1679616	2176782336	36·0000000	10·9027235	·0007716049
1297	1682209	2181825073	36·0138862	10·9055269	·0007710100
1298	1684804	2186875592	36·0277671	10·9083290	·0007704160
1299	1687401	2191933899	36·0416426	10·9111296	·0007698229
1300	1690000	2197000000	36·0555128	10·9139287	·0007692308

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
1301	1692601	2202073901	36·0693776	10·9167265	·0007686395
1302	1695204	2207155608	36·0832371	10·9195228	·0007680492
1303	1697809	2212245127	36·0970913	10·9223177	·0007674579
1304	1700416	2217342464	36·1109402	10·9251111	·0007668712
1305	1703025	2222447625	36·1247837	10·9279031	·0007662835
1306	1705636	2227560616	36·1386220	10·9306937	·0007656968
1307	1708249	2232681443	36·1524550	10·9334829	·0007651109
1308	1710864	2237810112	36·1662826	10·9362706	·0007645260
1309	1713481	2242946629	36·1801050	10·9390569	·0007639419
1310	1716100	2248091000	36·1939221	10·9418418	·0007633588
1311	1718721	2253243231	36·2077340	10·9446253	·0007627765
1312	1721344	2258403328	36·2215406	10·9475074	·0007621951
1313	1723969	2263571297	36·2353419	10·9501880	·0007616446
1314	1726596	2268747144	36·2491379	10·9529673	·0007610350
1315	1729225	2273930875	36·2626287	10·9557451	·0007604563
1316	1731856	2279122496	36·2767143	10·9585215	·0007598784
1317	1734489	2284322013	36·2904246	10·9612965	·0007593014
1318	1737124	2289529432	36·3042697	10·9640701	·0007587253
1319	1739761	2294744759	36·3180396	10·9668423	·0007581501
1320	1742400	2299968000	36·3318042	10·9696131	·0007575758
1321	1745041	2305199161	36·3455637	10·9723825	·0007570023
1322	1747684	2310438248	36·3593179	10·9751505	·0007564297
1323	1750329	2315685267	36·3730670	10·9779171	·0007558579
1324	1752976	2320940224	36·3868108	10·9806823	·0007552870
1325	1755625	2326203125	36·4005494	10·9834462	·0007547170
1326	1758276	2331473976	36·4142829	10·9862086	·0007541478
1327	1760929	2336752783	36·4280112	10·9889696	·0007535795
1328	1763584	2342039552	36·4417343	10·9917293	·0007530120
1329	1766241	2347334289	36·4554523	10·9944876	·0007524454
1330	1768900	2352637000	36·4691650	10·9972445	·0007518797
1331	1771561	2357947691	36·4828727	11·0000000	·0007513148
1332	1774224	2363266368	36·4965752	11·0027541	·0007507508
1333	1776889	2368593037	36·5102725	11·0055069	·0007501875
1334	1779556	2373927704	36·5239647	11·0082583	·0007496252
1335	1782225	2379270375	36·5376518	11·0110082	·0007490637
1336	1784896	2384621056	36·5513388	11·0137569	·0007485030
1337	1787569	2389979753	36·5650106	11·0165041	·0007479432
1338	1790244	2395346472	36·5786823	11·0192500	·0007473842
1339	1792921	2400721219	36·5923489	11·0219945	·0007468260
1340	1795600	2406104000	36·6060104	11·0247377	·0007462687
1341	1798281	2411494821	36·6196668	11·0274795	·0007457122
1342	1800964	2416893688	36·6333181	11·0302199	·0007451565
1343	1803649	2422300607	36·6469144	11·0329590	·0007446016
1344	1806336	2427715584	36·6606056	11·0356967	·0007440476
1345	1809025	2433138625	36·6742416	11·0384330	·0007434944
1346	1811716	2438569736	36·6878726	11·0411680	·0007429421
1347	1814409	2444008923	36·7014986	11·0439017	·0007423905
1348	1817104	2449456192	36·7151195	11·0466339	·0007418398
1349	1819801	2454911549	36·7287353	11·0493649	·0007412898
1350	1822500	2460375000	36·7423461	11·0520945	·0007407407
1351	1825201	2465846551	36·7559519	11·0548227	·0007401924
1352	1827904	2471326208	36·7695526	11·0575497	·0007396450

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
1353	1830609	2476813977	36·7831483	11·0602752	·0007390983
1354	1833316	2482309864	36·7967390	11·0629994	·0007385524
1355	1836025	2487813875	36·8103246	11·0657222	·0007380074
1356	1838736	2493326016	36·8239053	11·0684437	·0007374631
1357	1841449	2498846293	36·8374809	11·0711639	·0007369197
1358	1844164	2504374712	36·8510515	11·0738828	·0007363770
1359	1846881	2509911279	36·8646172	11·0766003	·0007358352
1360	1849600	2515456000	36·8781778	11·0793165	·0007352941
1361	1852321	2521008881	36·8917335	11·0820314	·0007347539
1362	1855044	2526569928	36·9052842	11·0847449	·0007342144
1363	1857769	2532139147	36·9188299	11·0874571	·0007336757
1364	1860496	2537716544	36·9323706	11·0901679	·0007331378
1365	1863225	2543302125	36·9459064	11·0928775	·0007326007
1366	1865956	2548895896	36·9594372	11·0955857	·0007320644
1367	1868689	2554497863	36·9729631	11·0982926	·0007315289
1368	1871424	2560108032	36·9864840	11·1009982	·0007309942
1369	1874161	2565726409	37·0000000	11·1037025	·0007304602
1370	1876900	2571353000	37·0135110	11·1064054	·0007299270
1371	1879641	2576987811	37·0270172	11·1091070	·0007293946
1372	1882384	2582630848	37·0405184	11·1118073	·0007288630
1373	1885129	2588282117	37·0540146	11·1145064	·0007283321
1374	1887876	2593941624	37·0675060	11·1172041	·0007278020
1375	1890625	2599609375	37·0899924	11·1199004	·0007272727
1376	1893376	2605285376	37·0944740	11·1225955	·0007267442
1377	1896129	2610969633	37·1079506	11·1252893	·0007262164
1378	1898884	2616662152	37·1214224	11·1279817	·0007256894
1379	1901641	2622362939	37·1348893	11·1306729	·0007251632
1380	1904400	2628072000	37·1483512	11·1333628	·0007246377
1381	1907161	2633789341	37·1618084	11·1360514	·0007241130
1382	1909924	2639514968	37·1752606	11·1387386	·0007235890
1383	1912689	2645248887	37·1887079	11·1414246	·0007230658
1384	1915456	2650991104	37·2021505	11·1441093	·0007225434
1385	1918225	2656741625	37·2155881	11·1467926	·0007220217
1386	1920996	2662500456	37·2290209	11·1494747	·0007215007
1387	1923769	2668267603	37·2424489	11·1521555	·0007209805
1388	1926544	2674043072	37·2558720	11·1548350	·0007204611
1389	1929321	2679826869	37·2692903	11·1575133	·0007199424
1390	1932100	2685619000	37·2827037	11·1601903	·0007194245
1391	1934881	2691419471	37·2961124	11·1628659	·0007189073
1392	1937664	2697228288	37·3095162	11·1655403	·0007183908
1393	1940449	2703045457	37·3229152	11·1682134	·0007178751
1394	1943236	2708870984	37·3363094	11·1708852	·0007173601
1395	1946025	2714704875	37·3496988	11·1735558	·0007168459
1396	1948816	2720547136	37·3630834	11·1762250	·0007163324
1397	1951609	2726397773	37·3764632	11·1788930	·0007158196
1398	1954404	2732256792	37·3898382	11·1815598	·0007153076
1399	1957201	2738124199	37·4032084	11·1842252	·0007147963
1400	1960000	2744000000	37·4165738	11·1868894	·0007142857
1401	1962801	2749884201	37·4299345	11·1895523	·0007137759
1402	1965604	2755776808	37·4432904	11·1922139	·0007132668
1403	1968409	2761677827	37·4566416	11·1948743	·0007127584
1404	1971216	2767587264	37·4699880	11·1975334	·0007122507

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
1405	1974025	2773505123	37·4833296	11·2001913	·0007117438
1406	1976836	2779431416	37·4966665	11·2028479	·0007112376
1407	1979649	2785366143	37·5099987	11·2055032	·0007107321
1408	1982464	2791309312	37·5233261	11·2081573	·0007102273
1409	1985281	2797260929	37·5366487	11·2108101	·0007097232
1410	1988100	2803221000	37·5499667	11·2134617	·0007092199
1411	1990921	2809189531	37·5632799	11·2161120	·0007087172
1412	1993744	2815166528	37·5765885	11·2187611	·0007082153
1413	1996569	2821151997	37·5898922	11·2214089	·0007077141
1414	1999396	2827145944	37·6031913	11·2240054	·0007072136
1415	2002225	2833148375	37·6164857	11·2267007	·0007067138
1416	2005056	2839159296	37·6297754	11·2293448	·0007062147
1417	2007889	2845178713	37·6430604	11·2319876	·0007057163
1418	2010724	2851206632	37·6563407	11·2346292	·0007052186
1419	2013561	2857243059	37·6696164	11·2372696	·0007047216
1420	2016400	2863288000	37·6828874	11·2399087	·0007042254
1421	2019241	2869341461	37·6961536	11·2425465	·0007037298
1422	2022084	2875403448	37·7094153	11·2451831	·0007032349
1423	2024929	2881473967	37·7226722	11·2478185	·0007027407
1424	2027776	2887553024	37·7359245	11·2504527	·0007022472
1425	2030625	2893640625	37·7491722	11·2530856	·0007017544
1426	2033476	2899736776	37·7624152	11·2557173	·0007012623
1427	2036329	2905841483	37·7756535	11·2583478	·0007007708
1428	2039184	2911954752	37·7888873	11·2609770	·0007002801
1429	2042041	2918076589	37·8021163	11·2636050	·0006997901
1430	2044900	2924207000	37·8153408	11·2662318	·0006993007
1431	2047761	2930345991	37·8285606	11·2688573	·0006988120
1432	2050624	2936493568	37·8417759	11·2714816	·0006983240
1433	2053489	2942649737	37·8549864	11·2741047	·0006978367
1434	2056356	2948814504	37·8681924	11·2767266	·0006973501
1435	2059225	2954987875	37·8813938	11·2793472	·0006968641
1436	2062096	2961169856	37·8945906	11·2819666	·0006963788
1437	2064969	2967360453	37·9077828	11·2845849	·0006958942
1438	2067844	2973559672	37·9209704	11·2872019	·0006954103
1439	2070721	2979767519	37·9341538	11·2898177	·0006949270
1440	2073600	2985984000	37·9473319	11·2924323	·0006944444
1441	2076481	2992209121	37·9605058	11·2950457	·0006939625
1442	2079364	3098442888	37·9736751	11·2976579	·0006934813
1443	2082249	3004685307	37·9868398	11·3002688	·0006930007
1444	2085136	3010936384	38·0000000	11·3028786	·0006925208
1445	2088025	3017196125	38·0131556	11·3054871	·0006920415
1446	2080916	3023464536	38·0263067	11·3080945	·0006915629
1447	2093809	3029741623	38·0394532	11·3107006	·0006910850
1448	2096704	3036027392	38·0525952	11·3133056	·0006906078
1449	2099601	3042321849	38·0657326	11·3159094	·0006901312
1450	2102500	3048625000	38·0788655	11·3185119	·0006896552
1451	2105401	3054936851	38·0919939	11·3211132	·0006891799
1452	2108304	3061257408	38·1051178	11·3237134	·0006887052
1453	2111209	3067586777	38·1182371	11·3263124	·0006882312
1454	2114116	3073924664	38·1313519	11·3289102	·0006877579
1455	2117025	3080271375	38·1444622	11·3315067	·0006872852
1456	2119936	3086626816	38·1575681	11·3341022	·0006868132

Number.	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
1457	2122849	3092990993	38·1706693	11·3366964	·0006863412
1458	2125764	3099363912	38·1837662	11·3392894	·0006858711
1459	2128681	3105745579	38·1968585	11·3418813	·0006854010
1460	2131600	3112136000	38·2099463	11·3444719	·0006849315
1461	2134521	3118535181	38·2230297	11·3470614	·0006844627
1462	2137444	3124943128	38·2361085	11·3496497	·0006839945
1463	2140369	3131359847	38·2491829	11·3522368	·0006835270
1464	2143296	3137785344	38·2622529	11·3548227	·0006830601
1465	2146225	3144219625	38·2753184	11·3574075	·0006825939
1466	2149156	3150662696	38·2883794	11·3599911	·0006821282
1467	2152089	3157114563	38·3014360	11·3625735	·0006816633
1468	2155024	3163575232	38·3144881	11·3651547	·0006811989
1469	2157961	3170044709	38·3275358	11·3677347	·0006807352
1470	2160900	3176523000	38·3405790	11·3703136	·0006802721
1471	2163841	3183010111	38·3536178	11·3728914	·0006798097
1472	2166784	3189506048	38·3666522	11·3754679	·0006793478
1473	2169729	3196010817	38·3796821	11·3780433	·0006788866
1474	2172676	3202524424	38·3927076	11·3806175	·0006784261
1475	2175625	3209046875	38·4057287	11·3831906	·0006779661
1476	2178576	3215578176	38·4187454	11·3857625	·0006775068
1477	2181529	3222118333	38·4317577	11·3883332	·0006770481
1478	2184484	3228667352	38·4447656	11·3909028	·0006765900
1479	2187441	3235225239	38·4577691	11·3934712	·0006761325
1480	2190400	3241792000	38·4707681	11·3960384	·0006756757
1481	2193361	3248367641	38·4837627	11·3986045	·0006752194
1482	2196324	3254952168	38·4967530	11·4011695	·0006747638
1483	2199289	3261545587	38·5097390	11·4037332	·0006743088
1484	2202256	3268147904	38·5227206	11·4062959	·0006738544
1485	2205225	3274759125	38·5356977	11·4088574	·0006734007
1486	2208196	3281379256	38·5486705	11·4114177	·0006729474
1487	2211169	3288008303	38·5616389	11·4139769	·0006724950
1488	2214144	3294646272	38·5746030	11·4165349	·0006720430
1489	2217121	3301293169	38·5875627	11·4190918	·0006715917
1490	2220100	3307949000	38·6005181	11·4206476	·0006711409
1491	2223081	3314613771	38·6134691	11·4242022	·0006706908
1492	2226004	3321287488	38·6264158	11·4267556	·0006702413
1493	2229049	3327970157	38·6393582	11·4293079	·0006697924
1494	2232036	3334661784	38·6522962	11·4318591	·0006693440
1495	2235025	3341362375	38·6652299	11·4344092	·0006688963
1496	2238016	3348071936	38·6781593	11·4369581	·0006684492
1497	2241009	3354790473	38·6910843	11·4395059	·0006680027
1498	2244004	3361517992	38·7040050	11·4420525	·0006675567
1499	2247001	3368254499	38·7169214	11·4445980	·0006671114
1500	2250000	3375000000	38·7298335	11·4471424	·0006666667
1501	2253001	3381754501	38·7427412	11·4496857	·0006662225
1502	2256004	3388518008	38·7556447	11·4522278	·0006657790
1503	2259009	3395290527	38·7685439	11·4547688	·0006553360
1504	2262016	3402072064	38·7814389	11·4573087	·0006648936
1505	2265025	3408862625	38·7943294	11·4598476	·0006644518
1506	2268036	3415662216	38·8072158	11·4623850	·0006640106
1507	2271049	3422470843	38·8200978	11·4649215	·0006635700
1508	2274064	3429288512	38·8329757	11·4674568	·0006631300

Number.	Squares.	Cubes.	√ Roots.	∛ Roots.	Reciprocals.
1509	2277081	3436115229	38·8458491	11·4699911	·0006626905
1510	2280100	3442951000	38·8587184	11·4725242	·0006622517
1511	2283121	3449795831	38·8715834	11·4750562	·0006618134
1512	2286144	3456649728	38·8844442	11·4775871	·0006613757
1513	2289169	3463512697	38·8973006	11·4801169	·0006609385
1514	2292196	3470384744	38·9101529	11·4826455	·0006605020
1515	2295225	3477265875	38·9230009	11·4851731	·0006600660
1516	2298256	3484156096	38·9358447	11·4876995	·0006596306
1517	2301289	3491055413	33·9486841	11·4902249	·0006591958
1518	2304324	3597963832	38·9615194	11·4927491	·0006587615
1519	2307361	3504881359	38·9743505	11·4952722	·0006583278
1520	2310400	3511808000	38·9871774	11·4977942	·0006578947
1521	2313441	3518743761	39·0000000	11·5003151	·0006574622
1522	2316484	3525688648	39·0128184	11·5028348	·0006570302
1523	2319529	3532642667	39·0256326	11·5053535	·0006565988
1524	2322576	3539605824	39·0384426	11·5078711	·0006561680
1525	2325625	3546578125	39·0512483	11·5103876	·0006557377
1526	2328676	3553559576	39·0640499	11·5129030	·0006553080
1527	2331729	3567549552	39·0768473	11·5154173	·0006548788
1528	2334784	3560558183	39·0896406	11·5179305	·0006544503
1529	2337841	3574558889	39·1024296	11·5204425	·0006540222
1530	2340900	3581577000	39·1152144	11·5229535	·0006535948
1531	2343961	3588604291	39·1279951	11·5254634	·0006531679
1532	2347024	3595640768	39·1407716	11·5279722	·0006527415
1533	2350089	3602686437	39·1535439	11·5304799	·0006523157
1534	2353156	3609741304	39·1663120	11·5329865	·0006518905
1535	2356225	3616805375	39·1790760	11·5354920	·0006514658
1536	2359256	3623878656	39·1918359	11·5379965	·0006510417
1537	2362369	3630961153	39·2045915	11·5404998	·0006506181
1538	2365444	3638052872	39·2173431	11·5430021	·0006501951
1539	2368521	3645153819	39·2300905	11·5455033	·0006497726
1540	2371600	3652264000	39·2428337	11·5480034	·0006493506
1541	2374681	3657983421	39·2555728	11·5505025	·0006489293
1542	2377764	3666512088	39·2683078	11·5530004	·0006485084
1543	2380849	3673650007	39·2810387	11·5554972	·0006480881
1544	2383936	3680797184	39·2937654	11·5579931	·0006476684
1545	2387025	3687953625	39·3064880	11·5604878	·0006472492
1546	2390116	3695119336	39·3192065	11·5629815	·0006468305
1547	2393209	3702294323	39·3319208	11·5654740	·0006464124
1548	2396304	3709478592	39·3446311	11·5679655	·0006459948
1549	2399401	3716672149	39·3573373	11·5704559	·0006455778
1550	2402500	3723875000	39·3700394	11·5729453	·0006451613
1551	2405601	3731087151	39·3827373	11·5754336	·0006447453
1552	2408704	3738308608	39·3954312	11·5779208	·0006443299
1553	2411809	3745539377	39·4081210	11·5804069	·0006439150
1554	2414916	3752779464	39·4208067	11·5828919	·0006435006
1555	2418025	3760028875	39·4334883	11·5853759	·0006430868
1556	2421136	3767287616	39·4461658	11·5878588	·0006426735
1557	2424249	3774555693	39·4588393	11·5903407	·0006422608
1558	2427364	3781833112	39·4715087	11·5928215	·0006418485
1559	2430481	3789119879	39·4841740	11·5953013	·0006414368
1560	2433600	3796416000	39·4968353	11·5977799	·0006410256

Number	Squares.	Cubes.	$\sqrt{\ }$ Roots.	$\sqrt[3]{\ }$ Roots.	Reciprocals.
1561	2436721	3803721481	39·5094925	11 6002576	·0006406150
1562	2439844	3811036328	39·5221457	11·6027342	·0006402049
1563	2442969	3818360547	39·5347948	11·6052097	·0006397953
1564	2446096	3825641444	39·5474399	11·6076841	·0006393862
1565	2449225	3833037125	39·5600809	11·6101575	·0006389776
1566	2452356	3840389496	39·5727179	11·6126299	·0006385696
1567	2455489	3847751263	39·5853508	11·6151012	·0006381621
1568	2458624	3855123432	39·5979797	11·6175715	·0006377551
1569	2461761	3862503009	39·6106046	11·6200407	·0006373486
1570	2464900	3869883000	39·6232255	11·6225088	·0006369427
1571	2468041	3877292411	39·6358424	11·6249759	·0006365372
1572	2471184	3884701248	39·6484552	11·6274420	·0006361323
1573	2474329	3892119157	39·6610640	11·6299070	·0006357279
1574	2477476	3899547224	39·6736688	11·6323710	·0006353240
1575	2480625	3906984375	39·6862696	11·6348339	·0006349206
1576	2483776	3914430976	39·6988665	11·6372957	·0006345178
1577	2486929	3921887033	39·7114593	11·6397566	·0006341154
1578	2490084	3929352552	39·7240481	11·6422164	·0006337136
1579	2493241	3936827539	39·7366329	11·6446751	·0006333122
1580	2496400	3944312000	39·7492138	11·6471329	·0006329114
1581	2499561	3951805941	39·7617907	11·6495895	·0006325111
1582	2502724	3959309368	39·7743636	11·6520452	·0006321113
1583	2505889	3966822287	39·7869325	11·6544998	·0006317119
1584	2509056	3974344704	39·7994976	11·6569534	·0006313131
1585	2512225	3981876625	39·8120585	11·6594059	·0006309148
1586	2515396	3989418056	39·8246155	11·6618574	·0006305170
1587	2518569	3996969003	39·8371686	11·6643079	·0006301197
1588	2521744	4004529472	39·8497177	11·6667574	·0006297229
1589	2524921	4012099469	39·8622628	11·6692058	·0006293266
1590	2528100	4014679000	39·8748040	11·6716532	·0006289308
1591	2531281	4027268071	39·8873413	11·6740996	·0006285355
1592	2534464	4034866688	39·8998747	11·6765449	·0006281407
1593	2537649	4042474857	39·9124041	11·6789892	·0006277464
1594	2540836	4050092584	39·9249295	11·6814325	·0006273526
1595	2544025	4057719875	39·9374511	11·6838748	·0006269592
1596	2547216	4065356736	39·9499687	11·6863161	·0006265664
1597	2550409	4073003173	39·9624824	11·6887563	·0006261741
1598	2553604	4080659192	39·9749922	11·6911955	·0006257822
1599	2556801	4088324799	39·9874980	11·6936337	·0006253909
1600	2560000	4096000000	40·0000000	11·6960709	·0006250000

To find the Square Root of Numbers exceeding 1600,

Example 4. Require the Square Root of 34698. In the column of *Squares* you will find,

$$\begin{array}{rl} +34969 = 187^2, & +34969 = 187^2, \\ -34698 = 186^2, + \ldots & -34596 = 186^2. \\ \hline 271 \text{ divided by} & 373 = 000.727, \end{array}$$

$$\sqrt{34698} = 186{\cdot}727 \text{ nearly.}$$

When the number contains Integer and Decimals.

Example 5. Required the Square Root of 7845·45? In the column of *Squares* you will find,

+7849·96 = 88·6²,	+7849·96 = 88·6²,
—7845·45 = 88·5²···,	—7832·25 = 88·5²,
451 divided by	1771 = 00·0256.
	$\sqrt{7845{\cdot}45}$ = 88·5256 nearly.

☞ When the number of ciphers in the integer is *even*, the number of ciphers taken in the Square column must also be *even*; but when the number of ciphers in the integer is *odd*, the number taken in the Square column must also be *odd.*

To find the Cube Root of Numbers exceeding 1600.

Example 6. Required the Cube Root of 5694958? In the *Cube* column you will find,

+5735339 = 179³	+5735339 = 179³.
—5694958 = 178³···	—5639752 = 178³.
40381 divided by	95587 = 000·4225,
	$\sqrt[3]{5694958}$ = 178·4225 nearly.

When the number contains Integer and Decimals.

Example 7. Required the Cube Root of 4186·586? In the column of *Cubes* you will find,

+4251·528 = 16·2³	4251·528 = 16·2³
—4186·585 = 16·1³···	4173·281 = 16·1³
64942	78247 = 00·083
	$\sqrt[3]{4186{\cdot}586}$ = 16·183 nearly.

☞ The following notice must be particularly attended to, when extracting *Cube Root* of numbers with decimals.

2	ciphers in the integer	must be	5, 8, or 11	ciphers in the	Cube column.
3	"	"	"	3, 6, or 9	"	"
4	"	"	"	4, or 7	"	"
5	"	"	"	5, or 8	"	"
6	"	"	"	6, or 9	"	"
7	"	"	"	7, or 10	"	"

Example 8. Required the Cube Root of 61358·75? In the *Cube* column and 8 ciphers you will find,

+61629·875 = 395³	+61629875 = 39·5³
—61358·750 = 394³···	—61162984 = 39·4³
271·125 divided by	466891 = 00·05807
	$\sqrt[3]{61358{\cdot}75}$ = 39·45807.

To find the Fourth Root.

Rule. Extract the Square Root of the number as before described, and of that root extract the Square Root again, then the last is the Fourth root of the number.

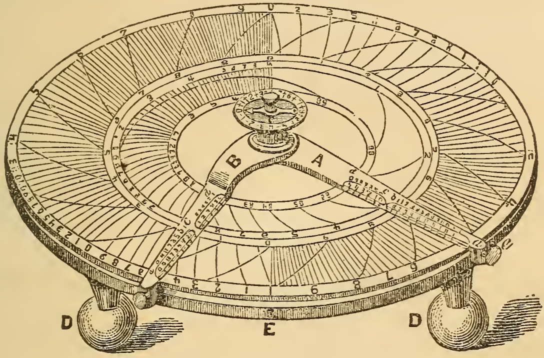
Example 9. Required the fourth root of 2469781?

$\sqrt[4]{2469781} = \sqrt{\sqrt{2469781}} = \sqrt{1571{\cdot}4463} = 39{\cdot}6467$, the answer.

To find the Sixth Root.

Rule. Find the Cube Root of the number as before described, and of that root extract the Cube Root again, and then the last is the Sixth root of the number.

NYSTROM'S CALCULATOR.



ALL calculations in this POCKET BOOK have been computed by this Instrument. It consists of a silvered brass plate on which are fixed two moveable arms, extending from the centre to the periphery. On the plate are engraved a number of curved lines in such form and divisions, that by their intersection with the arms, numbers are read and problems solved.

The arrangement for trigonometrical calculations is such that it is not necessary to notice *sine, cosine, tan,* &c., &c., operating only by the angles themselves expressed in degrees and minutes. This makes trigonometrical solutions so easy, that any one who understands Simple Arithmetic, will be able to solvo trigonometrical questions.

Calculations are performed by it almost instantly, no matter how complicated they may be, while there is nothing intricate or difficult in its use. The author of this book, who is the inventor, has thoroughly tested its practical utility. Without this instrument not one-tenth of the calculations and tables which he is continually bringing out, could be produced.

LOGARITHMS.

A Logarithm is an exponent of a power to which 10 must be raised to give a certain number, which will be understood by this

Number,.... Logarithm, Base,......... Exponent,...

$$\begin{array}{rlll} \log. 100 = & 2 & \text{because} & 10^2 = 100. \\ \log. 10000 = & 4 & \text{"} & 10^4 = 10000 \\ \log. 5012 = & 3{\cdot}7 & \text{"} & 10^{3.7} = 5012. \end{array}$$

The *unit* of the logarithm is called the *characteristic* or *index*, and the *decimal part* is called the *mantissa*, the sum of the *characteristic* and *mantissa* is the *Logarithm*. The invariable number 10 is the *base* for the system of Logarithms.

It is not necessary that the *base* should be 10, it can be any number, but all the tables of logarithms now in common use, are calculated with 10 to the base.

The nature of logarithms in connection with their numbers are such, that the *index* of the logarithm is always one less than the number of figures in the number, (when the base of the logarithm is 10,) as,

$$\begin{array}{r} \text{index } 5012 = 3 \\ \text{mantissa } 5012 = 0{\cdot}7 \\ \text{logarithm } 5012 = \overline{3{\cdot}7.} \end{array}$$

Let 10 be raised to any power x, and

the power of $10^x = a$ or $\log. a = x$,
the power of $10^z = b$ or $\log. b = z$.

Let the product of $ab = c$ and the quotient $\frac{a}{b} = c$.

$$10^x \times 10^z = 10^{x+z} = ab = c \qquad \text{or} \quad \log. c = x+z.$$

$$\frac{10^x}{10^z} = 10^{x-z} = \frac{a}{b} = d \qquad \text{or} \quad \log. d = x-z.$$

$$a = m^z \qquad \text{or } \log. m = z \times \log. a.$$

$$\sqrt[3]{a} = n \qquad \text{or } \log. n = \log. a : 3.$$

Any number represented by the letters a, b, c, or d, can be a power of 10, which exponent is the logarithm for the number. Logarithms are calculated for every number with three figures in the accompanying Table, by which any operation in Multiplication, Division, Involution and Evolution can be performed by simple Addition or Subtraction of Logarithms. Tables of Logarithms are commonly more extensive, and calculated for any number of four or five figures, which would occupy too much room in this book; but by the proportional parts, the logarithm can be found by this Table, to four or five figures. The *index* of the logarithms do not appear in the Table, only the *mantissa*. It is easily remembered that *the index is one less, than the number of figures in the number;* then when the number is only one figure, the index is 0; and when the number is a fraction, the index is negative.

When the logarithm is to be found for a fraction, we commonly have the fraction expressed in a decimal; and then the negative index is equal to the number of ciphers before the first figure, and commonly marked after the mantissa; thus explained in whole numbers and fractions:

$$\begin{array}{ll} \log. 365 = 2{\cdot}56229.. & \log. 0{\cdot}365 = {\cdot}56229-1. \\ \log. 46{\cdot}7 = 1{\cdot}66931 & \log. 0{\cdot}0467 = {\cdot}66931-2. \\ \log. 7{\cdot}59 = 0{\cdot}88024 & \log. 0{\cdot}00759 = {\cdot}88024-3. \end{array}$$

In the accompanying Table of Logarithms, for the trigonometrical lines the negative index is marked thus,

$$\log. \sin. 35^\circ 40' = \log. 0{\cdot}58306 = 1{:}76572.$$

To find the Logarithm of Numbers.

Example 1. Find the logarithm of 45.

To 45 in the first column of the Table answers 65321 in the next column, which is the mantissa; index = 1 because 45 is two figures.

Then, log. 45 = 1·65321, the answer.

Example 2. Find the logarithm of 768?

Opposite 76 in the first column, answers 88536 in the column marked 8 on the top or bottom. Index = 2 because 768 is three figures.

Then, log. 768 = 2·88536.

Example 3. Find the Logarithm of 6846?

	log. 6840 =	3·83505
Proportional part,	64×0·6 =	384
	log. 6846 =	3·835434 the answer.

To find the number for a given Logarithm.

Example 1. What number answers to the logarithm 3·87157?

In the Table you will find in the column of logarithms, that
log. 7440 = 3·87157.

Example 2. What number answers to the logarithm 3·801884?

Given logarithm	3·801884,	
Subt. nearest table log.	3·801400	= log. 6330,
Divided by proportional part,	69|484| - - - - - -	7,
		6337 the req. numb.

Multiplication by Logarithms.

Rule. Add together the logarithms of the factors, and the sum is the logarithm of the product.

Example 1. Multiply 425 by 48.

To	log. 425 =	2·62839,
Add	log. 48 =	1·68124,
The product,	log. 20400 =	4·30963.

Example 2. Multiply 79600 by 0·435.

To	log. 79600 =	4·90091,
Add,	log. 0·435 =	·63848—1,
The product	log. 34690 =	4·53939.

Division by Logarithms.

Rule. From the logarithm of the dividend subtract the logarithm of the divisor, and the difference is the logarithm of the quotient.

Example 1. Divide 43800 by 368.

From	log. 43800 =	4·64147,
Subtract	log. 368 =	2·56584,
The quotient	log. 119 =	2·07563.

Example 2. Divide 36 by 0.625.

From	log. 36 =	1·55636,
Subtract,	log. 0·625 =	·79588-1.
The quotient,	log. 57·6 =	1·76048.

A *negative index* follows an opposite operation of its mantissa, as if the mantissa is subtracted, add the negative index, and *vice versa*.

Envolution by Logarithms.

Rule. Multiply the logarithm of the number by its exponent, and the product is the logarithm of the power of the number.

Involution by Logarithms.

Rule. Divide the logarithm of the number by the index of the root, and the quotient is the logarithm of the root of the number.

LOGARITHM OF NUMBERS, FROM

0 TO 1000.

No.	0	1	2	3	4	5	6	7	8	9	Prop.
0	0	00000	30103	47712	60206	69897	77815	84510	90309	95424	
10	00000	00432	00860	01283	01703	02118	02530	02938	03342	03742	415
11	04139	04532	04921	05307	05690	06069	06445	06818	07188	07554	379
12	07918	08278	08636	08990	09342	09691	10037	10380	10721	11059	349
13	11394	11727	12057	12385	12710	13033	13353	13672	13987	14301	323
14	14612	14921	15228	15533	15836	16136	16435	16731	17026	17318	300
15	17609	17897	18184	18469	18752	19033	19312	19590	19865	20139	281
16	20412	20682	20951	21218	21484	21748	22010	22271	22530	22788	264
17	23044	23299	23552	23804	24054	24303	24551	24797	25042	25285	249
18	25527	25767	26007	26245	26481	26717	26951	27184	27415	27646	236
19	27875	28103	28330	28555	28780	29003	29225	29446	29666	29885	223
20	30103	30319	30535	30749	30963	31175	31386	31597	31806	32014	212
21	32221	32428	32633	32838	33041	33243	33445	33646	33845	34044	202
22	34242	34439	34635	34830	35024	35218	35410	35602	35793	35983	194
23	36172	36361	36548	36735	36921	37106	37291	37474	37657	37839	185
24	38021	38201	38381	38560	38739	38916	39093	39269	39449	39619	177
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330	171
26	41497	41664	41830	41995	42160	42324	42488	42651	42813	42975	164
27	43136	43296	43456	43616	43775	43933	44090	44248	44404	44560	158
28	44715	44870	45024	45178	45331	45484	45636	45788	45939	46089	153
29	46239	46389	46538	46686	46834	46982	47129	47275	47421	47567	148
30	47712	47856	48000	48144	48287	48430	48572	48713	48855	48995	143
31	49136	49276	49415	49554	49693	49831	49968	50105	50242	50379	138
32	50515	50650	50785	50920	51054	51188	51321	51454	51587	51719	134
33	51851	51982	52113	52244	52374	52504	52633	52763	52891	53020	130
34	53147	53275	53402	53521	53655	53781	53907	54033	54157	54282	126
35	54406	54530	54654	54777	54900	55022	55145	55266	55388	55509	122
36	55630	55750	55870	55990	56110	56229	56348	56466	56584	56702	119
37	56820	56937	57054	57170	57287	57403	57518	57634	57749	57863	116
38	57978	58092	58206	58319	58433	58546	58658	58771	58883	58995	113
39	59106	59217	59328	59439	59549	59659	59769	59879	59988	60097	110
40	60206	60314	60422	60530	60638	60745	60852	60959	61066	61172	107
41	61278	61384	61489	61595	61700	61804	61909	62013	62117	62221	104
42	62324	62428	62531	62634	62736	62838	62941	63042	63144	63245	102
43	63346	63447	63548	63648	63749	63848	63948	64048	64147	64246	99
44	64345	64443	64542	64640	64738	64836	64933	65030	65127	65224	98
45	65321	65417	65513	65609	65705	65801	65896	65991	66086	66181	96
46	66275	66370	66464	66558	66651	66745	66838	66931	67024	67117	94
47	67209	67302	67394	67486	67577	67669	67760	67851	67942	68033	92
48	68124	68214	68304	68394	68484	68574	68663	68752	68842	68930	90
49	69019	69108	69196	69284	69372	69460	69548	69635	69722	69810	88
50	69897	69983	70070	70156	70243	70329	70415	70500	70586	70671	86
51	70757	70842	70927	71011	71096	71180	71265	71349	71433	71516	84
52	71600	71683	71767	71850	71933	72015	72098	72181	72263	72345	82
53	72427	72509	72591	72672	72754	72835	72916	72997	73078	73158	81
54	73239	73319	73399	73480	73559	73639	73719	73798	73878	73957	80
55	74036	74115	74193	74272	74351	74429	74507	74585	74663	74771	78
No.	0	1	2	3	4	5	6	7	8	9	Prop.

No.	0	1	2	3	4	5	6	7	8	9	Prop.
56	74818	74896	74973	75050	75127	75204	75281	75358	75434	75511	77
57	75587	75663	75739	75815	75891	75966	76042	76117	76192	76267	75
58	76342	76417	76492	76566	76641	76715	76789	76863	76937	77011	74
59	77085	77158	77232	77305	77378	77451	77524	77597	77670	77742	73
60	77815	77887	77959	78031	78103	78175	78247	78318	78390	78461	72
61	78533	78604	78675	78746	78816	78887	78958	79028	79098	79169	71
62	79239	79309	79379	79448	79518	79588	79657	79726	79796	79865	70
63	79934	80002	80071	80140	80208	80277	80345	80413	80482	80550	69
64	80618	80685	80753	80821	80888	80956	81023	81090	81157	81224	68
65	81291	81358	81424	81491	81557	81624	81690	81756	81822	81888	67
66	81954	82020	82085	82151	82216	82282	82347	82412	82477	82542	66
67	82607	82672	82736	82801	82866	82930	82994	83058	83123	83187	65
68	83250	83314	83378	83442	83505	83569	83632	83695	83758	83821	64
69	83884	83947	84010	84073	84136	84198	84260	84323	84385	84447	63
70	84509	84571	84633	84695	84751	84818	84880	84941	85003	85064	62
71	85125	85187	85248	85309	85369	85430	85491	85551	85612	85672	61
72	85733	85793	85853	85913	85973	86033	86093	86153	86213	86272	60
73	86332	83991	86451	86510	86569	86628	86687	86746	86805	86864	59
74	86923	86981	87040	87098	87157	87215	87273	87332	87390	87448	58
75	87506	87564	87621	87679	87737	87794	87852	87906	87966	88024	57
76	88081	88138	88195	88252	88309	88366	88422	88479	88536	88592	56
77	88649	88705	88761	88818	88874	88930	88986	89042	89098	89153	56
78	89209	89265	89320	89376	89431	89487	89542	89597	89652	89707	55
79	89762	89817	89872	89927	89982	90036	90091	90145	90200	90254	54
80	90309	90363	90417	90471	90525	90579	90633	90687	90741	90794	54
81	90848	90902	90955	91009	91062	91115	91169	91222	91275	91328	53
82	91381	91434	91487	91540	91592	91645	91698	91750	91803	91855	53
83	91907	91960	92012	92064	92116	92168	92220	92272	92324	92376	52
84	92427	92479	92531	92582	92634	92685	92737	92788	92839	92890	51
85	92941	92993	93044	93095	93176	93196	93247	93298	93378	93399	51
86	93449	93500	93550	93601	93651	93701	93751	93802	93852	93902	50
87	93951	94001	94051	94101	94151	94200	94250	94300	94349	94398	49
88	94448	94497	94546	94596	94645	94694	94743	94792	94841	94890	49
89	94939	94987	95036	95085	95133	95182	95230	95279	95327	95376	48
90	95424	95472	95520	95568	95616	95664	95712	95760	95808	95856	48
91	95984	95951	95999	56047	96094	96142	96189	96236	96284	96331	48
92	96378	96426	96473	96520	96567	96614	96661	96708	96754	96801	47
93	96848	96895	96941	96988	97034	97081	97127	97174	97220	97266	47
94	97312	97359	97405	97451	97497	97543	97589	97635	97680	97726	56
95	97772	97818	97863	97909	97954	93000	98045	98091	98136	98181	56
96	98227	98272	98317	98362	98407	98452	98497	98542	98587	98632	55
97	98677	98721	98766	98811	98855	98900	98945	98989	99033	99078	55
98	99122	99166	99211	99255	99299	99343	99387	99431	99475	99519	54
99	99563	99607	99651	99694	99738	99782	99825	99869	99913	99956	54
No	0	1	2	3	4	5	6	7	8	9	Prop.

Examples. Find the Logarithm of

Log. 3 ? - - - - - - 0·47712, the answer.

Log. 54 ? - - - - - - 1·73239, "

Log. 867 ? - . . - - - 2·93802, "

Deg	0′	10′	20′	30′	40′	50′	60′	
0	0:00000	3:46372	3:26475	3:94084	2:06577	2:16268	2:24185	89
1	2:24185	2:30879	2:36677	2:41791	2:46366	2:50504	2:54281	88
2	2:54281	2:57756	2:60973	2:63968	2:66768	2:69399	2:71880	87
3	2:71880	2:74225	2:76451	2:78567	2:80585	2:82513	2:84358	86
4	2:84358	2:86128	2:87828	2:89464	2:91040	2:92560	2:94029	85
5	2:94029	2:95449	2:96824	2:98157	2:99449	1:00704	1:01923	84
6	1:01923	1:03108	1:04262	1:05385	1:06480	1:07548	1:08589	83
7	1:08589	1:09606	1:10599	1:11569	1:12518	1:13447	1:14355	82
8	1:14355	1:15249	1:16116	1:16970	1:17807	1:18628	1:19433	81
9	1:19433	1:20223	1:20999	1:21760	1:22509	1:23244	1:23967	80
10	1:23967	1:24677	1:25376	1:26063	1:26739	1:27404	1:28059	79
11	1:28059	1:28704	1:29339	1:29965	1:30581	1:31189	1:31787	78
12	1:31787	1:32378	1:32959	1:33533	1:34099	1:34657	1:35208	77
13	1:35208	1:35752	1:36288	1:36818	1:37341	1:37857	1:38367	76
14	1:38367	1:38871	1:39368	1:39860	1:40345	1:40825	1:41299	75
15	1:41299	1:41768	1:42231	1:42689	1:43142	1:43590	1:44033	74
16	1:44033	1:44472	1:44905	1:45334	1:45758	1:46178	1:46593	73
17	1:46593	1:47004	1:47411	1:47814	1:48212	1:48607	1:48998	72
18	1:48998	1:49385	1:49768	1:50147	1:50523	1:50895	1:51264	71
19	1:51264	1:51629	1:51991	1:52349	1:52704	1:53056	1:53405	70
20	1:53405	1:53750	1:54093	1:54432	1:54768	1:55102	1:55432	69
21	1:55432	1:55760	1:56085	1:56407	1:56726	1:57043	1:57357	68
22	1:57357	1:57668	1:57977	1:58284	1:58587	1:58889	1:59187	67
23	1:59187	1:59484	1:59778	1:60070	1:60359	1:60646	1:60931	66
24	1:60931	1:61214	1:61494	1:61772	1:62048	1:62322	1:62594	65
25	1:62594	1:62864	1:63132	1:63398	1:63662	1:63924	1:64184	64
26	1:64184	1:64442	1:64698	1:64952	1:65205	1:65455	1:65704	63
27	1:65704	1:65951	1:66197	1:66440	1:66682	1:66922	1:67160	62
28	1:67160	1:67397	1:67632	1:67866	1:68098	1:68328	1:68557	61
29	1:68557	1:68784	1:69009	1:69233	1:69456	1:69677	1:69897	60
30	1:69897	1:70115	1:70331	1:70546	1:70760	1:70773	1:71183	59
31	1:71183	1:71393	1:71601	1:71808	1:72014	1:72218	1:72421	58
32	1:72421	1:72622	1:72822	1:73021	1:73219	1:73415	1:73610	57
33	1:73610	1:73804	1:73997	1:74188	1:74379	1:74568	1:74756	56
34	1:74756	1:74942	1:75128	1:75312	1:75496	1:75678	1:75859	55
35	1:75859	1:76039	1:76217	1:76395	1:76572	1:76747	1:76921	54
36	1:76921	1:77095	1:77267	1:77438	1:77609	1:77778	1:77946	53
37	1:77946	1:78113	1:78279	1:78444	1:78608	1:78772	1:78934	52
38	1:78934	1:79095	1:79255	1:79415	1:79573	1:79730	1:79887	51
39	1:79887	1:80042	1:80197	1:80351	1:80503	1:80655	1:80806	50
40	1:80806	1:80956	1:81106	1:81254	1:81401	1:81548	1:81694	49
41	1:81694	1:81839	1:81983	1:82126	1:82268	1:82410	1:82551	48
42	1:82551	1:82691	1:82830	1:82968	1:83105	1:83242	1:83378	47
43	1:83378	1:83513	1:83647	1:83781	1:83914	1:84045	1:84177	46
44	1:84177	1:84307	1:84437	1:84566	1:84694	1:84821	1:84948	45
45	1:84948	1:85074	1:85199	1:85324	1:85448	1:85571	1:85693	44
	60′	50′	40′	30′	20′	10′	0′	Deg

LOGARITHM COSINE.

The negative index is noted by two points, and must always follow an opposite operation to that of the mantissa. If the mantissa is added, subtract the index, and *vice versa*.

Deg.	0′	10′	20′	30′	40′	50′	60′	
46	1:85693	1:85815	1:85936	1:86056	1:86175	1:86294	1:86412	43
47	1:86412	1:86530	1:86647	1:86763	1:86878	1:86993	1:87107	42
48	1:87107	1:87220	1:87333	1:87445	1:87557	1:87667	1:87778	41
49	1:87778	1:87887	1:87996	1:88104	1:88212	1:88319	1:88425	40
50	1:88425	1:88531	1:88636	1:88740	1:88844	1:88947	1:89050	39
51	1:89050	1:89152	1:89253	1:89354	1:89454	1:89554	1:89653	38
52	1:89653	1:89751	1:89849	1:89946	1:90043	1:90139	1:90234	37
53	1:90234	1:90329	1:90424	1:90517	1:90611	1:90703	1:90795	36
54	1:90795	1:90887	1:90978	1:91068	1:91158	1:91247	1:91336	35
55	1:91336	1:91424	1:91512	1:94599	1:91685	1:91771	1:91857	34
56	1:91857	1:91942	1:92026	1:92110	1:92194	1:92276	1:92359	33
57	1:92359	1:92440	1:92522	1:92602	1:92683	1:92762	1:92842	32
58	1:92842	1:92920	1:92998	1:93076	1:93153	1:93230	1:93306	31
59	1:95306	1:93382	1:93457	1:93532	1:93606	1:93679	1:93753	30
60	1:93753	1:93825	1:93898	1:93969	1:94040	1:94111	1:94181	29
61	1:94181	1:94251	1:94321	1:94389	1:94458	1:94526	1:94593	28
62	1:94593	1:94660	1:94726	1:94792	1:94858	1:94923	1:94988	27
63	1:94988	1:95052	1:95115	1:95179	1:95241	1:95304	1:95366	26
64	1:95366	1:95427	1:95488	1:95548	1:95608	1:95668	1:95727	25
65	1:95727	1:95786	1:95844	1:95902	1:95959	1:96016	1:96073	24
66	1:96073	1:96129	1:96184	1:96239	1:96294	1:96348	1:96402	23
67	1:96402	1:96456	1:96509	1:96561	1:96613	1:96665	1:96716	22
68	1:96716	1:96767	1:96817	1:96867	1:96917	1:96966	1:97015	21
69	1:97015	1:97063	1:97111	1:97158	1:97205	1:97252	1:97298	20
70	1:97298	1:97344	1:97389	1:97434	1:97479	1:97523	1:97567	19
71	1:97567	1:97610	1:97653	1:97695	1:97737	1:97779	1:97820	18
72	1:97820	1:97861	1:97901	1:97942	1:97981	1:98020	1:98059	17
73	1:98059	1:98098	1:98136	1:98173	1:98210	1:98247	1:98284	16
74	1:98284	1:98320	1:98355	1:98391	1:98425	1:98460	1:98494	15
75	1:98494	1:98528	1:98561	1:98594	1:98626	1:98658	1:98690	14
76	1:98690	1:98721	1:98752	1:98783	1:98813	1:98843	1:98872	13
77	1:98872	1:98901	1:98930	1:98958	1:98986	1:99013	1:99040	12
78	1:99040	1:99067	1:99093	1:99119	1:99144	1:99169	1:99194	11
79	1:99194	1:99219	1:99243	1:99266	1:99289	1:99312	1:99335	10
80	1:99335	1:99357	1:99378	1:99400	1:99421	1:99441	1:99462	9
81	1:99462	1:99481	1:99501	1:99520	1:99539	1:99557	1:99575	8
82	1:99575	1:99592	1:99610	1:99626	1:99643	1:99659	1:99675	7
83	1:99675	1:99690	1:99705	1:99719	1:99734	1:99748	1:99761	6
84	1:99761	1:99774	1:99787	1:99799	1:99811	1:99823	1:99834	5
85	1:99834	1:99845	1:99855	1:99865	1:99875	1:99885	1:99894	4
86	1:99894	1:99902	1:99911	1:99918	1:99926	1:99933	1:99940	3
87	1:99940	1:99946	1:99952	1:99958	1:99964	1:99968	1:99973	2
88	1:99973	1:99977	1:99981	1:99985	1:99988	1:99991	1:99993	1
89	1:99993	1:99995	1:99997	1:99998	1:99999	1:99999	1:99999	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

LOGARITHM COSINE.

Examples. Find the Logarithms,

Log:sine 35° 40′ ? - - - - - = 1:76572, the answer.

Log:cosine 18° 20′ ? - - - - - = 1:97737, "

Deg	0′	10′	20′	30′	40′	50′	60′	Deg.
0	3:00000	3:46372	3:76476	3:94085	2:06580	2:16272	2:24192	89
1	2:24192	2:30888	2:36689	2:41806	2:46384	2:50526	2:54308	88
2	2:54308	2:57787	2:61009	2:64009	2:66816	2:69452	2:71939	87
3	2:71939	2:74292	2:76524	2:78648	2:80674	2:82610	2:84464	86
4	2:84464	2:86243	2:87952	2:89598	2:91184	2:92715	2:94195	85
5	2:94195	2:95626	2:97013	2:98357	2:99662	1:00929	1:02162	84
6	1:02162	1:03360	1:04528	1:05665	1:06775	1:07857	1:08914	83
7	1:08914	1:09946	1:10955	1:11942	1:12908	1:13854	1:14780	82
8	1:14780	1:15687	1:16577	1:17449	1:18305	1:19146	1:19971	81
9	1:19971	1:20781	1:21578	1:22360	1:23130	1:23887	1:24631	80
10	1:24631	1:25364	1:26086	1:26796	1:27496	1:28185	1:28863	79
11	1:28865	1:29534	1:30195	1:30846	1:31488	1:32122	1:32747	78
12	1:32747	1:33364	1:33973	1:34575	1:35169	1:35756	1:36336	77
13	1:36336	1:36909	1:37475	1:38035	1:38588	1:39136	1:39677	76
14	1:39677	1:40212	1:40741	1:41265	1:41784	1:42297	1:42805	75
15	1:42805	1:43308	1:43805	1:44298	1:44787	1:45270	1:45749	74
16	1:45749	1:46224	1:46694	1:47160	1:47622	1:48080	1:48553	73
17	1:48533	1:48983	1:49429	1:49872	1:50310	1:50746	1:51177	72
18	1:51177	1:51605	1:52030	1:52452	1:52870	1:53285	1:53697	71
19	1:53697	1:54106	1:54511	1:54914	1:55314	1:55712	1:56106	70
20	1:56106	1:56498	1:56887	1:57273	1:57657	1:58038	1:58417	69
21	1:58417	1:58794	1:59168	1:59539	1:59909	1:60276	1:60641	68
22	1:60641	1:61003	1:61364	1:61722	1:62078	1:62433	1:62785	67
23	1:62785	1:63135	1:63483	1:63830	1:64174	1:64517	1:64858	66
24	1:64858	1:65197	1:65534	1·65870	1:66204	1:66536	1:66867	65
25	1:66867	1:67196	1:67523	1:67849	1:68174	1:68496	1:68818	64
26	1:68818	1:69138	1:69456	1:69773	1:70089	1:70403	1:70716	63
27	1:70716	1:71028	1:71338	1:71647	1:71955	1:72262	1:72567	62
28	1:72567	1:72871	1:73174	1:73476	1:73777	1:74076	1:74375	61
29	1:74374	1:74672	1:74968	1:75264	1:75558	1:75851	1:76143	60
30	1:76143	1:76435	1:76725	1:77014	1:77303	1:77590	1:77877	59
31	1:77877	1:78163	1:78447	1:78731	1:79015	1:79293	1:79578	58
32	1:79578	1:79859	1:80139	1:80418	1:80697	1:80974	1:81251	57
33	1:81251	1:81527	1:81803	1:82078	1:82352	1:82625	1:82898	56
34	1:82898	1:83170	1:83442	1:83713	1:83983	1:84253	1:84522	55
35	1:84522	1:84791	1:85059	1:85326	1:85593	1:85860	1:86126	54
36	1:86126	1:86391	1:86656	1:86920	1:87184	1:87448	1:87711	53
37	1:87711	1:87974	1:88236	1:88498	1:88759	1:89020	1:89281	52
38	1:89281	1:89541	1:89801	1:90060	1:90319	1:90578	1:90836	51
39	1:90836	1:91095	1:91352	1:91610	1:91867	1:92121	1:92381	50
40	1:92381	1:92637	1:92894	1:93149	1:93405	1:93661	1:93916	49
41	1:93916	1:94171	1:94497	1:94680	1:94935	1:95189	1:95443	48
42	1:95443	1:95697	1:95926	1:96205	1:96458	1:96712	1:96965	47
43	1:96965	1:97218	1:97471	1:97725	1:97978	1:98230	1:98483	46
44	1:98485	1:98736	1:98989	1:99242	1:99494	1:99747	1:00000	45
Deg.	60′	50′	40′	30′	20′	10′	0′	Deg.

LOGARITHM COTANGENT.

The negative index is noted by two points, and must always follow an opposite operation to that of the mantissa. If the mantissa is added, subtract the index, and *vice versa*.

Deg	0′	10′	20′	30′	40′	50′	60	
45	0·00000	0·00252	0·00505	0·00758	0·01010	0·01263	0·01516	44
46	0·01516	0·01769	0·02022	0·02275	0·02528	0·02781	0·03034	43
47	0·03034	0·03287	0·03541	0·03794	0·04048	0·04302	0·04556	42
48	0·04556	0·04810	0·05064	0·05319	0·05573	0·05828	0·06083	41
49	0·06083	0·06339	0·06594	0·06850	0·07106	0·07362	0·07618	40
50	0·07618	0·07875	0·08132	0·08389	0·08647	0·08904	0·09163	39
51	0·09163	0·09421	0·09680	0·09939	0·10199	0·10458	0·10716	38
52	0·10719	0·10979	0·11240	0·11502	0·11763	0·12025	0·12288	37
53	0·12288	0·12551	0·12815	0·13079	0·13343	0·13608	0·13873	36
54	0·13873	0·14139	0·14406	0·14673	0·14940	0·15208	0·15477	35
55	0·15477	0·15746	0·16016	0·16286	0·16557	0·16829	0·17101	34
56	0·17101	0·17374	0·17647	0·17921	0·18196	0·18472	0·18748	33
57	0·18748	0·19025	0·19302	0·19581	0·19860	0·20140	0·20421	32
58	0·20421	0·20702	0·20984	0·21268	0·21552	0·21836	0·22122	31
59	0·22122	0·22409	0·22696	0·22985	0·23274	0·23564	0·23856	30
60	0·23856	0·24148	0·24441	0·24735	0·25031	0·25327	0·25624	29
61	0·25624	0·25923	0·26222	0·26523	0·26825	0·27128	0·27432	28
62	0·27432	0·27737	0·28044	0·28352	0·28661	0·28971	0·29283	27
63	0·29283	0·29596	0·29910	0·30226	0·30543	0·30861	0·31181	26
64	0·31181	0·31503	0·31826	0·32150	0·32476	0·32803	0·33132	25
65	0·33132	0·33463	0·33795	0·34129	0·34465	0·34802	0·35141	24
66	0·35141	0·35482	0·35825	0·36169	0·36516	0·36864	0·37214	23
67	0·37214	0·37567	0·37921	0·38277	0·38635	0·38996	0·39359	22
68	0·39359	0·39723	0·40090	0·40460	0·40831	0·41205	0·41582	21
69	0·41582	0·41961	0·42342	0·42726	0·43112	0·43501	0·43893	20
70	0·43893	0·44287	0·44685	0·45085	0·45488	0·45893	0·46302	19
71	0·46302	0·46714	0·47129	0·47548	0·47969	0·48394	0·48822	18
72	0·48822	0·49254	0·49689	0·50127	0·50570	0·51016	0·51466	17
73	0·51466	0·51919	0·52377	0·52839	0·53305	0·53775	0·54250	16
74	0·54250	0·54729	0·55213	0·55701	0·56194	0·56692	0·57194	15
75	0·57194	0·57702	0·58215	0·58734	0·59258	0·59787	0·60322	14
76	0·60322	0·60864	0·61411	0·61964	0·62524	0·63090	0·63663	13
77	0·63663	0·64243	0·64830	0·65424	0·66026	0·66635	0·67252	12
78	0·67252	0·67877	0·68511	0·69153	0·69804	0·70465	0·71134	11
79	0·71134	0·71814	0·72503	0·73203	0·73913	0·74635	0·75368	10
80	0·75368	0·76112	0·76869	0·77639	0·78422	0·79218	0·80028	9
81	0·80028	0·80853	0·81694	0·82550	0·83422	0·84312	0·85219	8
82	0·85219	0·86145	0·87091	0·88057	0·89044	0·90053	0·91085	7
83	0·91085	0·92142	0·93224	0·94334	0·95471	0·96639	0·97838	6
84	0·97838	0·99070	1·00337	1·01642	0·02986	1·04373	1·05804	5
85	1·05804	1·07284	1·08815	1·10401	1·12047	1·13756	1·15535	4
86	1·15535	1·17389	1·19325	1·21351	1·23475	1·25707	1·28060	3
87	1·28060	1·30547	1·33184	1·35990	1·38990	1·42212	1·45691	2
88	1·45691	1·49473	1·53615	1·58193	1·63310	1·69111	1·75807	1
89	1·75807	1·83727	1·93419	2·05914	1·23523	2·53627	0·00000	0
	60′	50′	40′	30′	20′	10′	0′	Deg

LOGARITHM COTANGENT.

Example. Find the Logarithms,
Log.tan. 36° 40′? - - - - - - = 1:87185, the answer.
Log.tan. 58° 50′? - - - - - = 0:21836, "

ARITHMETICAL PROGRESSION.

Arithmetical Progression is a series of numbers, as 2, 4, 6, 8, 10, 12, &c., or 18, 15, 12, 9, 6, 3, in which every successive term is increased or diminished by a constant number.

Letters will denote,

a = the first term of the series.
b = any other term whose number from a is n.
n = number of terms within a and b.
δ = the difference between the terms.
S = the sum of all the terms.

In the series, 2, 5, 8, 11, $a = 2$, $b = 11$, $n = 4$, $\delta = 3$, and $S = 26$.

☞When the series is decreasing, take the first term = b and the last term = a.

The accompanying Table contains all the formulas or questions in Arithmetical Progressions, and the nature of the question will tell which formula is to be used.

Formulas for Arithmetical Progressions.

$a = b - \delta\,(n-1)$, . . 1,	$\delta = \dfrac{b-a}{n-1}$, ... 9,
$a = \dfrac{2S}{n} - b$, . . . 2,	$\delta = \dfrac{(b+a)(b-a)}{2S-a-b}$, . . 10,
$a = \dfrac{S}{n} - \dfrac{\delta}{2}(n-1)$, . . 3,	$\delta = \dfrac{2\,(S-an)}{n\,(n-1)}$, . . 11,
$b = a + \delta\,(n-1)$, . . 4,	$\delta = \dfrac{2\,(bn-S)}{n\,(n-1)}$, . . . 12,
$b = \dfrac{2S}{n} - a$, . . . 5,	$S = \dfrac{n\,(a+b)}{2}$, . . . 13,
$b = \dfrac{S}{n} + \dfrac{\delta}{2}(n-1)$, . . 6,	$S = \dfrac{(a+b)(b+\delta-a)}{2\delta}$, . . 14,
$n = \dfrac{b-a}{\delta} + 1$, . . . 7,	$S = n\left[a + \dfrac{\delta}{2}\,(n-1)\right]$ 15,
$n = \dfrac{2S}{a+b}$, ... 8,	$S = n\left[b - \dfrac{\delta}{2}(n-1)\right]$ . . 16,

$$a = \frac{\delta}{2} \pm \sqrt{\left(b+\frac{\delta}{2}\right)^2 - 2\delta S}, \quad . . . . \quad 17,$$

$$b = -\frac{\delta}{2} \pm \sqrt{\left(a-\frac{\delta}{2}\right)^2 + 2\delta S}, \quad . . . . \quad 18,$$

$$n = \frac{1}{2} - \frac{a}{\delta} \pm \sqrt{\left(\frac{1}{2}-\frac{a}{\delta}\right)^2 + \frac{2S}{\delta}}, \quad . . . \quad 19,$$

$$n = \frac{1}{2} + \frac{b}{\delta} \pm \sqrt{\left(\frac{1}{2}+\frac{b}{\delta}\right)^2 - \frac{2S}{\delta}}, \quad . . . \quad 20,$$

Example 1. A man was engaged to dig a well at one dollar ($1) for the first foot of the depth of the well, $1·84 for the second, and 84 cents more per every successive foot in depth, until he reached the water, which was found at a depth of 25 feet. How much money is due to the man?

This will be answered by the formula 15, in which $a=1$, $d=0{\cdot}84$, and $n=25$, then the sum,

$$S=25\left[1+\frac{0{\cdot}84}{2}(25-1)\right]=\$277 \text{ the answer.}$$

Example 2. A Propeller ship which is to run between Philadelphia and Charleston, cost $116500, of which the company agreed to pay on account $14075 at her first trip to Charleston; and per every successive trip, they paid $650 less than the former. How many trips must the vessel make until she is fully paid?

This will be answered by the formula 20, in which $b=\$14075$, $d=650$, and $S=116500$.

$$n=\frac{1}{2}+\frac{14075}{650}-\sqrt{\left(\frac{14075}{650}+\frac{1}{2}\right)^2-\frac{2\times116500}{650}}=10{\cdot}6 \text{ or } 11 \text{ trips.}$$

Arithmetical Progressions of a Higher Order.

Arithmetical Progressions are of the first order, when the difference δ is a constant number, but when the difference δ progresses itself with a constant number, the Progression is of the *second order.*

When the difference δ progresses in a second order, the Progression is of the *third order,* &c., &c., and is thus explained:

1, 2, 3, 4, 5, 6, ...	n,	Arith. Prog., first order.
1, 3, 6, 10, 15, 21, . . .	$\frac{n(n+1)}{2}$	2d. order.
1, 4, 10, 20, 35, 56, . .	$\frac{n(n+1)(n+2)}{2\times3}$,	3d. order.
1, 5, 15, 35, 70, 126, .	$\frac{n(n+1)(n+2)(n+3)}{2\times3\times4}$,	4th. order.

Here you will discover that the sum of n terms in one order, is equal to the same nth term in the next higher order. Arithmetical Progressions of the *first, second,* and *third orders,* are applied to

PILES OF BALLS AND SHELLS.

Triangular Piling.

Example 1. A complete triangular pile of balls has $n=12$ balls in each side. Require how many balls in the base, and how many in the whole pile?

In the base,	$=\frac{12(12+1)}{2}=78$ balls,	2d. order.
Whole pile,	$=\frac{12(12+1)(12+2)}{2\times3}=364$ balls,	3d. order.

Square Piling.

1, 4, 9, 16, 25, 36, ...	n^2	2d. order.
1, 5, 14, 30, 55, 91, . .	$\frac{n(n+1)(2n+1)}{2\times3}$,	3d. order.

[*See Examples* 2 *and* 3 *on page* 67.]

GEOMETRICAL PROGRESSION.

Geometrical Progression is a series of numbers, as 2 : 4 : 8 : 16 : 32 : &c., or 729 : 243 : 81 : 27 : 9 : &c., in which every successive term is multiplied or divided by a constant factor.

Letters will denote,

a = the first term of the series.
b = any other term whose number from a is n.
n = number of terms within a and b.
r = ratio, or the factor by which the terms are multiplied or divided.
S = Sum of the terms.

In the series 1 : 3 : 9 : 27 : $a = 1$, $b = 27$, $n = 4$, $r = 3$, $S = 40$.

The accompanying Table contains all the formulas or questions in Geometrical Progressions. The nature of the question will tell which formula is to be used.

Formulas for Geometrical Progressions.

$a = \frac{b}{r^{n-1}}$, . . . 1,	$r = \sqrt[n-1]{\frac{b}{a}}$, . . . 7,
$a = S - r(S - b)$, . . 2,	$r = \frac{S-a}{S-b}$, . . . 8,
$a = S\frac{r-1}{r^n-1}$, . . 3,	$ar^n + S - rS - a = 0$, . 9,
$b = ar^{n-1}$, . . . 4,	$S = \frac{br-a}{r-1}$, . . . 10,
$b = S - \frac{S-a}{r}$, . . 5,	$S = \frac{a(r^n-1)}{r-1}$, . . . 11,
$b = S\left(\frac{r-1}{r^n-1}\right)r^{n-1}$, 6,	$S = \frac{b(r^n-1)}{(r-1)r^{n-1}}$, . . 12,

$$n = 1 + \frac{\log.b - \log.a}{\log.r}, \quad 13,$$

$$n = 1 + \frac{\log.b - \log.a}{\log.(S+a) - \log.(S-b)}, \quad 14,$$

$$n = \frac{\log.[a + S(r-1)] - \log.a}{\log.r.}, \quad 15,$$

$$n = 1 + \frac{\log.b - \log.[br - S(r-1)]}{\log.r.}, \quad 16,$$

$$S = \frac{b\sqrt[n-1]{b} - a\sqrt[n-1]{a}}{\sqrt[n-1]{b} - \sqrt[n-1]{a}}, \quad 17,$$

Example 1. Required the 10th term in the Geometrical Progression 4 : 12 : 36.... ?
Given $a = 4$, $n = 10$, and $r = 3$. We have,
Formula 4. $b = ar^{n-1} = 4 \times 3^9 = 78732$, the tenth term.

Example 2. Required the sum of the 10 terms in the preceding example?

Formula 11, $S = \frac{a(r^n - 1)}{r - 1} = \frac{4(3^{10} - 1)}{2} = 118096$, the sum.

Example 3. Insert 6 proportional terms between 3 and 384?
Given $a = 3$, $b = 384$, and $n = 6 + 2 = 8$.

Formula 7, $r = \sqrt[n-1]{\frac{b}{a}} = \sqrt[7]{\frac{384}{3}} = 2$,

then 3 : 6 : 12 : 24 : 48 : 96 : 192 : 384, the answer.

Example 4. A man had 16 twenty dollar gold pieces, which he agreed to exchange for copper in such a way, that he gets one cent on the first \$20, two on the second, four on the third, and eight on the fourth, &c., &c.; until the sixteen \$20 pieces were covered. How many cents will come on the sixteenth gold piece, and what will be the whole amount of copper on the gold?

In the progression 1 : 2 : 4 : 8 : &c., we have,
Given $n = 16$, $r = 2$, and $a = 1$, then,

Formula 4. $b = 1 \times 2^{16-1} = \frac{2^{16}}{2} = \frac{4^8}{2} = \frac{16^4}{2} = \frac{256^2}{2} = 32768$ cents, on the sixteenth piece.

The total sum of cents will be found by the

Formula 10. $S = \frac{32768 \times 2 - 1}{2 - 1} = 65535$ cents $= \$655{\cdot}35$.

Piling of Balls and Shells.—[*From page* 64.]

Example 2. How many balls are contained in a complete square pile, $n = 10$ rows?

$$\frac{10(10+1)(2 \times 10+1)}{2 \times 3} = \frac{10 \times 11 \times 21}{6} = 385 \text{ balls.}$$

Rectangular Piling.

Let m be the number of balls on the top of the complete pile, and n = number of rows in the same, then the number of balls in the whole pile will be,

$$\frac{n(n+1)(2n+3m-2)}{2 \times 3}, \quad . \quad . \quad . \quad \text{3d. order.}$$

The number of balls in the longest bottom side will be $= m + n - 1$.

Example 3. The rectangular pile having 15 rows and 23 balls on the the top, how many in the whole pile?

$$\frac{15\,(15+1)(2 \times 15 + 3 \times 23 - 2)}{2 \times 3} = \frac{15 \times 16 \times 67}{6} = 2680 \text{ balls.}$$

COMPOUND INTEREST.

Compound Interest is when the Interest is added to the Capital for each year, and the sum is the Capital for the following year.

Amount, $a = c\,(1+p)^n$, ... 1,

Capital, $c = \dfrac{a}{(1+p)^n}$, ... 2,

Per centage, $p = \sqrt[n]{\dfrac{a}{c}} - 1$, ... 3,

Number of years, $n = \dfrac{log.a - log.c}{log.(1+p)}$, ... 4,

☞In these formulas p must be expressed in a fraction of 100.

Example 1. A capital $c = 8650$ standing with Compound Interest at $p = 5$ per cent. what will it amount to in $n = 9$ years.

Amount $a = 8650\,(1{\cdot}05)^9 = 13419$ dollars.

Example 2. A man commenced business with $c = 300$ dollars, after $n = 5$ years he had $a = 6875$ dollars At what rate did his money increase, and how soon will he have a fortune of 50000 dollars?

The first question, or the per centage, will be answered by the formula 3.

$$p = \sqrt[5]{\frac{6875}{300}} - 1 = \sqrt[5]{22{\cdot}9166} - 1 = 0{\cdot}87, \text{ or 87 per cent.}$$

The time from the commencement of business until the fortune is completed, will be answered from the formula 4.

$$n = \frac{log.50000 - log.300}{log.1{\cdot}87} = \frac{4{\cdot}69897 - 2{\cdot}47712}{0{\cdot}2720048.} = 8{\cdot}169 \text{ years,}$$

or 8 years and 2 months.

ANNUITIES.

Annuity is a certain sum of money to be paid at regular intervals.

A yearly payment or annuity b, is standing for n years, to find the whole amount a at p per cent. Interest.

Amount, $a = bn\left[1 + \dfrac{p}{2}(n+1)\right]$ Simple Int., . . . 1,

Amount, $a = \dfrac{b}{p}\left[(1+p)^n - 1\right]$ Comp. Int., ... 2,

A yearly payment or annuity b, is to be paid for n years, to find the present worth, or the amount a, which would pay it in full, at the beginning of the time n, deducting p per cent. Interest.

Amount, $a = bn\left[1 - \dfrac{p}{2}\left(\dfrac{n+1}{1+pn}\right)\right]$ Simp. Int., . . . 3,

Amount, $a = \dfrac{b}{p}\left[1 - \dfrac{1}{(1+p)^n}\right]$ Comp. Int., . . . 4,

A debt D, standing for Interest, is diminished yearly by a sum b; to find the debt d after n years, and the time n when it is fully paid?

The debt d after n years will be,

$$d = \frac{(Dp - b)(1+p)^n + b}{p} \text{ Comp. Int.,} \quad \ldots \quad 5,$$

The time n until fully paid will be,

$$n = \frac{log.b - log.(b - Dp)}{log.(1+p)} \quad \ldots \quad 6.$$

If $b = Dp$ then $n = \infty$, or the debt D will never be paid. If $b < Dp$, the debt D will be increased.

To find the yearly annuity b, which will pay a debt D in n years, at p per cent. Compound Interest?

$$b = \frac{Dp\,(1+p)^n}{(1+p)^n - 1} \quad \ldots \quad 7.$$

PAPER.

1 Ream = 20 quires = 480 sheets.
1 quire = 24 sheets.

Drawing Paper.

Cap,	13×16 inches.	Columbier,	34×23 inches.
Demy,	20×15 "	Atlas,	33×26 "
Medium,	22×17 "	Theorem,	34×28 "
Royal,	24×19 "	Double Elephant,	40×26 "
Super Royal,	27×19 "	Antiquarian,	52×31 "
Imperial,	30×21 "	Emperor,	40×60 "
Elephant,	28×22 "	Uncle Sam,	48×120 "

Continuous Colossal Drawing Paper, No. A, and No. B, 56 inches wide, and of any required length. No. A, of this paper is excellent for mechanical drawings. Price from 40 to 50 cents per yard.

Tracing Paper.

Double Crown,	30 by 20 inches.	Glazed or Crystal, Yellow or Blue Wove.
Double Double Crown,	40 " 30 "	
Double Double Double Crown,	60 " 40 "	

Finest French Vegetable Tracing Paper.

Grand Raisin (or Royal) 24 in. by 18. Grand Aigle 40 in. by 27.

Mounted Tracing Paper.

This paper is mounted on cloth, and is still transparent; it will take ink and water colours. It is 38 inches wide, and of any required length.

Vellum Writing Cloth,

Adapted for every description of tracing; it is transparent, durable, and strong. It is 18 to 38 inches wide, and of any required length.

SELECTION OF WATER COLOURS.

Blue. Real Ultramarine.
" French Blue.
" Indigo.
" Cobalt Blue.

Green. Olive Green.

Yellow. Cadmium.
" Gamboge.
" Ochre.

Red. Carmine.
" Crimson Lake.

Red. Rose Madder.
" Light Red.

Brown. Vandyke.
" Brown Madder.

Black. India Ink.
" Blue Black.
" Ivory Black.
" Lamp Black.

White. Chinese White.

UNITED STATES' STANDARD MEASURES AND WEIGHTS.

MEASURE OF LENGTH.

THE *Standard Measure of Length* is a brass rod = 1 yard at the temperature of 32° Fahrenheit. The length of a pendulum vibrating seconds in vacuo, at Philadelphia is 1·08614 yards, at + 32° Fahrenheit.

The **Surveying Chain** is = 22 yards = 66 feet. It consists of 100 links, and each link = 7·92 inches.

ROPES AND CABLES.

1 Cable length = 120 fathoms = 720 feet.
1 fathom = 6 feet.

GEOGRAPHICAL AND NAUTICAL MEASURES.

1 Degree of the great circle of the Earth round the Equator = 69·032 statute miles = 60 Nautical miles.

1 Statute mile = 5280 feet = 0·86875 Nautical miles.
1 Nautical mile = 6037·424 = 1·150 Statute miles.

LOG LINE.

The **Log Line** should be about 150 fathoms long, and 10 fathoms from the Log to the first knot on the line. If half a minute glass is used, it will be 51 feet between each succeeding knot. For 28 seconds glass it will be 47·6 feet = 7·93 fathoms per knot. This is the length of knot by calculation, but practically it is shortened to 7·5 fathoms per knot for 28 seconds glass.

MEASURE OF CAPACITY.

Gallon. The standard Gallon measures 231 cubic inches, and contains 8·3388822 pounds Avoirdupois = 58372·1757 grains Troy, of distilled water, at its maximum density 39·83° Fahrenheit, and 30 inches barometer height.

Bushel. The standard Bushel measures 2150·42 cubic inches = 77·627413 pounds Avoirdupois of distilled water at 39·83° Fahrenheit, barometer 30 inches. Its dimensions are 18½ inches inside diameter, 19½ inches outside, and 8 inches deep; and when heaped, the cone must not be less than 6 inches high, equal 2747·70 cubic inches for a true cone.

Pound. The standard Pound Avoirdupois is the weight of 27·7015 cubic inches of distilled water, at 39·83° Fahrenheit, barometer 30 inches, and weighed in the air.

MEASURE OF LENGTH.

Miles.	Furlongs.	Chains.	Rods.	Yards.	Feet.	Inches.
1	8	80	320	1760	5280	63360
0·125	1	10	40	220	660	7920
0·0125	0·1	1	4	22	66	792
0·003125	0·025	0·25	1	5.5	16·5	198
0·00056818	0·0045454	0·045454	0·181818	1	3	36
0·00018939	0·00151515	0·01515151	0·0606060	0·33333	1	12
0·000015783	0·000126262	0·001262626	0·00505050	0·0277777	0·083333	1

MEASURE OF SURFACE.

Sq. Miles.	Acres	S.Chains.	Sq. Rods.	Sq. Yards.	Sq. Feet.	Sq. Inches.
1	640	6400	102400	3097600	27878400	4014489600
0·001562	1	10	160	4840	43560	696960
0·0001562	0·1	1	16	484	4356	69696
0·000009764	0·00625	0·0625	1	30·25	272·25	39204
0·000000323	0·0002066	0·002066	0·0330	1	9	1296
0·0000000358	0·000002296	0·00002296	0·00367	0·1111111	1	144
0·00000000025	0·000000143	0·00000143	0·00002552	20·0007716	0·006944	1

MEASURE OF CAPACITY.

Cub. Yard.	Barrels.	Bushels.	Cub. Feet.	Pecks.	Gallons.	Cub. Inch
1	5·6103	25·2467	27	100·987	201·974	46656
0·1782	1	4·5	4·8125	18	36	8316
0·03961	0·2222	1	1·2438	4	8	2150·42
0·037037	0·2078	0·804	1	3·73809	7·47619	1728
0·009902	0·05555	0·25	0·26738	1	2	462
0·004951	0·02777	0·125	0·13369	0·5	1	231
0·00002143	0·0001202	0·000465	0·0005787	0·0021645	0·004329	1

MEASURE OF LIQUIDS.

Gallon.	Quarts.	Pints.	Gills.	Cub. inch.
1	4	8	32	231
0·25	1	2	8	57·75
0·125	0·5	1	4	28·875
0·03125	0·125	0·25	1	7·2175
0·004329	0·017315	0·03463	0·13858	1

MEASURES OF WEIGHTS.

AVOIRDUPOIS.

Ton.	Cwt.	Pounds.	Ounces.	Drams.
1	20	2240	35840	573440
0·05	1	112	1792	28672
0·00044642	0·0089285	1	16	256
0·00002790	0·000558	0·0625	1	16
0·00000174	0·0000348	0·0016	0·0625	1

TROY.

Pounds.	Ounces.	Dwt.	Grains.	Pound Avoir.
1	12	240	5760	0·822861
0·083333	1	20	480	0·068571
0·004166	0·05000	1	24	0·0034285
0·0001736	0·00208333	0·0416666	1	0·00020571
1·215275	14·58333	219·6666	7000	1

APOTHECARIES'.

Pounds.	Ounces.	Drams.	Scruples.	Grains.
1	12	96	288	5760
0·08333	1	8	24	480
0·01041666	0·125	1	3	60
0·0034722	0·0416666	0·3333	1	20
0·00017361	0·020833	0·16666	0·05	1

DIAMOND.

Carat.	Grain.	Parts.	Grains. Troy.
1	4	64	3·2
0·25	1	16	0·8
0·015625	0·0625	1	0·05
0·3125	1·25	20	1

GOLD COINS. U. S. STANDARD WEIGHT.

Name of the Coins	Dollars.	*WEIGHT TROY.*	
		Grains.	Ounces.
Double Eagle	$ 20	516	1·075
Eagle	$ 10	258	0·5375
Half Eagle	$ 5	129	0·26875
Three Dollar piece	$ 3	77·4	0·16125
Quarter Eagle	$ 2·50	64·5	0·134375
Dollar piece	$ 1	25·8	0·05375
Value per Grain	$ 0·0387596	1	0·00208333
Value per Ounce	$ 18·6046	480	1

SILVER COINS. U. S. STANDARD WEIGHT.

Name of the Coins.	Cents.	*WEIGHT TROY.*	
		Grains.	Ounces.
One Dollar	100	384	0·8
Half Dollar or five Dimes	50	192	0·4
Quarter Dollar or 2½ Dimes	25	96	0·2
One Dime	10	38·4	0·08
Half Dime	5	19·2	0·04
Three Cents piece	3	11·52	0·024
Value per Grain	0·26041668	1	0·00208333
Value per Ounce	125	480	1
Copper Cent	1	168	0·35
Half Cent	0·5	84	0·175
Value per Grain	0·00595238	1	0·00208333
Value per Ounce	2·8571424	480	1

The Standard fineness of Gold and Silver Coins is *one weight of alloy to nine weights of pure metal.* The alloy for Gold Coin is Silver and Copper, and Copper for Silver Coin.

Relative value of Foreign Gold and Silver Coins, fixed by the law of the United States.

1 *Pound Sterling* of Great Britain	$ 4·84
1 *Shilling*	0·242
1 *Pound Sterling* of Nova Scotia, New Brunswick, Newfoundland and Canada	4·00
1 *Dollar* of Mexico, Peru, and Central America	1·00
1 *Pagoda* of India	1·84
1 *Real Vellon* of Spain	0·05
1 *Real Plate* of Spain	0·10
1 *Rupee Company*	0·44½
1 *Rupee* of British India	0·44⅜
1 *Franc* of France and Belgium	0·18%/10
1 *Specie Dollar* of Sweden and Norway	1·06
1 *Ducat* of Sweden	2·15
1 *Specie Dollar* of Denmark	1·05
1 *Florin* of Netherland	0.40
1 *Florin* of Southern States of Germany	0.40
1 *Guilder* of Netherland	0.40
1 *Livre Tournoise* of France	0.18⅗

1 *Livre* of the Lombardy Venitian Kingdom	$0.16
1 *Livre* of Tuscany	0.16
1 *Livre* of Sardinia	0.18$\frac{6}{10}$
1 *Milrea* of Portugal	1.12
1 *Milrea* of Azores	0.83$\frac{1}{3}$
1 *Marc Banco* of Hamburg	0·35
1 *Rix Dollar* or *Thaler* of Prussia and the Northern States of Germany,	0.69
1 *Rix Dollar* of Bremen	0.78$\frac{3}{4}$
1 *Rouble Silver* of Russia	0.75
1 *Florin* of Austria	0.48$\frac{1}{2}$
1 *Ducat* of Naples	0.80
1 *Ounce* of Sicily	2.40
1 *Tael* of China	1.43
1 *Livre* of Leghorn	0.16

FOREIGN MEASURES OF LENGTH COMPARED WITH AMERICAN.

Places.	*Measures.*	*Inches.*	*Places.*	*Measures.*	*Inches.*
Amsterdam	Foot	11·14	Malta	Foot	11·17
Antwerp	"	11·24	Moscow	"	13·17
Bavaria	"	11·42	Naples	Palmo	10·38
Berlin	"	12·19	Prussia	Foot	12·36
Bremen	"	11·38	Persia	Arish	38·27
Brussels	"	11·45	Rhineland	Foot	12·35
China	" Mathematic	13·12	Riga	"	10·79
"	" Builder's	12·71	Rome	"	11·60
"	" Tradesman's	13·32	Russia	"	13·75
"	" Surveyor's	12·58	Sardinia	Palmo	9·78
Copenhagen	"	12·35	Sicily	"	9·53
Dresden	"	11·14	Spain	Foot	11·03
England	"	12·00	"	Toesas	66·72
Florence	Braccio	21·60	"	Palmo	8·34
France	Pied de Roi	12·79	Strasburgh	Foot	11·39
"	Metre	39·381	Sweden	"	11·69
Geneva	Foot	19·20	Turin	"	12·72
Genoa	Palmo	9·72	Venice	"	13·40
Hamburgh	Foot	11·29	Vienna	"	12·45
Hanover	"	11·45	Zurich	"	11·81
Leipsic	"	11·11	Utrecht	"	10·74
Lisbon	"	12·96	Warsaw	"	14·03
"	Palmo	8·64			

ENGLISH AND FRENCH MEASURES OF LENGTH.

BRITISH. Yard is referred to a natural standard, which is the length of a pendulum vibrating seconds in vacuo in London, at the level of the sea; measured on a brass rod, at the temperature of 62° Fahrenheit, = 39·1393 *Imperial inches.*

FRENCH. *Old System.*—

1 Line	= 12 points	= 0·08884	U. S. inches.
1 Inch	= 12 lines	= 1·06604	"
1 Foot	= 12 inches	= 12·7925	"
1 Toise	= 6 feet	= 76·755	"
1 League	= 2280 toises	(common.)	
1 League	= 2000 toises	(post.)	
1 Fathom	= 5 feet.		

New System.—

1 Millimetre	= ·03939	U. S. inches.
1 Centimetre	= ·39380	"
1 Decimetre	= 3·93809	"
1 *Metre*	= 39·38091	"
1 Decametre	= 393·80917	"
1 Hecatometre	= 3938·09171	"

FOREIGN ROAD MEASURES COMPARED WITH AMERICAN.

Places.	Measures	Yards.	Places.	Measures.	Yards.
Arabia . .	Mile	2148	Hungary .	Mile	9113
Bohemia .	"	10137	Ireland .	"	3038
China . .	Li	629	Netherlands	"	1093
Denmark .	Mile	8244	Persia . .	Parasang	6086
England .	" Statute	1760	Poland .	Mile, long	8101
" . .	" Geographical	2025	Portugal .	League	6760
Flanders .	"	6869	Prussia .	Mile	8468
France . .	League, marine	6075	Rome . .	"	2025
" . .	" common	4861	Russia . .	Verst	1167
" . .	" post	4264	Scotland .	Mile	1984
Germany .	Mile, long	10126	Spain . .	League, common	7416
Hamburgh .	"	8244	Sweden .	Mile	11700
Hanover .	"	11559	Switzerland	"	9153
Holland .	"	6395	Turkey .	Berri	1826

MEASURES OF SURFACE.

FRENCH. *Old System.*—1 Square Inch . . = 1·1364 U. S. inches.
1 Arpent (Paris) . . = 900 square toises.
1 Arpent (woodland) = 100 square royal perches.
New System.—1 Arc . . . . . . = 100 square metres.
1 Decare . . . . . = 10 ares.
1 Hecatare . . . . . = 100 ares.
1 Square Metre . . = 1550·85 square inches, or 10·7698 square feet.
1 Arc . . . . = 1076·98 "

FOREIGN MEASURES OF SURFACE COMPARED WITH AMERICAN.

Places.	Measures.	Sq. Yds.	Places.	Measures.	Sq. Yds.
Amsterdam	Morgen	9722	Portugal .	Geira	6970
Berlin . .	" great	6786	Prussia .	Morgen	3053
" . .	" small	3054	Rome . .	Pezza	3158
Canary Isles	Fanegada	2422	Russia . .	Dessetina	13066·6
England .	Acre	4840	Scotland .	Acre	6150
Geneva .	Arpent	6179	Spain . .	Fanegada	5500
Hamburgh .	Morgen	11545	Sweden .	Tunneland	5900
Hanover .	"	3100	Switzerland	Faux	7855
Ireland . .	Acre	7840	Vienna .	Joch	6889
Naples . .	Moggia	3998	Zurich .	Common acre	3875·0

FOREIGN MEASURES OF CAPACITY.

BRITISH. The *Imperial gallon* measures 277·274 cubic inches, containing 10 lbs. Avoirdupois of distilled water, weighed in air, at the temperature of 62° degrees, the barometer at 30 inches.
For Grain. 8 bushels = 1 quarter.
1 quarter = 10·2694 cubic feet.
Coal, or heaped measure. 3 bushels = 1 sack.
12 sacks = 1 chaldron.
Imperial bushel = 2218·192 cubic inches.
**Heaped bushel.* 19½ ins. diam., cone 6 ins. high = 2812·4872 cubic ins.
1 chaldron = 58·658 cubic feet, and weighs 3136 pounds.
1 chaldron (Newcastle) = 5936 pounds.

FRENCH. *New System.*—1 *Litre* = 1 cub. decimetre, or 61·074 U. S. cubic inches.
Old System.—1 Boisseau = 13 litres = 793·964 cub. ins., or 3·43 galls.
1 Pinte = 0·931 litres, or 56·817 cubic inches.

SPANISH. 1 Wine Arroba = 4·2455 gallons.
1 Fanega (common measure) = 1·593 bushels.

* When heaped in the form of a true cone.

FOREIGN LIQUID MEASURES COMPARED WITH AMERICAN.

Places.	Measures.	Cub. In.	Places.	Measures.	Cub. In.
Amsterdam	Anker	2331	Naples	Wine Barille	2541
"	Stoop	146	"	Oil Stajo	1133
Antwerp	"	194	Oporto	Almude	1555
Bordeaux	Barrique	14033	Rome	Wine Barille	2560
Bremen	Stubgens	194·5	"	Oil "	2240
Canaries	Arrobas	949	"	Boccali	80
Constantinople	Almud	319	Russia	Weddras	752
Copenhagen	Anker	2355	"	Kunkas	94
Florence	Oil Barille	1946	Scotland	Pint	103·5
"	Wine "	2427	Sicily	Oil Caffiri	662
France	Litre	61·07	Spain	Azumbres	22·5
Geneva	Setier	2760	"	Quartillos	30·5
Genoa	Wine Barille	4530	Sweden	Eimer	4794
"	Pinte	90·5	"	Kanna	159·57
Hamburgh	Stubgen	221	Trieste	Orne	4007
Hanover	"	231	Tripoli	Mattari	1376
Hungary	Eimer	4474	Tunis	Oil "	1157
Leghorn	Oil Barille	1942	Venice	Secchio	628
Lisbon	Almude	1040	Vienna	Eimer	3452
Malta	Caffiri	1270	"	Maas	86·33

FOREIGN DRY MEASURES COMPARED WITH AMERICAN.

Places.	Measures.	Cub. In.	Places.	Measures.	Cub. In.
Alexandria	Rebele	9587	Malta	Salme	16930
"	Kislos	10418	Marseilles	Charge	9411
Algiers	Tarrie	1219	Milan	Moggi	8444
Amsterdam	Mudde	6596	Naples	Tomoli	3122
"	Sack	4947	Oporto	Alquiere	1051
Antwerp	Viertel	4705	Persia	Artaba	4013
Azores	Alquiere	731	Poland	Zorzec	3120
Berlin	Scheffel	3180	Riga	Loop	3978
Bremen	"	4339	Rome	Rubbio	16904
Candia	Charge	9288	"	Quarti	4226
Constantinople	Kislos	2023	Rotterdam	Sach	6361
Copenhagen	Toende	8489	Russia	Chetwert	12448
Corsica	Stajo	6014	Sardinia	Starelli	2988
Florence	Stari	1449	Scotland	Firlot	2197
Geneva	Coupes	4739	Sicily	Salme gros	21014
Genoa	Mina	7382	"	" generale	16886
Greece	Medimni	2390	Smyrna	Kislos	2141
Hamburgh	Scheffel	6426	Spain	Catrize	41269
Hanover	Malter	6868	Sweden	Tunna	8940
Leghorn	Stajo	1501	Trieste	Stari	4521
"	Sacco	4503	Tripoli	Caffiri	19780
Lisbon	Alquiere	817	Tunis	"	21855
"	Fanega	3268	Venice	Stajo	4945
Madeira	Alquiere	684	Vienna	Metzen	3753
Malaga	Fanaga	3783			

FRENCH MEASURES OF SOLIDITY.

FRENCH.	1 Cubic Foot	=	2093·470 U. S. inches.
	Decistre	=	3·5375 cubic feet.
	Stere (a cubic metre)	=	25·375 "
	Decastere	=	353·75 "
	1 Stere	=	61074·664 cubic inches.

For the *Square* and *Cubic Measures* of other countries, take the length of the measure in Table, page 72, and square or cube it as required.

ENGLISH AND FRENCH MEASURES OF WEIGHT.

BRITISH.	1 troy Grain	=	·003961 cubic inches of distilled water.
	1 troy Pound	=	22·815689 cubic inches of water.
FRENCH.	*Old System.*—1 Grain . . .	=	0·8188 grains troy.
	1 Gros . . .	=	58·9548 "
	1 Ounce . . .	=	1·0780 oz. avoirdupois.
	1 Livre . . .	=	1·0780 lbs. "
	New System.—Milligramme . .	=	·01543 troy grains.
	Centigramme . .	=	·15433 "
	Decigramme . .	=	1·54331 "
	Gramme . . .	=	15·43315 "
	Decagramme . .	=	154·33159 "
	Hecatogramme . .	=	1543·3159 "
	1 Millier = 1000 Kilogrammes	=	1 ton sea weight.
	1 Kilogramme . .	=	2·204737 lbs. avoirdupois.
	1 Pound avoirdupois .	=	0·4535685 Kilogramme
	1 Pound troy .	=	0·3732223 "

NOTE.—*In the new French system, the values of the base of each measure, viz., Metre, Litre, Stere, Are, and Gramme, are decreased or increased by the following words prefixed to them. Thus,*

Milli	expresses the	1000th part.	Hecato	expresses	100 times the value.
Centi	"	100th "	Chilio	"	1000 "
Deci	"	10th "	Myrio	"	1000 "
Deca	"	10 times the value.			

FOREIGN WEIGHTS COMPARED WITH AMERICAN.

Places.	*Weights.*	*Number equal to 100 avoirdupois pounds.*	*Places.*	*Weights.*	*Number equal to 100 avoirdupois pounds.*
Aleppo	Rottoli	20·46	Hanover .	Pound	93·20
" . .	Oke	35·80	Japan . .	Catty	76·92
Alexandria .	Rottoli	107·	Leghorn .	Pound	133·56
Algiers . .	"	84·	Leipsic . .	" (common)	97·14
Amsterdam	Pound	91·8	Lyons . .	" (silk)	98·81
Antwerp .	"	96·75	Madeira . .	"	143·20
Barcelona .	"	112·6	Mocha . .	Maund	33·33
Batavia . .	Catty	76·78	Morea . .	Pound	90·79
Bengal . .	Seer	53·57	Naples . .	Rottoli	50·91
Berlin . .	Pound	96.8	Rome . .	Pound	133·69
Bologna . .	"	125·3	Rotterdam .	"	91·80
Bremen . .	"	90·93	Russia . .	"	110·86
Brunswick .	"	97·14	Sicily . .	"	142·85
Cairo . .	Rottoli	105.	Smyrna . .	Oke	36·51
Candia . .	"	85·9	Sumatra . .	Catty	35·56
China . .	Catty	75·45	Sweden . .	Pound	106·67
Constantinople	Oke	35·55	" .	"	120·68
Copenhagen	Pound	90·80	Tangiers .	" (miner's)	94·27
Corsica . .	"	131·72	Tripoli . .	Rottoli	89·28
Cyprus . .	Rottoli	19·07	Tunis . .	"	90·09
Damascus .	"	25·28	Venice . .	Pound (heavy)	94·74
Florence .	Pound	133·56	" . .	" (light)	150.
Geneva . .	" (heavy)	82·35	Vienna . .	"	81·
Genoa . .	" "	92·86	Warsaw . .	"	112·25
Hamburgh .	" "	93·63			

Component parts of the Navy Ration.	Pounds.						Ounces		Either.					Fractions of a Pint.			
	Beef.	Pork.	Flour.	Raisins or dried fruit.	Pickles.	Rice.	Biscuits.	Sugar.	Tea.	Coffee.	Cocoa.	Butter.	Cheese.	Beans.	Molasses.	Vinegar.	Spirits.
Sunday.	1		½	¼			14	2	¼	1	1						¼
Monday.		1					14	2	¼	1	1			½			¼
Tuesday.	1					½	14	2	¼	1	1	2	2				¼
Wednesday.		1			¼		14	2	¼	1	1			½			¼
Thursday.	1		½	¼			14	2	¼	1	1						¼
Friday.	1					½	14	2	¼	1	1	2	2		½		¼
Saturday.		1			¼		14	2	¼	1	1			½		½	¼
Total Rations per week.	4	3	1	½	½	1	98	14	1¾	7	7	4	4	1½	½	½	1¾

By the Differential Calculus we ascertain the simultaneous progress of variable quantities depending on one another. The variable quantities are designated by the last letters u, v, x, y, z, and the constant quantities by the first, a, b, c, e, f, of the alphabet. The letter d is placed before variables to denote the instantaneous progress of that quantity, as dx, and called the differential of x. $d\cdot$ reads differential. Let the side of a square be denoted by x and the area by z; when x increases uniformly, z will increase more rapidly. When $x=1$, $z=1$, but when $x=2$, $z=4$. When we know the instantaneous increase of x, what will be that of z? If we add, say only a point to the side x, there will be added two lines or $2x$ to the square. We know that $z=x^2$, the $d\cdot$ or increment of the square will be $dz=2x\,dx$, of which dx is the point added to x and $2x$ is the two lines added to the square, called the differential coefficient. Let v denote the volume of a cube, and x its side, we have $v=x^3$ and $dv=3x^2dx$, which shows that if a point dx is added to x there will be $3x^2$ or three squares added to the cube.

The $d\cdot$ of any power of a variable is equal to the power diminished by 1, multiplied by the primitive exponent and the product by the $d\cdot$ of the variable. The $d\cdot$ of a constant is $=o$. When the constant is a factor to the variable it appear unchanged in the $d\cdot$ coefficient, but when a term it disappears.

I. The $d\cdot$ of length u of any line defined by a formula of rectangular co-ordinates x and y, is $du=\sqrt{dx^2+dy^2}$. II. The $d\cdot$ of area z of any plan figure bounded by a curved line and rectangular co-ordinates in $dz=y\,dx$, $y=$ordinate, $x=$abscissa. III. The $d\cdot$ of solidity v of any figure bounded by a plan rotating round its abscissa x, is $dv=\pi y^2 dx$, $y=$ordinate of the outer line of the plan. IV. The $d\cdot$ of surface z of any solid bounded by a plan rotating round its abscissa x, is $dz=2\pi y\,du$, in which $u=$length of the outer edge of the plan.

Successive $d\cdot s$ is when the first $d\cdot$ coeff. is considered a function of a new function.

$u=ax^4$. 1st. $d\cdot$ coeff. $\frac{du}{dx}=4ax^3$, 2nd. $d\cdot$ coeff. $d\left(\frac{du}{dx}\right)=\frac{d^2u}{dx^2}=12ax^2$,

3d. $d\cdot$ coeff. $d\left(\frac{d^2u}{dx^2}\right)=\frac{d^3u}{dx^3}=24ax$, etc., etc.

d^2y means the second, d^3y the third $d\cdot$ coefficient of u. dx^2 means the square of the $d\cdot$ of x, etc.

Example 1. The diameter of a sphere increases at a rate of $dx=2{\cdot}31$ inches per second, when $x=9{\cdot}5$ inches, at what rate ($dv=$? does the volume v increase? $v=\frac{\pi x^3}{6}=0{\cdot}523x^3$. $dv=0{\cdot}523\times3x^2dx=1{\cdot}569\times2{\cdot}31$ $=327{\cdot}1$ cubic inches, the answer.

Example 2. It is found that the displacement of a ship increases as $x^{1\cdot5}$ the draft of water. At the load draft $a=18$ feet the displacement is $T=2000$ tons. Required the displacement $t=$?) when $x=12$ ft. and how much ($dt.$?) can the vessel be loaded per $dx=1$ inch or 1–12 foot, at that draft.

$$t=\frac{x^{1\cdot5}T}{a^{1\cdot5}}=\frac{12^{1\cdot5}\times2000}{18^{1\cdot5}}=1088{\cdot}6 \text{ tons, at } x=12 \text{ feet draft.}$$

$$dt=\frac{1{\cdot}5\,Tx^{\frac{1}{2}}dx}{a^{1\cdot5}}=\frac{1{\cdot}5\times2000\times\sqrt{12}}{18^{1\cdot5}\times12 \text{ in.}}\,11{\cdot}34 \text{ tons per inch.}$$

The following page contains the differentials of formulas and trigonometrical functions. l means the Naperian logarithm. The common logarithm log. multiplied by 2·302585 gives the Naperian logarithm l.

FORMULAS.		DIFFERENTIALS.	
$y=x$		$dy=dx,$	1
$y=ax^2$		$dy=2\,a\,x\,dx,$	2
$y=x^n$		$dy=u\,x^{n-1}\,dx,$	3
$3\,a\,b\,x^3$	$=$	$9\,a\,b\,x^2\,dx,$	4
$4\,a\,b^2\,x^n$	$=$	$4\,n\,a\,b^2\,x^{n-1}\,dx,$	5
$a+x^3$	$=$	$3\,x^2\,dx,$	6
$(a+b\,x^2$	$=$	$2\,x\,(a+b)\,dx,$	7
$6a\,b^4\,x^3-c$	$=$	$18\,a\,b^4\,x^2\,dx,$	8
$x+3\,z^2-v$	$=$	$dx+6\,z\,dz-dv,$	9
$6x^3+4ax^2-3x$	$=$	$(18\,x^2+8\,a\,x-3)dx,$	10
$x\,v^2$	$=$	$v\,dx+2\,x\,v\,dv,$	11
$x\,v\,z$	$=$	$x\,v\,z\left(\frac{dx}{x}+\frac{dv}{v}+\frac{dz}{z}\right)$	12
$x(x^2-x\,b^2)$	$=$	$(3\,x^2-b^2\,x)\,dx,$	13
$\frac{x^2}{v}$	$=$	$\frac{2\,x\,v\,dx-x^2\,dv}{v^2},$	14
$\frac{a}{x}$	$=$	$\frac{a\,dx}{x^2}$	15
$\frac{a}{x^n}$	$=$	$\frac{n\,a\,x^{n-1}\,dx}{x^{2n}}$	16
$(a+\sqrt{x})^3$	$=$	$\frac{3(a+\sqrt{x})^2dx}{2\sqrt{x}}$	17
$\left(a+\sqrt[n]{x}\right)^m$	$=$	$\frac{m\left(a+\sqrt[n]{x}\right)^{m-1}dx}{n\sqrt[n]{x}}$	
$\frac{1}{4(a-x)^n}$	$=$	$\frac{dx}{(a-x)^{n+1}}$	19
$\frac{2\sqrt{2\,a\,x-x^2}}{x}$	$=$	$\frac{2\,a\,dx}{x\sqrt{2\,ax-x^2}}$	20
a^x	$=$	$a^x\,l\cdot a\,dx,$	21
$d\cdot l\cdot x$	$=$	$\frac{dx}{x}$	22
$x\,l\cdot x$	$=$	$(1+l\cdot x)dx,$	23
$\frac{l\cdot x}{x^n}$	$=$	$\frac{(1-l\cdot x)dx,}{x^{n+1}}$	24
$\frac{x}{l\cdot x}$	$=$	$\frac{(l\cdot x-1)dx}{(l\cdot x)^2}$	25
$\frac{a\,y}{\sqrt{x^2+y^2}}$	$=$	$\frac{a\,y\,x\,dx-a\,x^2\,dy}{\sqrt{(x^2+y^2)^3}}$	26
$\frac{a-2\,b\,x}{(a+b\,x)^2}$	$=$	$\frac{2\,b^2\,x\,dx}{(a+bx)^3}$	27
$\sqrt{x}=x^{\frac{1}{2}}$	$=$	$\frac{dx}{2\sqrt{x}}$	28
$(a\,x+x^2)^n$	$=$	$n(a\,x+x^2)^{n-1}(a+2\,x)dx$	29
$\sqrt{a^2+bx^2}$	$=$	$\frac{b\,x\,d\,x}{\sqrt{a^2+b\,x^2}}$	30
$d^2(a\,x^3)$	$=$	$6\,a\,x\,dx^2,$	31
$d^3(a\,x^3)$	$=$	$6\,a\,dx^3,$	32
$d^4(a\,x^3)$	$=$	$o\,b\,a\,x^{o-1}dx^4=o,$	33
$\sin.\,v$	$=$	$+\cos.\,v\,d\,v,$	34
cas. v	$=$	$-\sin.\,v\,dv,$	35
$\tan.\,v$	$=$	$+\frac{dv}{\cos.^2 v}$	36
$\cot.\,v$	$=$	$-\frac{dv}{\sin.^2 v}$	37
$\sec.\,v$	$=$	$+\frac{\cos.\,v\,dv}{\cos.^2 v}$	38
cosec. v	$=$	$-\frac{\cos.\,v\,dv}{\sin.^2 v.}$	39
Tant. for any curve		$t=y\sqrt{1+\frac{dx^2}{dy^2}}$	40

The Integral Calculus is the reverse of the Differential, or to find the original formula of a given differential. The symbol $\int$ is placed before the $d\cdot$ to denote that the integral is to be taken out of it, or that the original formula is to be found.

The $d\cdot$ of $a\,x^3 = 3\,a\,x^2\,dx$, and $\int 3\,a\,x^2\,dx = \dfrac{3\,a\,x^{2+1}}{3} = a\,x^3$.

Rule to find the integral. Add 1 to the exponent of the variable x in the $d\cdot$, divide the $d\cdot$ by the new exponent, dx will disappear, and the quotient is the integral. The integral $\int$ does not effect a constant. A constant term in a formula disappears in its $d\cdot$, consequently any integral may have a constant term, whose value is determined by making the variable in the integral $= o$, when the first member in the formula will be a constant. It is therefore customary to add a constant C to the integral. When it is known that the first member is $= o$ at the same time the variable in the $\int$ is $= o$, then $C = o$. When a differential is to be integrated between two limits of the invariable, say $x = a$ and $x = b$, it is indicated by $\int_a^b$. or $\int_a^b 3\,c\,x^2\,dx = c(b^3 - a^3)$.

Successive differentials are accompanied with the same order of integrals, as $\int\int 6\,x\,dx^2 = \int 3\,x^2\,dx = x^3$.

The integrals of the differentials gives the formula for the problem.

Example 3. It is required to find by the calculus a formula for the area z of a rightangled triangle. Proposition II, page 78. $dz = y\,dx$, the formula for the hypothenuse is $y = a\,x$, $dz = a\,x\,dx$, $z = \int a\,x\,dx = \dfrac{a\,x^2}{2} = \dfrac{y\,x}{2}$, or the area z is half the rectangle of the sides x and y.

Example 4. Find a formula for the convex surface z of a cone, whose side is u, and $r =$ radius of the base? Prop. IV, page 78. $dz = 2\,\pi\,r\,du$, and $z = \int 2\,\pi\,r\,du = \pi\,r\,u$, the answer.

Example 5. Find a formula for the area z of a circle, when it is known that the circumference $y = 2\,\pi\,x$? Prop. II, page 78. $dz = y\,dx = 2\,\pi\,x\,dx$, and $z = \int 2\,\pi\,x\,dx = \pi\,x^2$ the answer, $x =$ radius of the circle.

Example 6. Find a formula for the area z of a parabola of $x =$ abscissa or height, and $y =$ ordinate, or half the base? Formula for a parabola $y = \sqrt{p\,x}$, in which $p =$ the constant parametric diameter, or $p = \dfrac{y^2}{x}$. $x = \dfrac{y^2}{p}$, $dx = \dfrac{2\,y\,dy}{p}$. Prop. II, $dz = y\,dx = \dfrac{2\,y^2\,dy}{p}$, and $z = \displaystyle\int \frac{2\,y^2\,dy}{p} = \frac{2\,y^3}{3\,p} = \frac{2\,x\,y}{3}$, the answer, or the area of a parabola is ⅔ of the base by the height.

Example 7. Find a formula for the volume v of a paraboloid? Prop. III. $dv = \pi\,y^2\,dx = \dfrac{2\,\pi\,y^3 dy}{p}$, and $v = \displaystyle\int \frac{2\,\pi\,y^3\,dy}{p} = \frac{\pi\,y^4}{2p}$, but $p = \dfrac{y^2}{x}$ and $v = \dfrac{\pi}{2} y^2\,x$, the answer.

V. The center of gravity s from the origin of x, of any plan figure bounded by a curved line and rectangular co-ordinates is $s = \dfrac{\int x\,y\,ds}{z}$, when z area of the plan.

VI. The center of gravity s from the origin of x of any solid figure is $s = \dfrac{\int x\,z\,dx}{v}$, when $z =$ ordinate cross section and v volume of the same.

Example 8. Find a formula for the centre of gravity ($s = ?$) of a cone. The ordinate cross-section $z = \pi y^2$, and $v = \dfrac{\pi}{3} y^2\,x$, when $x =$ height and y radius of the base of the cone. Prop. VI. $s = \dfrac{\int x\,z\,dx}{v} = \dfrac{3 \int \pi\,x\,y^2\,dx}{\pi\,y^2\,x}$

As the center of gravity is not influenced by the proportion of x and y, we can make $y = x$, when $s = \dfrac{3 \int \pi\,x^3\,dx}{\pi\,x^3} = \dfrac{3\,\pi\,x^4}{4\,\pi\,x^3} = ¾\,x$, from the top.

DIFFERENTIALS.	INTEGRALS.	
$\int dx = x + c$	$\int x\,dx = \frac{x^2}{2} + C,$	1
$\int 4\,ax^3\,dx = 4\,a\int x^3\,dx =$	$a\,x^4 + C,$	2
$\int x^n\,dx, \quad =$	$\frac{x^{n+1}}{n+1} + C,$	3
$\int\sqrt{x\,dx}\int x^{\frac12}\,dx =$	$\frac{2\,x^{1\frac12}}{3} + C,$	4
$\int\frac{dx}{\sqrt{x}} = \int x^{-\frac12}\,dx =$	$2\sqrt{x} + C,$	5
$\int\frac{dx}{x^2} = \int x^{-1}\,dx =$	$l\cdot x + C,$	6
$\int\frac{dx}{x^2} = \int x^{-2}\,dx =$	$-\frac{1}{x} + C,$	7
$\int\frac{dx}{x^3} = \int x^{-3}\,dx =$	$-\frac{1}{2\,x^2} + C,$	8
$\int\left(a\,x^3 + \frac{b}{2\sqrt{x}}\right)dx =$	$\frac{a\,x^4}{4} + b\sqrt{x} + C,$	9
$\int\frac{a\,dx}{x} \quad =$	$a\,l\cdot x + C,$	10
$\int\frac{b\,dx}{a+x} \quad =$	$b\,l\cdot(a+x) + C,$	11
$\int\frac{3\,a\,x^2\,dx}{b + a\,x^3} \quad =$	$l\cdot(b + a\,x^3) + C,$	12
$\int axdx + 3x^2dx - b^2dx =$	$\frac{ax^2}{2} + x^3 - b^2x + C,$	
$\int(a^2+b^2)\,dx \quad =$	$x(a^2+b^2) + C,$	14
$\int(ax - 2x^2)^2dx =$	$x^3\left(\frac{a^2}{3} - ax + \frac{4x^2}{5}\right) + C,$	
$\int 3(a\,x - x^2)^2(a - 2\,x(dx =$	$(ax - x^2)^3 + C,$	
$\int\frac{n(x^{n-1}dx)}{\sqrt{a+x^n}} \quad =$	$\sqrt{a^2+x^n} + C,$	17
$\int\frac{2\,a\,dx}{a^2 - x^2} \quad =$	$l.\frac{a+x}{a-x} + C,$	18
$\int\sqrt{a^2 \mp x^2}\,dx =$	$\frac{x}{2}\sqrt{a^2 \mp x^2} + \frac{a^2}{4}\,l\cdot(x + \sqrt{a^2 - x^2}),$	
$\int\sqrt{a \mp b\,x}\cdot dx \quad =$	$\frac{2}{3}b(\sqrt{a \mp bx})^3 + C,$	20

DIFFERENTIALS.	INTEGRALS.	
$\int\frac{dx}{\sqrt{a^2+x^2}} \quad =$	$l\cdot(x + \sqrt{a^2+x^2}),$	21
$\int_a^b 3\,m\,x^2\,dx \quad =$	$m\,b^3 - m\,a^3,$	22
$\int_a^b m\,x\,dx \quad =$	$\frac{m}{2}(b^2 - a^2),$	23
$\int_0^\infty\frac{dx}{a^2+x^2} \quad =$	$\frac{\pi}{2\,a},$	24
$\int_0^a\frac{dx}{\sqrt{a^2-x^2}} \quad =$	$\frac{\pi}{2},$	25
$\int_a^b = -\int_b^a$	$\int_a^c = \int_a^b + \int_b^c$	26
$\int\sin.\,x\,dx \quad = -$	$\cos.x + C,$	27
$\int\cos.\,x\,dx \quad =$	$\sin.x + C,$	28
$\int\tan.\,x\,dx \quad =$	$-\,l\cdot\cos.x + C,$	29
$\int\cot.\,x\,dx \quad =$	$-\,l\cdot\sin.x + C,$	30
$\int\frac{dx}{\sin.x} \quad =$	$l\cdot\tan.\frac{x}{2} + C,$	31
$\int\frac{dx}{\cos.x} \quad =$	$l\cdot\tan.\left(\frac{\pi}{4} + \frac{x}{2}\right) + C,$	32
$\int\sin.x\,\cos.x\,dx =$	$\frac{1}{2}\sin^2.\,x + C,$	33
$\int^\infty\frac{\sin.b\,x}{x}dx \quad =$	$\frac{\pi}{2}$	34
$\int_0^\infty\frac{\cos.b\,x}{x}dx \quad =$	$\infty,$	35
$\int\frac{dt}{1+t^2} =$	circle arc of which t=tant.	
$\int\frac{dx}{\sqrt{2\,x - x^2}} \quad =$	circle arc of which x = sin.versus.	37
$\int\int\int 6adx^3 = \int\int 6axdx^2 = \int 3ax^2dx = ax^3 + C$		
$\int\int 2\,(a+b)\,dx^2 \quad =$	$(a+b)x^2 + C,$	39
$\int\int 2\,v^2\,dx^2 + 8\,v\,x\,dx\,dv + 2\,x^2dv^2 = x\,v^2,$		40

Two variable quantities x and y depended on one another, to find the value of one, when the other is a *maxima* or *minima*.

$$\left.\begin{matrix} x \\ y \end{matrix}\right\} \text{is a maxima or minima when its first differential coefficient} \left\{\begin{matrix} \frac{dx}{dy} = o. \\ \frac{dy}{dx} = o. \end{matrix}\right.$$

When the second $d\cdot$ coef. $\frac{d^2y}{dx^2}$ is positive, y is a minimum, and when negative y is a maximum. The variables may have both maximums and minimums, as formulas will indicate.

Example 1. Find the value of x when y is a maximum or minimum, in the formula $y = x^3 - 12x + 22$? $dy = (3x^2 - 12)\,dx$, $\frac{dy}{dx} = 3x^2 - 12 = o.$

Of which $x = \sqrt{\frac{12}{3}} = 2$ the answer. $\frac{d^2y}{dx^2} = 6x$, which is positive, consequently $y = 2^3 - 12 \times 2 + 22 = 6$, a minimum, when $x = 2$.

Example 2. It is required to cut out the strongest possible beam of height h and breadth b, from a log of diameter D, fig. 221 page 174? The strength of a beam is in proportion to bh^2 which is to be a maximum.

$D^2 = b^2 + h^2$, $h^2 = D^2 - b^2$, $bh^2 = b(D^2 - h^2)$, $d(bh^2) = (D^2 - 3b^2)\,db$.

$\frac{d(bh^2)}{db} = D^2 - 3b^2 = o$, of which the breadth $b = D\sqrt{\frac{1}{3}} = 0{\cdot}577\ D$, and height $h = \sqrt{D^2 - \frac{1}{3}} = D\sqrt{0{\cdot}6666} = 0{\cdot}8164\ D$, the answer. The second $d\cdot$ coef. $\frac{d^2(bh^2)}{db^2} = -6b$, which is negative, and therefore bh^2 is a maximum when $b = 0{\cdot}577\,D$.

Example 3. It is required to know the proportion of heighth h and diameter D of a cylinder, having the greatest cubic containt v, with the smallest surface z including top and bottom? $z = \frac{\pi D^2}{2} + \pi D h = \frac{2v}{h} + \pi D h$, which is to be a minimum. Set $v = 1$ and $D = 1$, then $z = \frac{2}{h} + \pi h$, and $dz = \left(\pi - \frac{4}{h^2}\right) dh$, $\frac{dz}{dh} = \pi - \frac{4}{h^2} = o$, when $h = \sqrt{\frac{4}{\pi}} = 1{\cdot}1284\,D$, the answer. The second $d\cdot$ coef. $\frac{d^2z}{dh^2} = +\frac{4 \times 2}{h^4}$ which is positive, and z a minimum when $h = 1{\cdot}1284\ D$.

Maclaurin's Theorem.

Maclaurin's Theorem, explains how to develop into a series a function with one variable, as

$$u = (u) + \frac{x}{1}\left(\frac{du}{dx}\right) + \frac{x^2}{2}\left(\frac{d^2u}{dx^2}\right) + \frac{x^3}{2 \times 3}\left(\frac{d^3u}{dx^3}\right) \dots + \frac{x^n}{2 \times 3 \times \dots n}\left(\frac{d^nu}{dx^n}\right) \text{ etc.}$$

where the factors in the parenthesis is that which it assumes when $x = o$.

The function $u = \frac{1}{a + x}$ developed into a series will be

$$\frac{1}{a + x} = \frac{1}{a} - \frac{x}{a^2} + \frac{x^2}{a^3} - \frac{x^3}{a^4} \dots \frac{x^n}{a^{n+1}}. \text{ etc.}$$

Taylor's Theorem.

Taylor's Theorem, explains how to develop into a series a function of the sum or difference of two variable as $u = x \pm y$.

$$F(x \pm y) = u \pm \frac{du}{dx} y + \frac{d^2u}{dx^2} \cdot \frac{y^2}{2} \pm \frac{d^3u}{dx^3} \cdot \frac{y^3}{2 \times 3} + \dots \frac{d\,u}{dx_n} \cdot \frac{y^n}{2 \times 3 \dots \times n}.$$

where u represents the value of the function when $y = o$.

Interpolation is to insert numerical values between given datas, for constructing tables or empirical formulas expressing the probable relative variation of quantities. Let x and y be two variable quantities depending on one another and measured in simultaneous stages of their progress, as

$$x_1 \ x_2 \ x_3 \ x_4 \text{ and } x_5$$

$$y_1 \ y_2 \ y_3 \ y_4 \text{ and } y_5$$

We have $y = Ay_1 + By_2 + Cy_3 + Dy_4 + Ey_5 + \&c.$ - - - - - 1

2, 3, 4, 5 given datas.

$$A = \frac{(x-x_2)\,(x-x_3)\,(x-x_4)\,(x-x_5)}{(x_1-x_2)\,(x_1-x_3)\,(x_1-x_4)\,(x_1-x_5)}$$

$$B = \frac{(x-x_1)\,(x-x_3)\,(x-x_4)\,(x-x_5)}{(x_2-x_1)\,(x_2-x_3)\,(x_2-x_4)\,(x_2-x_5)}$$

2 >

$$C = \frac{(x-x_1)\,(x-x_2)\,(x-x_4)\,(x-x_5)}{(x_3-x_1)\,(x_3-x_2)\,(x_3-x_4)\,(x_3-x_5)}$$

3 >

$$D = \frac{(x-x_1)\,(x-x_2)\,(x-x_3)\,(x-x_5)}{(x_4-x_1)\,(x_4-x_2)\,(x_4-x_3)\,(x_4-x_5)}$$

4 >

$$E = \frac{(x-x_1)\,(x-x_2)\,(x-x_3)\,(x-x_4)}{(x_5-x_1)\,(x_5-x_2)\,(x_5-x_3)\,(x_5-x_4)}$$

5 >

Number of given datas.

The values of the coefficients A, B, C, D, and E, with their given datas, inserted in formula 1 gives an empirical formula for the variation of x and y. The number of observations or given datas of x and y should be one more than the order of progression. In arithmetical progression two observations are sufficient for a correct formula. For all curves in the conic sections, or others which are of the second order, there should be at least three observations. Pressure of steam progresses with the temperature in the 6th order, for which requires seven observations to make a correct formula. When the order of progression is not known, the more observations gives the most correct result.

Example. Let y represent the boiling-point of salt water and x the percentage of salt in solution. It is found in three experiments,

that $x_1 = 3$, $x_2 = 18$, $x_3 = 36$ per cent. salt.
when $y_1 = 213^\circ$, $y_2 = 219^\circ$, $y_3 = 226^\circ$ boiling-point.

Find a formula that will give any intermediate value of x and y?

$$A = \frac{(x-18)(x-36)}{(3-18)(3-36)}, \quad B = \frac{(x-3)(x-36)}{(18-3)(18-36)}, \quad C = \frac{(x-3)(x-18)}{(36-3)(36-18)},$$

$$y = 213\,A + 219\,B + 226\,C. \qquad y = 0{\cdot}40729x + 211{\cdot}78.$$

GEOMETRY.

DEFINITIONS.

Demonstration is a course of reasoning by which a truth is established. It consists of,

Thesis, the truth to be established, and,

Hypothesis, the foundation for the demonstration.

Axiom is that which is self-evident and requires no demonstration.

Theorem is something to be proved by demonstration.

Postulate is something to be *done*, but is self evident and requires no demonstration.

Problem is something proposed to be *done*, and requires demonstration.

Proposition is either a Theorem or a Problem.

Corolary is an obvious conseqence deduced from something that has gone before.

Scolium is a remark on preceding propositions, commonly demonstrated by algebraical formulæ.

Lemma is something premised for a following demonstration.

Geometrical Quantities.

Point is a position, but no magnitude.

A Line is length, without breadth or thickness.

A Straight Line is the shortest distance between two points.

Curved line is a length which in every point changes its direction.

Superficies, Surface, Area, is that which has length and breadth, but no thickness.

Plane surface is a plane which coincides with a straight line in every direction.

Curved surface is a plane which coincides with a curved line.

Solid has length, breadth and thickness.

Circle.

Circle, Cirumference, Periphery, is a curved line drawn on a plane surface, and bounded at a common distance from one point in the plane, (centre.)

Radius is a line* drawn from the centre in a circle to the periphery.

Diameter is a line drawn through the centre to the periphery, or the longest line in a circle.

Chord is any line extending its both ends to the periphery of a circle. and does not go through the centre.

Arc is a part of a periphery.

Circle plane, is a plane surface bounded within a circumference.

Sector is a part of a circle-plane bounded within an arc and two radii.

Segment is a part of a circle plane bounded within a chord and an arc.

Zone is a part of a circle included between two parallel chords.

Lune is the space between the intersecting arcs of two eccentric circles.

Oval is a round figure having one long and one short diameter at right angles to one another.

Semicircle is a half circle.

Quadrant is a quarter of a circle.

Angles.

Angle is the opening or inclination of two lines which meet in one point.

If two radii being drawn from the extremities of a circle arc, to the centre; the arc, is a measure of the angle at the centre.

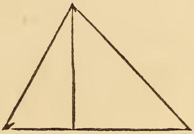
Right angle is when the opening is a quarter of a circle.

Acute angle is less than a right angle.

Obtuse angle is greater than a right angle.

* *Line* by itself means a straight line.

Triangles.



Triangle is a figure of three sides.
Equilateral Triangle has all its sides equal.
Isosceles Triangle has two of its sides equal.
Scalene Triangle has all its sides unequal.
Right-angled triangle has one right angle.
Obtuse-angled triangle, has one obtuse angle.
Acute-angled triangle has all its angles acute.

Quadrangles.

Quadrangle is a figure of four sides.

Parallelogram having its opposite sides parallel, and the opposite angles equal.

Square, having its four sides equal, and four right angles.

Rectangle, having its opposite sides equal, and four right angles.

Rhombus, having four equal sides, and opposite angles equal but not right.

Rhomboid, same as a parallelogram.

Trapezium, having four unequal sides.

Trapezoid, having only two opposite sides parallel.

Gnomon is the space included between the lines forming two similar parallelograms, of which the smaller is inscribed in the larger, so as to have one common angle.

Polygons.

Polygons are plane and rightlined figures.

Regular Polygons are plane figures which inscribe, or circumscribe a circle, and their sides being equal. Polygons are named according to their number of sides, thus,

Trigon	has	three	sides.	*Octagon*	has	eight	sides.
Tetragon	"	four	"	*Nonagon*	"	nine	"
Pentagon	"	five	"	*Decagon*	"	ten	"
Hexagon	"	six	"	*Undecagon*	"	eleven	"
Heptagon	"	seven	"	*Dodecagon*	"	twelve	"

For properties of Polygons see page 103.

Solids.

Sphere is a solid bounded within a half circle rotating round its diameter.

Spherical segment, (zone) is a part of a sphere cut off by a plane.

Spheroid is a sphere flatted or longed at two opposite sides ; as our earth is flatted at the poles, and having one diameter shortest; an egg is longed, and having one diameter longest.

Spindle is a solid bounded within a curved line rotating round its base.

Cylinder is a solid bounded within a rectangle rotating round one of its sides, (axis.) A cylinder has a circle plane to its base.

Cone is bounded within a right-angled triangle rotating round one of its sides that forms the right angle.

Ungula is the bottom part of a Cone or Cylinder, cut off by a plane passing obliquely through the base.

Cube is bounded within six squares.

Parallelopiped is bounded within six parallelograms.

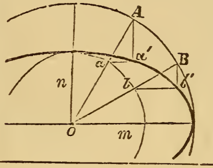
Prism is a solid described by a rightlined plane moving in a straight line, so that the plane forms an angle to its direction line.

Prismoid is a prism cut obliquely at the ends.

Pyramid is bounded between a rightlined plane, and one point at a distance from the plane. The sides of the rightlined plane, are bases of triangles determinating at the aforesaid point, (vertex.)

Perimeter is the sum of all the sides in a figure, plane or solid.

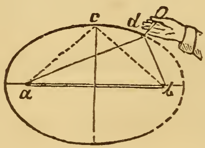
Polyhedrons. See page 95, for properties and names of the five regular polyhedrons.



37.

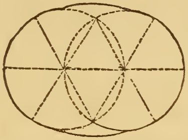
To construct an ellipse.

With o as a centre, draw two concentric circles with diameters equal to the long and short axes of the desired ellipse. Draw from o any number of radii, A, B, &c. Draw the line $B\ b'$ parallel to n and $b\ b'$ parallel to m, then b' is a point in the desired ellipse.



38. *To draw an ellipse with a string.*

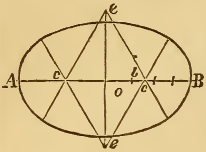
Having given the two axes, set off from c half the great axis at a and b, which are the two focuses in the ellipse. Take an endless string as long as the three sides in the triangle a, b, c, fix two pins or nails in the focuses one in a, and one in b, lay the string round a, and b, stretch it with a pencil d, which then will describe the desired ellipse.



39.

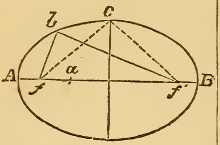
To draw an ellipse by circle arcs.

Divide the long axis into three equal parts, draw the two circles and where they intersect one another are the centres for the tangent arcs of the ellipse as shown by the figure.



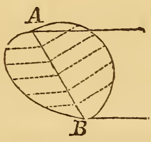
40. *To draw an ellipse by circle arcs.*

Given the two axes, set off the short axis from A to b, divide $b\ B$ into three equal parts, set off two of these parts from o towards c and c which are the centres for the ends of the ellipse. Make equilateral triangles on c c, when $e\ e$ will be the centres for the sides of the ellipse. If the long axis is more than twice the short one, this construction will not make a good ellipse.



41. *To construct an ellipse.*

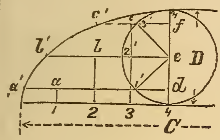
Given the two axes, set off half the long axis from c to $f\ f'$, which will be the two focuses in the ellipse. Divide the long axis into any number of parts, say a to be a division point. Take $A\ a$ as radius and f as centre and describe a circle arc about b, take $a\ B$ as radius and f' as centre describe another circle arc about b, then the intersection b is a point in the ellipse, and so the whole ellipse can be constructed.



42.

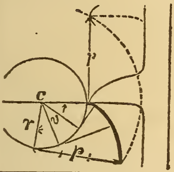
To draw an ellipse that will tangent two parallel lines in A and B.

Draw a semicircle on $A\ B$, draw ordinates in the circle at right angle to $A\ B$, the corresponding and equal ordinates for the ellipse to be drawn parallel to the lines, and thus the elliptic curve is obtained as shown by the figure.



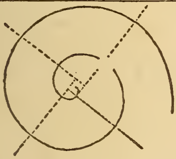
43. *To construct a cycloid.*

The circumference $C = 3{\cdot}14\,D$. Divide the rolling circle and base line C into a number of equal parts, draw through the division point the ordinates and abscissas, make $a\,a' = 1'\,d$, $b\,b' = 2'\,e$, $c\,c' = 3'\,f$, then a' b' and c' are points in the cycloid. In the *Epicycloid* and *Hypocycloid* the abscissas are circles and the ordinates are radii to one common centre.



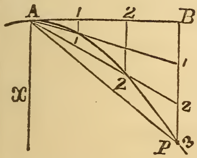
44. *Evolute of a circle.*

Given the pitch p, the angle v, and radius r. Divide the angle v into a number of equal parts, draw the radii and tangents for each part, divide the pitch p into an equal number of equal parts, then the first tangent will be one part, second two parts, third three parts, &c., and so the *Evolute* is traced.



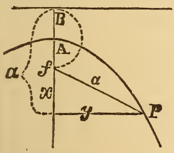
45. *To construct a spiral with compasses and four centres.*

Given the pitch of the spiral, construct a square about the centre, with the four sides together equal to the pitch. Prolong the sides in one direction as shown by the figure, the corners are the centres for each arc of the external angles.



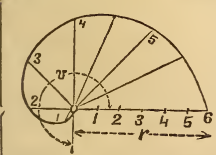
46. *To construct a Parabola.*

Given the vertex A, axis x, and a point P. Draw $A\,B$ at right angle to x, and $B\,P$ parallel to x, divide $A\,B$ and $B\,P$ into an equal number of equal parts. From the vertex A draw lines to the divisions on $B\,P$, from the divisions on $A\,B$ draw the ordinates parallel to x, the corresponding intersections are points in the *parabola.*



47. *To construct a Parabola.*

Given the axis of ordinate B, and vertex A. Take A as a centre and describe a semicircle from B which gives the focus of the *parabola* at f. Draw any ordinate y at right angle to the abscissa $A\,x$, take a as radius and the focus f as a centre, then intersect the ordinate y, by a circle-arc in P which will be a point in the *parabola.* In the same manner the whole Parabola is constructed.



48.

To draw an arithmetic spiral.

Given the pitch p and angle v, divide them into an equal number of equal parts say 6. make $0\,1 = 0\,1$, $0\,2 = 0\,2$, $0\,3 = 0\,3$, $0\,4 = 0\,4$, $0\,5 = 0\,5$, and $0\,6 =$ the pitch p; then join the points 1, 2, 3, 4, 5, and 6, which will form the spiral required.

CIRCLE.

The periphery of a circle is divided into 360° (degrees) equal parts, each called a degree.

One degree = 60′ (minutes.)
One minute = 60″ (seconds.)
Half a circle (hemisphere) = 180°.
Quarter of a circle (quadrant) = 90°.
By the accompanying formula any part of the circle can be calculated.

Formula for the Circle.

$p = \pi d = 3{\cdot}14d$, ... 1,

$p = 2\pi r = 6{\cdot}28r$, ... 2,

$p = 2\sqrt{\pi a} = 3{\cdot}54\sqrt{a}$, ... 3,

$d = \frac{p}{\pi} = \frac{p}{3{\cdot}14}$, ... 4,

$d = 2\sqrt{\frac{a}{\pi}} = 1{\cdot}128\sqrt{a}$, . . 5,

$r = \frac{p}{2\pi} = \frac{p}{6{\cdot}28}$ 6,

$r = \sqrt{\frac{a}{\pi}} = 0{\cdot}564\sqrt{a}$, . . . 7,

$a = \frac{\pi d^2}{4} = 0{\cdot}785d^2$, ... 8,

$a = \pi r^2 = 3{\cdot}14r^2$, ... 9,

$a = \frac{p^2}{4\pi} = \frac{p^2}{12{\cdot}56}$, ... 10,

$a = \frac{pr}{2} = \frac{pd}{4}$, ... 11,

$a = \frac{\pi rd}{2} = 1.57rd$, ... 12.

$\pi = 3{\cdot}14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798214808651328230664709384 46$. . . . .

$2\pi = 6{\cdot}28218530710000$.
$3\pi = 9{\cdot}42477796070000$.
$4\pi = 12{\cdot}5663706143000$.
$5\pi = 15{\cdot}7079632679000$.
$6\pi = 18{\cdot}8495559215000$.
$7\pi = 21{\cdot}9911485751000$.
$8\pi = 25{\cdot}1327412887000$.
$9\pi = 28{\cdot}2743338823000$.
$\frac{1}{2}\pi = 1{\cdot}57079632679000$.
$\frac{1}{4}\pi = 0{\cdot}78539816339700$.
$\frac{1}{3}\pi = 1{\cdot}04719755119600$.
$\frac{1}{6}\pi = 0{\cdot}52359877559800$.
$\frac{1}{8}\pi = 0{\cdot}39269908169800$.
$\frac{1}{12}\pi = 0{\cdot}26179938779900$.
$\frac{1}{360}\pi = 0{\cdot}00872667621060$.
$\frac{1}{\pi} = 0{\cdot}31830988618370$.
$\frac{2}{\pi} = 0{\cdot}63661977236740$.

$\frac{4}{\pi} = 1{\cdot}27323954473480$.
$\frac{3}{\pi} = 0{\cdot}95492965855110$.
$\frac{6}{\pi} = 1{\cdot}90965931710220$.
$\frac{12}{\pi} = 3{\cdot}81971863420440$.
$\frac{360}{\pi} = 114{\cdot}591559026122$.
$\pi^2 = 9{\cdot}86965000000000$.
$\sqrt{\pi} = 1{\cdot}77245300000000$.
$\sqrt{\frac{1}{\pi}} = 0{\cdot}56418900000000$.
$\frac{1}{4\pi} = 0{\cdot}07957747154500$.

Letters denote,

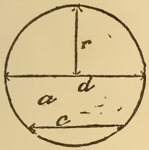
r = radius of the circle.
d = diameter.
p = periphery.
a = area of a circle, or part thereof.
b = circle-arc, length of.
c = chord of a segment, length of.
h — height of a segment.
s = side of a regular polygon.
v = centre angle.
w = polygon angle.

☞ Be careful to express all the dimensions by the same unit, as miles, rods, yards, feet, or inches, &c., &c., or else the calculation will be wrong.

Example 1. Fig. 49. The diameter of a circle is 8 feet, 8 inches, how long is the circumference?

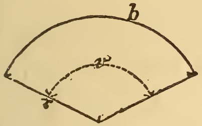
Formula 1. $p = \pi d = 3{\cdot}14 \times 8{\cdot}666 = 27{\cdot}211$ feet, the answer.

49.



The periphery of a Circle is commonly expressed by the *Greek* letter $\pi = 3{\cdot}14$ when the diameter $d = 1$ or the unit. For any other value of the diameter d, we will denote the periphery by the letter p, r = radius, and a = area of the circle. The periphery of a circle is equal to $3\frac{14}{100}$ times its diameter, c = chord.

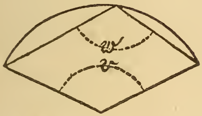
50.



$$b = \frac{\pi r v}{180} = 0{\cdot}0175 rv,$$

$$v = \frac{180 b}{\pi r} = 5{\cdot}74\frac{b}{r}.$$

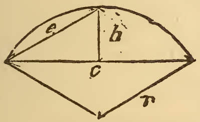
51.



$$w = 180 - \frac{v}{2},$$

$$v = 2(180^\circ - w).$$

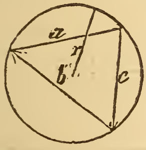
52.



$$r = \frac{c^2+4h^2}{8h} = \frac{e^2}{2h},$$

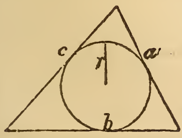
$$c = 2\sqrt{2hr - h^2}.$$

53.

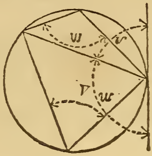


$$r = \frac{ac}{2\sqrt{a^2 - \left(\frac{a^2+b^2-c^2}{2b}\right)^2}}.$$

54.

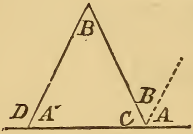


$$r = \frac{b\sqrt{a^2 - \left(\frac{a^2+b^2-c^2}{2b}\right)^2}}{a+b+c}.$$



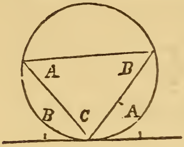
55.

$$v = v, \quad w = w,$$
$$w + v = 180°, \; w > v.$$



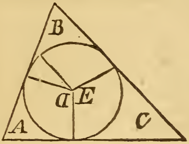
56.

$$D = B + C, \quad A' + B' + C = 180°,$$
$$B = D - C, \quad A + B + C = 180°,$$
$$A' = A, \quad B' = B'.$$



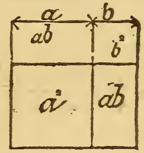
57.

$$A + B + C = 180°,$$
$$A' = A, \quad B' = B.$$



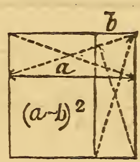
58.

$$E + C = A + D = 180°,$$
$$D = B + c,$$
$$E = A + B.$$



59.

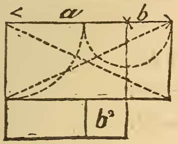
$$(a + b)^2 = a^2 + 2ab + b^2.$$



60.

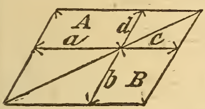
$$(a - b)^2 = a^2 - 2ab + b^2.$$

61.



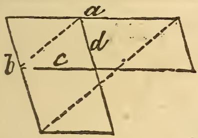
$$(a + b)(a - b) = a^2 - b^2.$$

62.



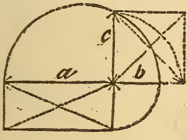
$$a : b = c : d,$$
$$ad = bc,$$
$$A = B.$$

63.



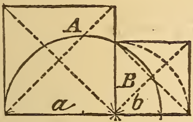
$$a : b = c : d,$$
$$ad = bc.$$

64.



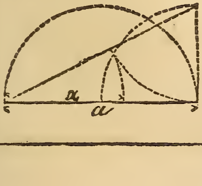
$$a : c = c : b,$$
$$ab = c^2,$$
$$c = \sqrt{ab}.$$

65.

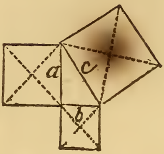


$$A : B = a : b.$$

66.

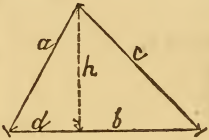


$$a : x = x : a - x,$$
$$x = \sqrt{a^2 + \left(\frac{a}{2}\right)^2} - \frac{a}{2}.$$



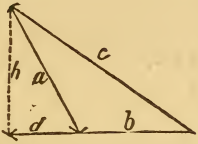
67.

$$c^2 = a^2 + b^2,$$
$$a^2 = c^2 - b^2,$$
$$b^2 = c^2 - a^2.$$



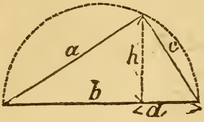
68.

$$c^2 = a^2 + b^2 - 2bd,$$
$$h = \sqrt{a^2 - d^2}.$$
$$d = \frac{a^2 + b^2 - c^2}{2b}.$$



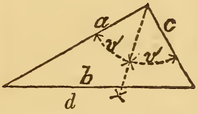
69.

$$c^2 = a^2 + b^2 + 2bd,$$
$$h^2 = \sqrt{a^2 - d^2},$$
$$d = \frac{c^2 - a^2 - b^2}{2b}.$$



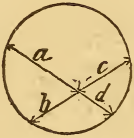
70.

$$a : b = h : c,$$
$$h = \frac{ac}{b} = \frac{ad}{c},$$
$$d = \frac{c^2}{b} = \frac{ch}{a}.$$



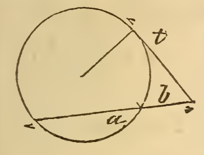
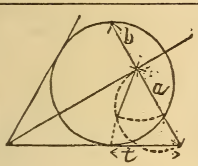
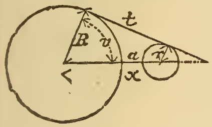
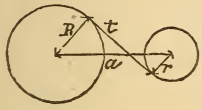
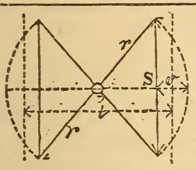
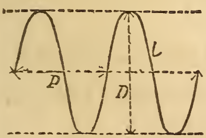
71.

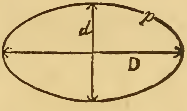
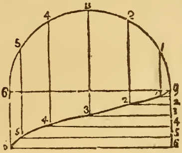
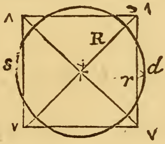
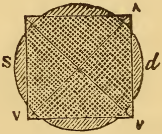
$$a : c = d : (b - d),$$
$$d = \frac{ab}{c + a},$$
$$v = v.$$

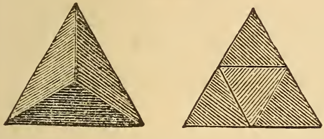


72.

$$a : c = b : d,$$
$$ad = bc.$$

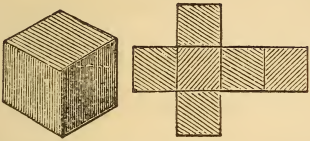
	73. $a : t = t : b,$ $t^2 = ab.$
	74. $t^2 = (a + b)(a - b),$ $t = \sqrt{a^2 - b^2}.$
	75. $x = \frac{aR}{R - r},\quad a = \sqrt{t^2 + (R - r)^2},$ $t = \sqrt{a^2 - (R - r)^2},\quad \sin.v = \frac{t}{a}.$
	76. $t = \sqrt{a^2 - (R + r)^2},$ $a = \sqrt{t^2 + (R + r)^2}.$
	77. $V = r - \sqrt{r^2 - \frac{S^2}{4}}\quad l = 2r - V,$ $S = 2\sqrt{r^2 - (r - V)^2}.\quad r = \frac{1}{2}(l + V).$
	78. $P = \sqrt{\frac{l^2}{n^2} - \pi^2 d^2},$ $l = n\sqrt{\pi^2 d^2 + P^2},$ $n = \frac{l}{\sqrt{\pi^3 d^2 + P}}.$

	79. *To find the length of a Spiral.* $l = \pi r n = \frac{\pi r^2}{P}$, $n = \frac{l}{\pi r} = \frac{r}{P}$, $P = \frac{\pi r^2}{l} = \frac{r}{n}$. $P = Pitch.$
	80. *To find the length of a Spiral.* $l = \pi n\,(R + r)$, $l = \frac{\pi}{P}\,(R^2 - r^2)$.
	81. *Periphery of an Ellipse.* $p = 2\sqrt{D^2 + 1{\cdot}4674 d^2}$.
	82. To construct a screw *Helix.*
	83. *To square a Circumference.* $R = 0{\cdot}556355\, d = 1{\cdot}1127\, r = 0{\cdot}7071\, S.$ $S = 0{\cdot}785398\, d = 1{\cdot}57079\, r = 1{\cdot}4142\, R.$ $d = 1{\cdot}27322\, S = 1{\cdot}79740\, R = 2\, r.$
	84. *To square a Circleplane.* $R = 0{\cdot}650744\, d = 1{\cdot}30148\, r = 0{\cdot}7071\, S.$ $S = 0{\cdot}886226\, d = 1{\cdot}77245\, r = 1{\cdot}4142\, R.$ $d = 1{\cdot}12831\, S = 1{\cdot}5367\, R = 2\, r.$



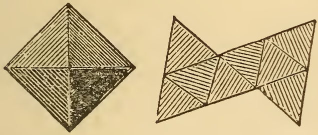
85. *Tetrahedron.*

$r = 0{\cdot}20413\,\mathbf{s}.$
$R = 0{\cdot}60725\,\mathbf{s}.$
$\mathbf{a} = 1{\cdot}73205\,\mathbf{s}^2.$
$\mathbf{c} = 0{\cdot}11785\,\mathbf{s}^3.$



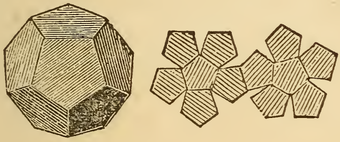
86. *Hexahedron.*

$r = 0{\cdot}50000\,\mathbf{s}.$
$R = 0{\cdot}86602\,\mathbf{s}.$
$\mathbf{a} = 6{\cdot}00000\,\mathbf{s}^2.$
$\mathbf{c} = 1{\cdot}00000\,\mathbf{s}^3.$



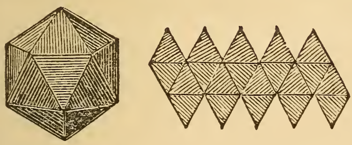
87. *Octahedron.*

$r = 0{\cdot}40721\,\mathbf{s}.$
$R = 0{\cdot}71710\,\mathbf{s}.$
$\mathbf{a} = 3{\cdot}46410\,\mathbf{s}^2.$
$\mathbf{c} = 0{\cdot}47140\,\mathbf{s}^3$



88. *Dodecahedron.*

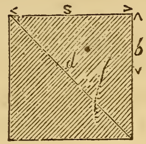
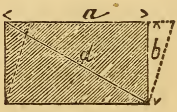
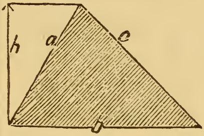
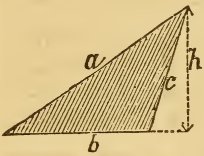
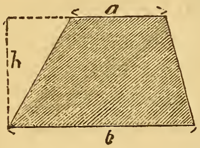
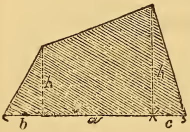
$r = 1{\cdot}11350\,\mathbf{s}.$
$R = 1{\cdot}36428\,\mathbf{s}.$
$\mathbf{a} = 20{\cdot}5457\,\mathbf{s}^2.$
$\mathbf{c} = 7{\cdot}66312\,\mathbf{s}^3.$

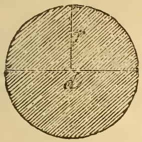
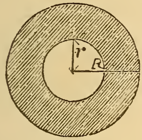
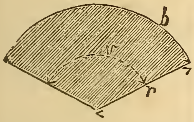
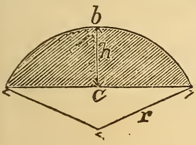
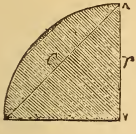
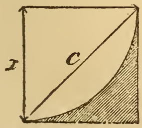


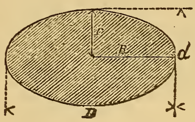
89. *Icosahedron.*

$r = 0{\cdot}7558\,\mathbf{s}.$
$R = 0{\cdot}9510\,\mathbf{s}.$
$\mathbf{a} = 8{\cdot}66025\,\mathbf{s}^2.$
$\mathbf{c} = 2{\cdot}18169\,\mathbf{s}^3.$

r = Radius of an inscribed Sphere.
R = Radius of circumscribed Sphere.
$\mathbf{a}$ = Area of the Polyhedrons.
$\mathbf{c}$ = Cubic contents of the Polyhedrons.
$\mathbf{s}$ = Side or edge of the Polyhedrons.

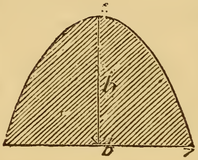
	90. *Square.* $\mathbf{a} = s^2 = 4b^2.$ $\mathbf{a} = 0{\cdot}7071d^2 = 2{\cdot}8284\ c^2.$
	91. *Rectangle.* $\mathbf{a} = a\,b,$ $\mathbf{a} = b\,\sqrt{d^2 - b^2}.$
	92. *Triangle.* $\mathbf{a} = \frac{b\,h}{2} = \frac{1}{2}b\,h,$ $\mathbf{a} = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2\,b}\right)^2}.$
	93. *Triangle.* $\mathbf{a} = \frac{1}{2}b\,h,$ $\mathbf{a} = \frac{b}{2}\sqrt{a^2 - \left(\frac{c^2 - a^2 - b^2}{2\,b}\right)^2}.$
	94. *Quadrangle.* $\mathbf{a} = \frac{1}{2}h(a + b).$
	95. *Quadrangle.* $\mathbf{a} = \frac{1}{2}(a\,[h + h'] + b\,h' + c\,h).$

	96. *Circle Plane.* $\mathbf{a} = \pi r^2 = 0{\cdot}785\, d^2,$ $\mathbf{a} = \frac{p\,r}{2} = 0.0794\, P^2.$
	97. *Circle Ring.* $\mathbf{a} = \pi(R^2 - r^2) = \pi(R + r)(R - r),$ $\mathbf{a} = 0{\cdot}785(D^2 - d^2).$
	98. *Sector.* $\mathbf{a} = \tfrac{1}{2} b\, r,$ $\mathbf{a} = \frac{\pi r^2 v}{360} = \frac{r^2 v}{114{\cdot}5}.$
	99. *Segment.* $\mathbf{a} = \tfrac{1}{2}[b\, r - c\,(r - h)],$ $\mathbf{a} = \frac{\pi r^2 v}{360} \mp \frac{c}{2}(r - h).$
	100. *Quadrant.* $\mathbf{a} = 0{\cdot}785\, r^2 = 0{\cdot}3916\, c^2.$
	101. $\mathbf{a} = 0{\cdot}215\, r^2 = 0{\cdot}1075\, c^2.$



102. *Ellipse.*

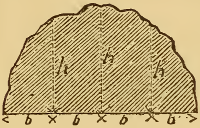
$$\mathbf{a} = \pi R r = 0{\cdot}785\, D\, d.$$



103. *Parabola.*

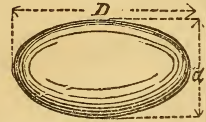
$$\mathbf{a} = \tfrac{2}{3}\, b\, h = \tfrac{1}{6}\, b^2,$$

$$\mathbf{a} = \tfrac{2}{3}\, h \sqrt{p\, h}.$$



104. *Irregular Figure.*

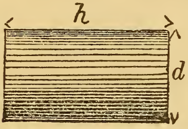
$$\mathbf{a} = b\,(h + h' + h'').$$



105. *Ellipsoid.*

$$\mathbf{a} = 8{\cdot}88\, r \sqrt{R^2 + r^2},$$

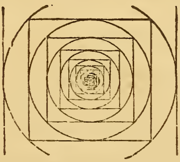
$$\mathbf{a} = 2{\cdot}22\, d \sqrt{D^2 + d^2}.$$



106. *Cylinder.*

$$\mathbf{a} = 2\,\pi\, r\, h = \pi\, d\, h,$$

$$h = \frac{\mathbf{a}}{2\,\pi\, r} = \frac{\mathbf{a}}{\pi\, d}.$$



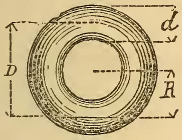
The road to Extremity of Space.

1st. Draw a Circle and inscribe a Square, and in that Square a Circle, &c., &c., &c. . . . . . . The last figure that can be drawn, is one extremity of Space Required if the last one is a Circle or a Square?

2nd. Draw a Circle and circumscribe a Square, and around that Square a Circle, &c., &c., &c. . . . . . The last one that can be circumscribed is the other extremity of space. Required if the last figure is a Circle or a Square?

107. *Sphere.*

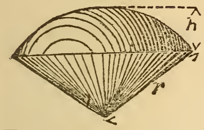
$$\mathbf{a} = 4\,\pi\,r^2 = 12{\cdot}56\,r^2 = \pi\,d^2.$$



108. *Forus.*

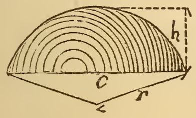
$$\mathbf{a} = 4\,\pi^2\,R\,r = 39{\cdot}44\,R\,r,$$

$$\mathbf{a} = 9{\cdot}86\,D\,d.$$



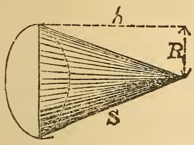
109. *Sphere Sector.*

$$\mathbf{a} = \frac{\pi\,r}{2}(4\,h + c).$$



110 *Circle Zone.*

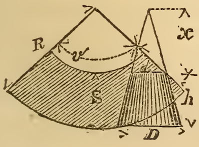
$$\mathbf{a} = 2\,\pi\,r\,h = \frac{\pi}{4}(c^2 + h^2).$$



111. *Cone.*

$$\mathbf{a} = \pi\,R\,s,$$

$$\mathbf{a} = \pi\,R\,\sqrt{R^2 + h^2}.$$



112. *Cone.*

$$x = \frac{d\,h}{D - d}, \qquad R = s + \frac{d\,s}{D - d},$$

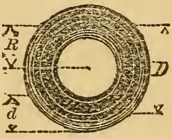
$$\mathbf{a} = \frac{\pi\,s}{2}(D + d),$$

$$v = \frac{180\,D}{R} = \frac{180(D - d)}{s}.$$

113. *Sphere.*

$$\mathbf{C} = \frac{4\,\pi\,r^3}{3} = 4{\cdot}189\,r^3,$$

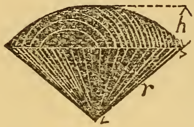
$$\mathbf{C} = \frac{\pi\,d^3}{6} = 0{\cdot}523\,d^3.$$



114. *Forus.*

$$\mathbf{C} = 2\,\pi_2\,R\,r^2 = 19{\cdot}72\,R\,r^3,$$

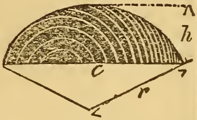
$$\mathbf{C} = 2{\cdot}463\,D\,d^2.$$



115. *Sphere Sector.*

$$\mathbf{C} = \tfrac{2}{3}\,\pi\,r^2\,h = 2{\cdot}0944\,r^2\,h,$$

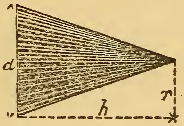
$$\mathbf{C} = \tfrac{2}{3}\,\pi\,r^2\left(r \mp \sqrt{r^2 - \tfrac{1}{4}\,c^2}\right).$$



116. *Zone.*

$$\mathbf{C} = \pi\,h^2(r - \tfrac{1}{3}\,h),$$

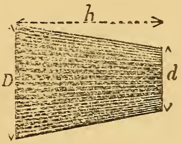
$$\mathbf{C} = \pi\,h^3\left(\frac{c^2 + 4h^2}{8\,h} - \tfrac{1}{3}\,h\right).$$



117. *Cone.*

$$\mathbf{C} = \frac{\pi\,r^2\,h}{3} = 1{\cdot}046\,r^2\,h,$$

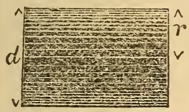
$$\mathbf{C} = 0{\cdot}2616\,d^2\,h.$$



118. *Conic Frustrum.*

$$\mathbf{C} = \tfrac{1}{3}\,\pi\,h(R^2 + R\,r + r^2),$$

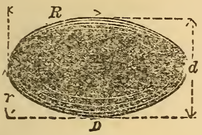
$$\mathbf{C} = \tfrac{1}{12}\,\pi\,h(D^2 + D\,d + d^2)$$



119. *Cylinder.*

$$\mathbf{c} = \pi r^2 h = 0.785\, d^2 h,$$

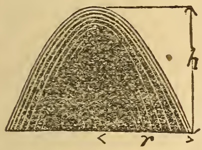
$$\mathbf{c} = \frac{p^2 h}{4\pi} = 0{\cdot}0796\, p^2 h.$$



120. *Ellipsoid.*

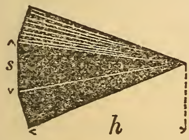
$$\mathbf{c} = 0{\cdot}424\, \pi^2 R r^2 = 4{\cdot}1847\, R r^2,$$

$$\mathbf{c} = 0{\cdot}053\, \pi^2 D d^2 = 0{\cdot}5231\, D d^2.$$



121. *Paraboloid.*

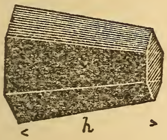
$$\mathbf{c} = \tfrac{1}{2} \pi r^2 h = 1{\cdot}5707\, r^2 h.$$



122. *Pyramid.*

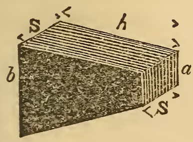
$$\mathbf{c} = \tfrac{1}{3}\, \mathbf{a}\, h,$$

$$\mathbf{c} = \frac{n s h}{6} \sqrt{r^2 - \frac{s^2}{4}}.$$



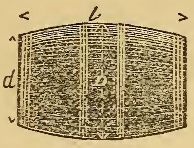
123. *Pyramidic Frustrum.*

$$\mathbf{c} = \frac{h}{3}(A + \mathbf{a} + \sqrt{A\,\mathbf{a}}).$$



124. *Wedge Frustrum.*

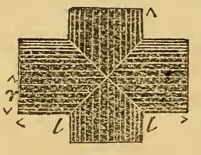
$$\mathbf{c} = \frac{h s}{2}(a + b).$$



125. *Cask.*

$$c = 1{\cdot}0453\, l\left(0{\cdot}4\, D^2 + 0{\cdot}2\, D\, d + 0{\cdot}15 d^2\right),$$

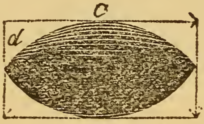
$$Gallon = \frac{l}{2200}\left(4\, D^2 + 2\, Dd + 1{\cdot}5 d^2\right).$$



126. *Cylinder Sections.*

$$\mathbf{c} = \pi\, r^2(l + l' - \tfrac{2}{3} r),$$

$$\mathbf{c} = \pi\, r^2(l + l') - 2{\cdot}1\, r^3.$$



127. *Circular Spindle.*

$$\mathbf{c} = \pi\left(\tfrac{1}{6}\, c^3 - 0{\cdot}2\, d[c + \tfrac{2}{3}\sqrt{c^2 + d^2}]\sqrt{d^2 + c^2}\right)$$

Example 1. Fig. 92. The base of a Triangle is $b = 8$ feet, 3 inches, and the height, $h = 5$ feet, 6 inches. What is the area $\mathbf{a} = ?$

$$\mathbf{a} = \frac{b\, h}{2} = \frac{8{\cdot}25 \times 5{\cdot}5}{2} = 22{\cdot}6875 \text{ square feet.}$$

Example 2. Fig. 98. A Circle Sector having an angle $v = 39°$ and the radius $r = 67\tfrac{3}{4}$ inches. What is the area of the sector $\mathbf{a} = ?$

$$\mathbf{a} = \frac{\pi r^2 v}{360} = \frac{3{\cdot}14 \times 67{\cdot}75^2 \times 39°}{360} = 1562{\cdot}1 \text{ square feet.}$$

Example 3. Fig. 110. A Spherical Zone having its diameter $c = 18\tfrac{1}{2}$ inches and height $h = 7\tfrac{3}{4}$ inches. What is the convex surface of the Zone?

$$\mathbf{a} = \frac{\pi}{4}\left(c^2 + h^2\right) = \frac{3{\cdot}14}{4}\left(18{\cdot}5^2 + 7{\cdot}75^2\right) = 315{\cdot}96 \text{ square inches.}$$

Example 4. Fig. 88. Require the radius R of a Sphere that will circumscribe a Dodecahedron with the side $s = 9$ inches.

$$R = 1{\cdot}36428 \times 9 = 12{\cdot}27852 \text{ inches, the answer.}$$

Example 5. Fig. 118. A Frustrum of a Cone having its bottom diameter $D = 13$ inches, the top diameter $d = 5\tfrac{1}{4}$ inches, and the height $h = 25$ inches. What is the cubic contents $\mathbf{c} = ?$

$\mathbf{c} = \frac{1}{12}\pi h\left(D^2 + D\, d + d^2\right) = 0{\cdot}2618 \times 25\left(13^2 + 13 \times 5{\cdot}25 + 5{\cdot}25^2\right) = 20995$ cubic inches.

Example 6. Fig. 125. A Cask having its bung diameter $D = 36$ inches, head diameter $d = 28$ inches, and length $l = 56$ inches, (inside measurement) how many gallons of liquid can be contained in the cask? (The gallon = 231 cub. in.)

$$\text{Gallon} = \frac{56}{2200}\left(4 \times 36^2 + 2 \times 36 \times 28 + 1{\cdot}5 \times 28^2\right) = 214 \text{ gallons.}$$

Example 7. Fig. 50. Require the length of the circle-arc b, when the angle $v = 42^\circ$, and the radius $r = 4$ feet, 3 inches?

$$b = \frac{\pi\, r\, v}{180} = \frac{3{\cdot}14 \times 4{\cdot}25 \times 42}{180} = 3{\cdot}113 \text{ feet.}$$

Example 8. Fig. 52. Require the radius of a circle-arc, whose chord is 9 feet, 4 inches, and height, $h = 1$ foot, 8 inches?

$$r = \frac{c^2 + 4h^2}{8h} = \frac{9{\cdot}33^2 + 4 \times 1{\cdot}66^2}{8 \times 1{\cdot}66} = \frac{98{\cdot}0711}{13{\cdot}28} = 7{\cdot}384 \text{ feet.}$$

Example 9. Fig. 68. The three sides in a triangle being, $a = 6{\cdot}42$, $b = 7{\cdot}75$, and $c = 8{\cdot}66$ feet. How high is the triangle over the base b?

$$d = \frac{a^2 + b^2 - c^2}{2b} = \frac{6{\cdot}42^2 + 7{\cdot}75^2 - 8{\cdot}66^2}{2 \times 8{\cdot}66} = 1{\cdot}5175 \text{ feet,}$$

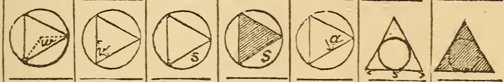
the height $h = \sqrt{a^2 - d^2} = \sqrt{6{\cdot}42^2 - 1{\cdot}5175^2} = 6{\cdot}24$ feet, the answer.

Example 10. Fig. 77. The radius of a walking beam is, $r = 8{\cdot}36$ feet, the stroke $S = 5{\cdot}5$ feet. How much is the vibration $V = ?$

Vibration, $$V = r - \sqrt{r^2 - \frac{S^2}{4}} = 8{\cdot}36 - \sqrt{8{\cdot}36^2 - \frac{5{\cdot}5^2}{4}}$$

$$= 0{\cdot}471 \text{ feet} = 5{\cdot}65 \text{ inches} = 5\tfrac{21}{32}, \text{ the answer.}$$

TABLE OF POLYGONS.



Number of sides in the Polygon.		Centre Angle w.	Polygon Angle v.	Side $= k\,R$.	Area $= k\,S^2$.	Apotem $= k\,R$.	Side $= k\,r$.	Area $= k\,r^2$.
Trigon.	3	120°	60°	1·732	0·4330	0·5000	3·4641	5·1961
Tetragon.	4	90°	90°	1·4142	1·0000	0·7071	2·0000	4·0000
Pentagon.	5	72°	108°	1·1755	1·7205	0·8090	1·4536	3·6327
Hexagon.	6	60°	120°	1·0000	2·5980	0·8660	1·1547	3·4640
Heptagon.	7	51°43′	128°17′	0·8677	3·6339	0·9009	0·9631	3·3710
Octagon.	8	45°	135°	0·7653	4·8284	0·9238	0·8284	3·3130
Nonagon.	9	40°	140°	0·6840	6·1820	0·9396	0·7279	3·2750
Decagon.	10	36°	144°	0·6180	7·6942	0·9510	0·6498	3·2490
Undecagon.	11	32°13′	147°47′	0·5634	9·3656	0·9595	0·5872	3·2290
Dodecagon.	12	30°	150°	0·5176	11·196	0·9659	0·5359	3·2152
	14	25°43′	154°17′	0·4450	15·334	0·9762	0·4562	3·1935
	15	24°	156°	0·4158	17·642	0·9781	0·4250	3·1882
	16	22°30′	157°30′	0·3900	20·128	0·9807	0·4068	3·1824
	18	20°	160°	0·3472	25·534	0·9848	0·3526	3·1737
	20	18°	162°	0·3130	40·634	0·9877	0·3166	3·1676
	24	15°	165°	0·2610	45·593	0·9914	0·2632	3·1596

Explanation of the Table for Polygons.

The number of sides in the polygon is noted in the first column.

k = *tabular coefficient*, to be multiplied as noted on the top of the columns.

Example 1. How long is the side of an inscribed Pentagon, when the radius of the circle is 3 feet, and 4 inches? (4 inches = 0·333 feet.)

$$3{\cdot}333 \times 1{\cdot}1755 = 3{\cdot}9179 \text{ feet, the answer.}$$

Example 2. What is the area of a Heptagon when one of its sides is 13·75 inches

$$13{\cdot}75^2 \times 3{\cdot}6339 = 687{\cdot}02 \text{ square inches.}$$

Diameter.	Circ.	Area.
$\frac{1}{32}$	·0981	·00076
$\frac{1}{16}$	·1963	.00306
$\frac{1}{8}$	·3926	·01227
$\frac{3}{16}$	·5890	·02761
$\frac{1}{4}$	·7854	·04908
$\frac{5}{16}$	·9817	·07669
$\frac{3}{8}$	1·178	·1104
$\frac{7}{16}$	1·374	·1503
$\frac{1}{2}$	1·570	·1963
$\frac{9}{16}$	1·767	·2485
$\frac{5}{8}$	1·963	·3067
$\frac{11}{16}$	2·159	·3712
$\frac{3}{4}$	2·356	·4417
$\frac{13}{16}$	2·552	·5184
$\frac{7}{8}$	2.748	·6013
$\frac{15}{16}$	2·945	·6902
1	3·141	·7854
	3·534	·9940
$\frac{1}{4}$	3·927	1·227
	4·319	1·484
$\frac{1}{2}$	4·712	1·767
	5·105	2·073
$\frac{3}{4}$	5·497	2·405
	5·890	2·761
2	6·283	3·141
	6·675	3·546
$\frac{1}{4}$	7·068	3·976
	7·461	4·430
$\frac{1}{2}$	7·854	4·908
	8·246	5·411
$\frac{3}{4}$	8·639	5·939
	9·032	6·491
3	9·424	7·068
	9·817	7·669
$\frac{1}{4}$	10·21	8·295
	10·60	8·946
$\frac{1}{2}$	10·99	9·621
	11·38	10·320
$\frac{3}{4}$	11·78	11·044
	12·17	11·793
4	12·56	12·566
	12·95	13·364
$\frac{1}{4}$	13·35	14·186
	13·74	15·033
$\frac{1}{2}$	14·13	15·904
	14·52	16·800
$\frac{3}{4}$	14·92	17·720
	15.31	18·665

Diameter.	Circ.	Area
5	15·70	19·635
	16·10	20·629
$\frac{1}{4}$	16·49	21·647
	16·88	22·690
$\frac{1}{2}$	17·27	23·758
	17·67	24·850
$\frac{3}{4}$	18·06	25·967
	18·45	27·108
6	18·84	28·274
	19·24	29·464
$\frac{1}{4}$	19·63	30·679
	20·02	31·919
$\frac{1}{2}$	20·42	33·183
	20·81	34·471
$\frac{3}{4}$	21·20	35·784
	21·57	37·122
7	21·99	38·484
	22·38	39·871
$\frac{1}{4}$	22·77	41·282
	23·16	42·718
$\frac{1}{2}$	23·56	44·178
	23·95	45·663
$\frac{3}{4}$	24·34	47·173
	24·74	48·707
8	25·13	50·265
	25·52	51·848
$\frac{1}{4}$	25·91	53·456
	26·31	55·088
$\frac{1}{2}$	26·70	56·745
	27·09	58·426
$\frac{3}{4}$	27·48	60·132
	27·88	61·862
9	28·27	63·617
	28·66	65·396
$\frac{1}{4}$	29·05	67·200
	29·45	69·029
$\frac{1}{2}$	29·84	70·882
	30·23	72·759
$\frac{3}{4}$	30·63	74·662
	31·02	76·588
10	31·41	78·539
	31·80	80·515
$\frac{1}{4}$	32·20	82·516
	32·59	84·540
$\frac{1}{2}$	32·98	86·590
	33·37	88·664
$\frac{3}{4}$	33·77	90·762
	34·16	92·885

Diameter.	Circ.	Area.
.11	34.55	95·033
	34·95	97·205
$\frac{1}{4}$	35·34	99·402
	35·73	101·62
$\frac{1}{2}$	36·12	103·86
	36·52	106·13
$\frac{3}{4}$	36·91	108·43
	37·30	110·75
12	37·69	113·09
	38·09	115·46
$\frac{1}{4}$	38·48	117·85
	38·87	120·27
$\frac{1}{2}$	39·27	122·71
	39·66	125·18
$\frac{3}{4}$	40·05	127·67
	40·44	130·19
13	40·84	132·73
	41·23	135·29
$\frac{1}{4}$	41·62	137·88
	42·01	140·50
$\frac{1}{2}$	42·41	143·13
	42·80	145·80
$\frac{3}{4}$	43·19	148·48
	43·58	151·20
14	43·98	153·93
	44·37	156·69
$\frac{1}{4}$	44·76	159·48
	45·16	162·29
$\frac{1}{2}$	45·55	165·13
	45·94	167·98
$\frac{3}{4}$	46·33	170·87
	46·73	173·78
15	47·12	176·71
	47·51	179·67
$\frac{1}{4}$	47·90	182·65
	48·30	185·66
$\frac{1}{2}$	48·69	188·69
	49·08	191·74
$\frac{3}{4}$	49·48	194·82
	49·87	197·93
16	50·26	201·06
	50·65	204·21
$\frac{1}{4}$	51·05	207·39
	51·44	210·59
$\frac{1}{2}$	51·83	213·82
	52·22	217·07
$\frac{3}{4}$	52·62	220·35
	53·01	223·65

Diameter.	Circ.	Area.	Diameter.	Circ.	Area	Diameter.	Circ.	Area.
17	53·40	226·98	23	72·25	415·47	29	91·10	660·52
	53·79	230·33		72·64	420·00		91·49	666·22
¼	54·19	233·70	¼	73·04	424·55	¼	91·89	671·95
	54·58	237·10		73·43	429·13		92·28	677·71
½	54·97	240·52	½	73·82	433·73	½	92·67	683·49
	55·37	243·97		74·21	438·30		93·06	689·29
¾	55·76	247·45	¾	74·61	443·01	¾	93·46	695·12
	56·16	250·94		75·	447·69		93·85	700·98
18	56·54	254·46	24	75·39	452·39	30	94·24	706·86
	56·94	258·01		75·79	457·11		94·64	712·76
¼	57·33	261·58	¼	76·18	461·86	¼	95·03	718·69
	57·72	265·18		76·57	466·63		95·42	724·64
½	58·11	268·80	½	76·96	471·43	½	95·81	730·61
	58·51	272·44		77·36	476·25		96·21	736·61
¾	58·90	276·11	¾	77·75	481·10	¾	96·60	742·64
	59·29	279·81		78·14	485·97		96·99	748·69
19	59·69	283·52	25	78·54	490·87	31	97·38	754·76
	60·08	287·27		78·93	495·79		97·78	760·86
¼	60·47	291·03	¼	79·32	500·74	¼	98·17	766·99
	60·86	294·83		79·71	505·71		98·56	773·14
½	61·26	298·64	½	80·10	510·70	½	98·96	779·31
	61·65	302·48		80·50	515·72		99·35	785·51
¾	62·04	306·35	¾	80·89	520·70	¾	99·74	791·73
	62·43	310·24		81·28	525·83		100·1	797·97
20	62·83	314·16	26	81·68	530·93	32	100·5	804·24
	63·22	318·09		82·07	536·04		100·9	810·54
¼	63·61	322·06	¼	82·46	541·18	¼	101·3	816·86
	64·01	326·05		82·85	546·35		101·7	823·21
½	64.40	330·06	½	83·25	551·54	½	102·1	829·57
	64·79	334·10		83·64	556·76		102·4	835·97
¾	65·18	338·16	¾	84·03	562·00	¾	102·8	842·39
	65·58	342·25		84·43	567·26		103·2	848·83
21	65·97	346·36	27	84·82	572·55	33	103·6	855·30
	66·36	350·49		85·21	577·87		104·	861·79
¼	66·75	354·65	¼	85·60	583·20	¼	104·4	868·30
	67·15	358·84		86·	588·57		104·8	874·84
½	67·54	363·05	½	86·39	593·95	½	105·2	881·41
	67·93	367·28		86·78	599·37		105·6	888·00
¾	68·32	371·54	¾	87·17	604·80	¾	106·	894·61
	68·72	375·82		87·57	610·26		106·4	901·25
22	69·11	380·13	28	87·96	615·75	34	106·8	907·92
	69·50	384·46		88·35	621·26		107·2	914·61
¼	69·90	388·82	¼	88·75	626·79	¼	107·5	921·32
	70·29	393·20		89·14	632·35		107·9	928·06
½	70·68	397·60	½	89·53	637·94	½	108·3	934·82
	71·07	402·03		89·92	643·54		108·7	941·60
¾	71·47	406·49	¾	90·32	649·18	¾	109·1	948·41
	71·86	410·97		90·71	654·83		109·5	955·25

Diameter.	Circ.	Area.	Diameter.	Circ.	Area.	Diameter.	Circ.	Area.
35	109·9	962·11	41	128.8	1320·2	47	147·6	1734·9
	110·3	968·99		129·1	1328·3		148·	1744·1
¼	110·7	975·90	¼	129·5	1336·4	¼	148·4	1753·4
	111·1	982·84		129·9	1344·5		148·8	1762·7
½	111·5	989·80	½	130·3	1352·6	½	149·2	1772·0
	111·9	996·78		130·7	1360·8		149·6	1781·3
¾	112·3	1003·7	¾	131·1	1369·0	¾	150·	1790·7
	112·7	1010·8		131·5	1377·2		150·4	1800·1
36	113·	1017·8	42	131·9	1385·4	48	150·7	1809·5
	113·4	1024·9		132·3	1393·7		150·1	1818·9
¼	113·8	1032·0	¼	132·7	1401·9	¼	151·5	1828·4
	114·2	1039·1		133·1	1410·2		151·9	1837·9
½	114·6	1046·3	½	133·5	1418·6	½	152·3	1847·4
	115·	1053·5		133·9	1426·9		152·7	1856·9
¾	115·4	1060·7	¾	134·3	1435·3	¾	153·1	1866·5
	115·8	1067·9		134·6	1443·7		153·5	1876·1
37	116·2	1075·2	43	135·	1452·2	49	153·9	1885·7
	116·6	1082·4		135·4	1460·6		154·3	1895·3
¼	117·	1089·7	¼	135·8	1469·1	¼	154·7	1905·0
	117·4	1097·1		136·2	1477·6		155·1	1914·7
½	117·8	1104·4	½	136·6	1486·1	½	155·5	1924·4
	118·2	1111·8		137·	1494·7		155·9	1934·1
¾	118·6	1119·2	¾	137·4	1503·3	¾	156·2	1943·9
	118·9	1126·6		137·8	1511·9		156·6	1953·6
38	119·3	1134·1	44	138·2	1520·5	50	157·	1963·5
	119·7	1141·5		138·6	1529·1		157·4	1973·3
¼	120·1	1149·0	¼	139·	1537·8	¼	157·8	1983·1
	120·5	1156·6		139·4	1546·5		158·2	1993·0
½	120·9	1164·1	½	139·8	1555·2	½	158·6	2002·9
	121·3	1171·7		140·1	1564·0		159·	2012·8
¾	121·7	1179·3	¾	140·5	1572·8	¾	159·4	2022·8
	122·1	1186·9		140·9	1581·6		159·8	2032·8
39	122·5	1194·5	45	141·3	1590·4	51	160·2	2042·8
	122·9	1202·2		141·7	1599·2		160·6	2052·8
¼	123·3	1209·9	¼	142·1	1608·1	¼	161·	2062·9
	123·7	1217·6		142·5	1617·0		161·3	2072·9
½	124·	1225·4	½	142·9	1625·9	½	161·7	2083·0
	124·4	1233·1		143·3	1634·9		162·1	2093·2
¾	124·8	1240·9	¾	143·7	1643·8	¾	162·5	2103·3
	125·2	1248·7		144·1	1652·8		162·9	2113·5
40	125·6	1256·6	46	144·5	1661·9	52	163·3	2123·7
	126·	1264·5		144·9	1670·9		163·7	2133·9
¼	126·4	1272·3	¼	145·2	1680·0	¼	164·1	2144·1
	126·8	1280·3		145·6	1689·1		164·5	2154·4
½	127·2	1288·2	½	146·	1698·2	½	164·9	2164·7
	127·6	1296·2		146·4	1707·3		165·3	2175·0
¾	128·	1304·2	¾	146·8	1716·5	¾	165·7	2185·4
	128·4	1312·2		147·2	1725·7		166·1	2195·7

Diameter.	Circ.	Area.	Diameter.	Circ.	Area.	Diameter.	Circ.	Area.
53	166·5	2206·1	59	185·3	2733·9	65	204·2	3318·3
	166·8	2216·6		185·7	2745·5		204·5	3331·0
¼	167·2	2227·0	¼	186·1	2757·1	¼	204·9	3343·8
	167·6	2237·5		186·5	2768·8		205·3	3356·7
½	168·	2248·0	½	186·9	2780·5	½	205·7	3369·5
	168·4	2258·5		187·3	2792·2		206·1	3382·4
¾	168·8	2269·0	¾	187·7	2803·9	¾	206·5	3395·3
	169·2	2279·6		188·1	2815·6		206·9	3408·2
54	169·6	2290·2	60	188·4	2827·4	66	207·3	3421·2
	170·	2300·8		188·8	2839·2		207·7	3434·1
¼	170·4	2311·4	¼	189·2	2851·0	¼	208·1	3447·1
	170·8	2322·1		189·6	2862·8		208·5	3460·1
½	171·2	2332·8	½	190·	2874·7	½	208·9	3473·2
	171·6	2343·5		190·4	2886·6		209·3	3486·3
¾	172·	2354·2	¾	190·8	2898·5	¾	209·7	3499·3
	172·3	2365·0		191·2	2910·5		210·	3512·5
55	172·7	2375·8	61	191·6	2922·4	67	210·4	3525·6
	173·1	2386·6		192·	2934·4		210·8	3538·8
¼	173·5	2397·4	¼	192·4	2946·4	¼	211·2	3552·0
	173·9	2408·3		192·8	2958·5		211·6	3565·2
½	174·3	2419·2	½	193·2	2970·5	½	212·	3578·4
	174·7	2430·1		193·6	2982·6		212·4	3591·7
¾	175·1	2441·0	¾	193·9	2994·7	¾	212·8	3605·0
	175·5	2452·0		194·3	3006·9		213·2	3618·3
56	175·9	2463·0	62	194·7	3019·0	68	213·6	3631·6
	176·3	2474·0		195·1	3031·2		214·	3645·0
¼	176·7	2485·0	¼	195·5	3043·4	¼	214·4	3658·4
	177·1	2496·1		195·9	3055·7		214·8	3671·8
½	177·5	2507·1	½	196·3	3067·9	½	215·1	3685·2
	177·8	2518·2		196·7	3080·2		215·5	3698·7
¾	178·2	2529·4	¾	197·1	3092·5	¾	215·9	3712·2
	178·6	2540·5		197·5	3104·8		216·3	3725·7
57	179·	2551·7	63	197·9	3117·2	69	216·7	3739·2
	179·4	2562·9		198·3	3129·6		217·1	3752·8
¼	179·8	2574·1	¼	198·7	3142·0	¼	217·5	3766·4
	180·2	2585·4		199·	3154·4		217·9	3780·0
½	180·6	2596·7	½	199·4	3166·9	½	218·3	3793·6
	181·	2608·0		199·8	3179·4		218·7	3807·3
¾	181·4	2619·3	¾	200·2	3191·9	¾	219·1	3821·0
	181·8	2630·7		200·6	3204·4		219·5	3834·7
58	182·2	2642·0	64	201·	3216·9	70	219·9	3848·4
	182·6	2653·4		201·4	3229·5		220·3	3862·2
¼	182·9	2664·9	¼	201·8	3242·1	¼	220·6	3875·9
	183·3	2676·3		202·2	3254·8		221·	3889 8
½	183·7	2687·8	½	202·6	3267·4	½	221·4	3903·6
	184·1	2699·3		203·	3280·1		221·8	3917·4
¾	184·5	2710·8	¾	203·4	3292·8	¾	222·2	3931·3
	184·9	2722·4		203·8	3305·5		222·6	3945·2

Diameter.	Circ.	Area.
71	223·	3959·2
	223·4	3973·1
¼	223·8	3987·1
	224·2	4001·1
½	224·6	4015·1
	225·	4029·2
¾	225·4	4043·2
	225·8	4067·3
72	226·1	4071·5
	226·5	4085·6
¼	226·9	4099·8
	227·3	4114·0
½	227·7	4128·2
	228·1	4142·5
¾	228·5	4156·7
	228·9	4171·0
73	229·3	4185·3
	229·7	4199·7
¼	230·1	4214·1
	230·5	4228·5
½	230·9	4242·9
	231·3	4257·3
¾	231·6	4271·8
	232·	4286·3
74	232·4	4300·8
	232·8	4315·3
¼	233·2	4329·9
	233·6	4344·5
½	234·	4359·1
	234·4	4373·8
¾	234·8	4388·4
	235·2	4403·1
75	235·6	4417·8
	236·	4432·6
¼	236·4	4447·3
	236·7	4462·1
½	237·1	4476·9
	237·5	4491·8
¾	237·9	4506·6
	238·3	4521·5
76	238·7	4536·4
	239·1	4551·4
¼	239·5	4566·3
	239·9	4581·3
½	240·3	4596·3
	240·7	4611·3
¾	241·1	4626·4
	241·5	4641·5
77	241·9	4656·6
	242·2	4671·7
¼	242·6	4686·9
	243·	4702·1
½	243·4	4717·3
	243·8	4732·5
¾	244·2	4747·7
	244·6	4763·0
78	245·	4778·3
	245·4	4793·7
¼	245·8	4809·0
	246·2	4824·4
½	246·6	4839·8
	247·	4855·2
¾	247·4	4870·7
	247·7	4886·1
79	248·1	4901·6
	248·5	4917·2
¼	248·9	4932·7
	249·3	4948·3
½	249·7	4963·9
	250·1	4979·5
¾	250·5	4995·1
	250·9	5010·8
80	251·3	5026·5
	251·7	5042·2
¼	252·1	5058·0
	252·5	5073·7
½	252·8	5089·5
	253·2	5105·4
¾	253·6	5121·2
	254·	5137·1
81	254·4	5153·0
	254·8	5168·9
¼	255·2	5184·8
	255·6	5200·8
½	256·	5216·8
	256·4	5232·8
¾	256·8	5248·8
	257·2	5264·9
82	257·6	5281·0
	258·	5297·1
¼	258·3	5313·2
	258·7	5329·4
½	259·1	5345·6
	259·5	5361·8
¾	259·9	5378·0
	260·3	5394·3
83	260·7	5410·6
	261·1	5426·9
¼	261·5	5443·2
	261·9	5459·6
½	262·3	5476·0
	262·7	5492·4
¾	263·1	5508·8
	263·5	5525·3
84	263·8	5541·7
	264·2	5558·2
¼	264·6	5574·8
	265·	5591·3
½	265·4	5607·9
	265·8	5624·5
¾	266·2	5641·1
	266·6	5657·8
85	267·	5674·5
	267·4	5691·2
¼	267·8	5707·9
	268·2	5724·6
½	269·6	5741·4
	268·9	5758·2
¾	269·3	5775·0
	269·7	5791·9
86	270·1	5808·8
	270·5	5825·7
¼	270·9	5842·6
	271·3	5859·5
½	271·7	5876·5
	272·1	5893·5
¾	272·5	5910·5
	272·9	5927·6
87	273·3	5944·6
	273·7	5961·7
¼	274·1	5978·9
	274·4	5996·0
½	274·8	6013·2
	275·2	6030·4
¾	275·6	6047·6
	276·	6064·8
88	276·4	6082·1
	276·8	6099·4
¼	277·2	6116·7
	277·6	6134·0
½	278·	6151·4
	278·4	6168·8
¾	278·8	6186·2
	279·2	6203·6

Diameter.	Circ.	Area.	Diameter.	Circ.	Area.	Diameter.	Circ.	Area.
89	279·6	6221·1	93	292·1	6792·9	97	304·7	7389·8
	279·9	6238·6		292·5	6811·1		305·1	7408·8
¼	280·3	6256·1	¼	292·9	6829·4	¼	305·5	7427·9
	280·7	6273·6		293·3	6847·8		305·9	7447·0
½	281·1	6291·2	½	293·7	6866·1	½	306·3	7466·2
	281·5	6308·8		294·1	6884·5		306·6	7485·3
¾	281·9	6326·4	¾	294·5	6902·9	¾	307·	7504·5
	282·3	6344·0		294·9	6921·3		307·4	7523·7
90	282·7	6361·7	94	295·3	6939·7	98	307·8	7542·9
	283·1	6379·4		295·7	6958·2		308·2	7562·2
¼	283·5	6397·1	¼	296·	6976·7	¼	308·6	7581·5
	283·9	6414·8		296·4	6995·2		309·0	7600·8
½	284·3	6432·6	½	296·8	7013·8	½	309·4	7620·1
	284·7	6450·4		297·2	7032·3		309·8	7639·4
¾	285·1	6468·2	¾	297·6	7050·9	¾	310·2	7658·8
	285·4	6486·0		298·	7069·5		310·6	7678·2
91	285·8	6503·8	95	298·4	7088·2	99	311·0	7697·7
	286·2	6521·7		298·8	7106·9		311·4	7717·1
¼	286·6	6539·6	¼	299·2	7125·5	¼	311·8	7736·6
	287·	6557·6		299·6	7144·3		312·1	7756·1
½	287·4	6575·5	½	300·	7163·0	½	312·5	7775·6
	287·8	6593·5		300·4	7181·8		312·9	7795·2
¾	288·2	6611·5	¾	300·8	7200·5	¾	313·3	7814·7
	288·6	6629·5		301·2	7219·4		313·7	7834·3
92	289·	6647·6	96	301·5	7238·2	100	314·1	7853·9
	289·4	6665·7		301·9	7257·1		314·5	7853·6
¼	289·8	6683·8	¼	302·3	7275·9	¼	314·9	7893·3
	290 2	6701·9		302·7	7294·9		315·3	7913·1
½	290·5	6720·0	½	303·1	7313·8	½	315·7	7932·7
	290·9	6738·2		303·5	7332·8		316·0	7942·4
¾	291·3	6756·4	¾	303·9	7351·7	¾	316·4	7972·2
	291·7	6776·4		304·3	7370·7		316·8	7991·9

EXPLANATION OF THE TABLE FOR SEGMENTS, &c.

The *chord divided by the height* is the gauge in the Table, the quotient in the first column.

k = tabular coefficient, always to be multiplied by the chord.

To find the angle of an arc of a circle.

RULE. Divide the base (chord) of the arc by its height, (*sine verse*) and find the quotient in the first column. The corresponding number in the second column is the angle of the arc in degrees of the circle.

To find the radius of an arc of a circle.

RULE. Divide the chord of the arc by its height, and find the quotient in the first column. The corresponding number in the third column, multiplied by the chord, is the radius of the arc.

Chord div. by height.	Centre Angle v.	Radius $r = k\ c$.	Cir. Arc. $b = k\ c$.	Area Seg. $a = k\ c^2$.	Surface $a = k\ c^2$.	Solidity $C = k\ c^3$.	Chord $c = k\ r$.
458·08	1	57·296	1·0000	·01091	·78539	·00085	·01744
229·18	2	28·649	1·0000	·00218	·78549	·00172	·03490
152·77	3	19·101	1·0000	·00327	·78462	·00255	·05234
114·57	4	14·327	1·0000	·00436	·78574	·00310	·06978
84·747	5	11·462	1·0001	.00647	·78586	·00401	·08722
76·375	6	9·5530	1·0003	·00741	·78599	·00514	·10466
65·943	7	8·1902	1·0004	·00910	·78621	·00592	·12208
57·273	8	7·1678	1·0006	·01089	·78630	·00686	·13950
50·902	9	6·3728	1·0008	·01254	·78665	·00772	·15690
45·807	10	5·7368	1·0011	·01407	·78695	·00857	·17430
41·203	11	5·2167	1·0013	·01552	·78730	·00964	·19168
38·133	12	4·7834	1·0016	·01695	·78725	·01031	·20904
35·221	13	4·4168	1·0019	·01841	·78794	·01114	·22640
32·742	14	4·1027	1·0023	·02000	·78832	·01199	·24372
30·514	15	3·8307	1·0027	·02157	·78889	·01288	·26104
28·601	16	3·5927	1·0029	·02269	·78909	·01375	·27834
26·915	17	3·3827	1·0034	·02434	·78969	·01462	·29560
25·412	18	3·1962	1·0039	·02592	·79028	·01542	·31286
24·068	19	3·0293	1·0044	·02744	·79084	·01635	·33008
22·860	20	2·8793	1·0048	·02878	·79140	·01722	·34728
21·760	21	2·7440	1·0054	·03040	·79234	·01802	·36446
20·777	22	2·6222	1·0059	·03178	·79300	·01897	·38160
19·862	23	2·5080	1·0066	·03343	·79340	·01984	·39872
19·028	24	2·4050	1·0072	·03493	·79416	·02072	·41582
18·261	25	2·3101	1·0078	·03639	·79486	·02159	·43286
17·553	26	2·2233	1·0084	·03784	·79530	·02248	·44990
16·970	27	2·1418	1·0091	·03970	·79639	·02315	·46688
16·288	28	2·0673	1·0101	·04115	·79748	·02424	·48384
15·721	29	1·9969	1·0105	·04230	·79811	·02511	·50076
15·191	30	1·9319	1·0113	·04385	·79907	·02600	·51762
14·970	31	1·8710	1·0121	·04476	·78530	·02692	·53446
14·230	32	1·8140	1·0129	·04710	·80098	·02778	·55126
13·796	33	1·7605	1·0138	·04842	·80181	·02866	·56802
13·382	34	1·7102	1·0146	·04989	·80300	·02956	.58479
12·994	35	1·6628	1·0155	·05137	·80405	·03046	·60140
12·733	36	1·6184	1·0167	·05311	·80531	·03137	·61802
12·473	37	1·5758	1·0174	·05401	·80622	·03226	·63460
11·931	38	1·5358	1·0184	·05628	·80713	·03328	·65112
11·621	39	1·4979	1·0194	·05755	·80850	·03418	·66760
11·342	40	1·4619	1·0204	·05899	·80987	·03506	·68404
11·060	41	1·4266	1·0207	·06001	·81046	·03589	·70040
10·791	42	1·3952	1·0226	·06196	·81240	·03680	·71672
10·534	43	1·3643	1·0237	·06359	·81377	·03773	·73300
10·289	44	1·3347	1·0248	·06574	·81505	·03864	·74920
10·043	45	1·3066	1·0260	·06628	·81756	·03890	·76536
9·8303	46	1·2797	1·0272	·06826	.81795	·04050	·78146
9·6153	47	1·2539	1·0290	·06998	·81939	·04143	·79748
9·4092	48	1·2289	1·0297	·09138	·82064	·04247	·81346

Chord div. by height.	Centre Angle v.	Radius $r = k\ c$.	Cir. Arc. $b = k\ c$.	Area Seg. $a = k\ c^2$.	Surface $a = k\ c^2$.	Solidity $C = k\ c^3$.	Chord $c = k\ r$.
							
9·2113	49	1·2057	1·0309	·07290	·82244	·04330	·82938
9·0214	50	1·1831	1·0323	·07453	·82384	·04424	·84522
8·8387	51	1·1614	1·0336	·07611	·82562	·04519	·86102
8·6629	52	1·1406	1·0349	·07758	·82729	·04614	·87674
8·4462	53	1·1206	1·0364	·07959	·83363	·04685	·89238
8·3306	54	1·1014	1·0378	·08083	·83072	·04805	·90798
8·1733	55	1·0828	1·0393	·08246	·83249	·04901	·92348
8·0215	56	1·0650	1·0407	·08400	·83422	·05002	·93894
7·8750	57	1·0478	1·0422	·08579	·83602	·05098	·95430
7·7334	58	1·0313	1·0431	·08680	·83796	·05191	·96960
7·5895	59	1·0154	1·0454	·08891	·84064	·05299	·98484
7·4565	60	1·0000	1·0470	·09106	·84266	·05400	1·0000
7·3358	61	·98515	1·0486	·09209	·84380	·05466	1·0150
7·2118	62	·97080	1·0503	·09375	·84581	·05583	1·0300
7·0914	63	·95694	1·0520	·09540	·84791	·05684	1·0450
6·9748	64	·94352	1·0537	·09697	·84996	·05784	1·0598
6·8616	65	·93058	1·0555	·09865	·85215	·05885	1·0746
6·7512	66	·91804	1·0573	·10036	·85441	·05987	1·0892
6·6453	67	·90590	1·0591	·10201	·85640	·06088	1·1038
6·5469	68	·89415	1·0610	·10367	·85815	·06181	1·1184
6·4902	69	·88276	1·0629	·10520	·85464	·06201	1·1328
6·3431	70	·87172	1·0648	·10710	·86350	·06396	1·1471
6·2400	71	·86102	1·8668	·10887	·86699	·06515	1·1614
6·1553	72	·85065	1·0687	·11046	·86834	·06604	1·1755
6·0652	73	·84058	1·0708	·11225	·87081	·06709	1·1896
5·9773	74	·83082	1·0728	·11385	·87935	·06815	1·2036
5·8918	75	·82134	1·0749	·11563	·87590	·06921	1·2175
5·8084	76	·81213	1·0770	·11736	·87853	·07037	1·2313
5·7271	77	·80319	1·0792	·11910	·88120	·07136	1·2450
5·6478	78	·79449	1·0814	·12072	·88389	·07244	1·2586
5·5704	79	·78606	1·0836	·12281	·88677	·07352	1·2721
5·4949	80	·77786	1·0859	·12441	·88949	·07462	1·2855
5·4254	81	·76988	1·0882	·12660	·89161	·07512	1·2989
5·3492	82	·76212	1·0905	·12793	·89520	·07683	1·3121
5·2705	83	·75458	1·0920	·12958	·89958	·07819	1·3252
5·2101	84	·74724	1·0953	·13157	·90095	·07907	1·3383
5·1429	85	·74009	1·0977	·13330	·90420	·07960	1·3512
5·0772	86	·73314	1·1012	·13546	·90734	·08102	1·3639
5·0134	87	·72637	1·1027	·13704	·91036	·08440	1·3767
4·9501	88	·71978	1·1054	·13893	·91363	·08836	1·3893
4·8886	89	·71336	1·1079	·14078	·91696	·08450	1·4818
4·8216	90	·70710	1·1105	·14279	·92210	·08621	1·4142
4·7694	91	·70101	1·1132	·14449	·92352	·08716	1·4265
4·7117	92	·69508	1·1159	·14643	·92476	·08798	1·4387
4·6615	93	·68930	1·1186	·14817	·92914	·08932	1·4507
4·5999	94	·68366	1·1211	·15009	·93385	·09076	1·4627
4·5453	95	·67817	1·1242	·15211	·93746	·09197	1·4745
4·4845	96	·67282	1·1271	·15375	·94272	·09348	1·4863

Chord div. by height.	Centre Angle v.	Radius $r = k\,c$.	Cir. Arc. $b = k\,c$.	Area Seg. $a = k\,c^2$.	Surface $a = k\,c^2$.	Solidity $C = k\,c^3$.	Chord $c = k\,r$.
4·4398	97	·66760	1·1300	·15600	·94470	·09442	1·4979
4·3859	98	·66250	1·1329	·15801	·94852	·09567	1·5094
4·3383	99	·65754	1·1359	·15995	·95236	·09693	1·5208
4·2862	100	·65270	1·1382	·16180	·95682	·09831	1·5321
4·2406	101	·64798	1·1420	.16393	·96011	·09856	1·5432
4·1930	102	·64338	1·1451	·16610	·96412	·10076	1·5543
4·1570	103	·63889	1·1483	·16925	·96568	·10557	1·5652
4·1006	104	·63450	1·1515	·17001	·97246	·10273	1·5760
4·0555	105	·63023	1·1547	·17204	·97643	·10471	1·5867
4·0113	106	·62607	1·1580	·17414	·98067	·10601	1·5973
3·9679	107	·62200	1·1614	·17619	·98495	·10735	1·6077
3·9252	108	·61803	1·1648	·17832	·98931	·10870	1·6180
3·8832	109	·61416	1·1682	·18041	·99376	·11007	1·6282
3·8419	110	·61039	1·1716	·18257	·98827	·11149	1·6383
3·8013	111	·60670	1·1752	·18472	1·0028	·11284	1·6482
3·7612	112	·60325	1·1790	·18696	1·0077	·11426	1·6581
3·7221	113	·59960	1·1823	·18900	1·0122	·11566	1·6677
3·6837	114	·59618	1·1859	·19117	1·0169	·11709	1·6773
3·6454	115	·59284	1·1897	·19339	1·0218	·11853	1·6867
3·6086	116	·58959	1·1934	·19559	1·0266	·11995	1·6961
3·5712	117	·58641	1·1972	·19787	1·0317	·12145	1·7053
3·5349	118	·58331	1·2011	·20009	1·0368	·12294	1·7143
3·4992	119	·58030	1·2050	·20227	1·0417	·12444	1·7232
3·4641	120	·57735	1·2089	·20453	1·0472	·12596	1·7320
3·4296	121	·57450	1·2130	·20678	1·0525	·12748	1·7407
3·3953	122	·57168	1·2177	·20945	1·0578	·12903	1·7492
3·3616	123	·56895	1·2213	·21175	1·0634	·13060	1·7576
3·3285	124	·56628	1·2253	·21399	1·0690	·13218	1·7659
3·2940	125	·56370	1·2295	·21538	1·0753	·13391	1·7740
3·2637	126	·56116	1·2338	·21859	1·0803	·13558	1·7820
3·2319	127	·55870	1·2381	·22121	1·0862	·13701	1·7898
3·2006	128	·55630	1·2425	·22370	1·0921	·13866	1·7976
3·1716	129	·55396	1·2470	·22617	1·0974	·14028	1·8051
3·1393	130	·55169	1·2515	·22865	1·1040	·14202	1·8126
3·1093	131	·54947	1·2561	·23113	1·1104	·14371	1·8199
3·0805	132	·54732	1·2607	·23372	1·1164	·14537	1·8271
3·0555	133	·54522	1·2654	·23603	1·1212	·14676	1·8341
3·0216	134	·54318	1·2701	·23892	1·1295	·14894	1·8410
2·9777	135	·54120	1·2749	·24198	1·1420	·15209	1·8477
2·9651	136	·53927	1·2798	·24364	1·1428	·15252	1·8543
2·9374	137	·53740	1·2847	·24676	1·1495	·15422	1·8608
2·9115	138	·53557	1·2897	·24938	1·1558	·15605	1·8671
2·8829	139	·53380	1·2948	·25222	1·1634	·15807	1·8733
2·8562	140	·53209	1·2999	·25485	1·1705	·15996	1·8794
2·8299	141	·53042	1·3051	·25759	1·1777	·16201	1·8853
2·8038	142	·52881	1·3065	·25936	1·1851	·16381	1·8910
2·7781	143	·52724	1·3157	·26320	1·1925	·16577	1·8966
2·7527	144	·52573	1·3211	·26604	1·2000	·16776	1·9021

Chord div. by height.	Centre Angle v.	Radius $r = k\ c$.	Cir. Arc. $b = k\ c$.	Area Seg. $a = k\ c^2$.	Surface $a = k\ c^2$.	Solidity $c = k\ c^3$.	Chord $c = k\ r$.
2·7276	145	·52426	1·3265	·26889	1·2077	·16965	1·9074
2·7002	146	·52284	1·3320	·27196	1·2166	·17209	1·9126
2·6816	147	·52147	1·3377	·27449	1·2219	·17205	1·9176
2·6533	148	·52015	1·3433	·27772	1·2318	·17605	1·9225
2·6301	149	·51887	1·3491	·28168	1·2396	·17809	1·9272
2·6064	150	·51764	1·3549	·28369	1·2476	·18023	1·9318
2·5830	151	·51645	1·3608	·28674	1·2563	·18666	1·9363
2·5598	152	·51530	1·3668	·28983	1·2648	·18751	1·9406
2·5239	153	·51420	1·3729	·29397	1·2801	·18845	1·9447
2·5143	154	·51315	1·3790	·29607	1·2824	·18913	1·9487
2·4919	155	·51214	1·3852	·29928	1·2914	·19147	1·9526
2·4699	156	·51117	1·3919	·30259	1·3004	·19374	1·9563
2·4478	157	·51014	1·3973	·30560	1·3094	·19607	1·9598
2·4262	158	·50936	1·4043	·30905	1·3191	·20029	1·9632
2·4047	159	·50851	1·4109	·31239	1·3287	·20095	1·9663
2·3835	160	·50771	1·4175	·31575	1·3368	·20342	1·9696
2·3613	161	·50695	1·4243	·31931	1·3490	·20609	1·9725
2·3417	162	·50623	1·4311	·32263	1·3583	·20847	1·9753
2·3211	163	·50555	1·4380	·32618	1·3682	·21105	1·9780
2·3004	164	·50491	1·4450	·32969	1·3791	·21371	1·9805
2·2805	165	·50431	1·4520	·33327	1·3895	·21634	1·9829
2·2605	166	·50374	1·4592	·33684	1·4021	·21904	1·9851
2·2408	167	·50323	1·4665	·34048	1·4111	·22177	1·9871
2·2212	168	·50275	1·4739	·34422	1·4222	·21946	1·9890
2·2013	169	·50231	1·4813	·34802	1·4344	·22766	1·9908
2·1826	170	·50191	1·4889	·35230	1·4476	·23028	1·9924
2·1636	171	·50154	1·4966	·35563	1·4565	·23266	1·9938
2·1447	172	·50122	1·5044	·35953	1·4684	·23650	1·9951
2·1271	173	·50093	1·5123	·36337	1·4797	·23900	1·9962
2·1075	174	·50068	1·5202	·36747	1·4927	·24225	1·9972
2·0892	175	·50047	1·5283	·37152	1·5052	·24537	1·9981
2·0710	176	·50030	1·5365	·37562	1·5179	·24856	1·9988
2·0530	177	·50017	1·5448	·37974	1·5308	·25179	1·9993
2·0352	178	·50007	1·5533	·38401	1·5439	·25531	1·9996
2·0175	179	·50002	1·5618	·38828	1·5573	·25840	1·9999
2·0000	180	·50000	1·5707	·39269	1·5708	·26179	2·0000

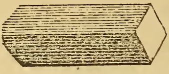
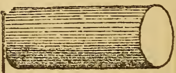
To find the length of an arc of a circle.

RULE. Divide the chord of the arc by its height, and find the quotient in the first column. The corresponding number in the fourth column multiplied by the chord is the length of the arc.

To find the area of a segment of a circle,

RULE. Divide the chord of the segment by its height, and find the quotient in the first column. The corresponding number in the fifth column multiplied by the square of the chord, is the area of the segment.

Coefficient for Capacity and Weight,


Names of Substances.	*FFF.*	*Fii.*	*iii.*	*FF²*.	*Fi²*.	*ii²*.	*F³*.	*i³*.
Cubic inches, - -	1728	12	1	1356	9·42	0·785	903·7	0·523
Cubic feet, - - -	1	·..694	·....58	0·785	·..549	·....44	0·523	·.... 3
Gallons, - - - -	7·476	0·052	·...433	5·868	·.408	·....34	3·91	·..226
Water, fresh, - -	62·5	0·433	0·036	49	0·34	·.283	32·7	0·019
Water, salt, - - -	64·3	0·445	0·037	50·4	0·35	0·029	33·6	0·02
Oil, - - - - - -	57·5	0·4	0·033	45·1	0·313	0·026	30	0·017
Cast-iron, - - -	450	3·12	0·26	353	2·45	0·204	235	0·136
Wrought-iron, - -	487	3·37	0·281	382	2·65	0·221	255	0·147
Steel, - - - - -	490	3·4	0·283	385	2·67	0·222	257	0·149
Brass, - - - - -	532	3·68	0·307	417	2·9	0·241	278	0·161
Tin, - - - - -	456	3·16	0·263	358	2·48	0·207	239	0·138
Lead, - - - - -	710	4·92	0·41	557	3·87	0·322	371	0·215
Zinc, - - - - -	440	3·05	0·254	345	2·4	0·2	230	0·133
Copper, - - - -	556	3·85	0·321	436	3·03	0·252	291	0·168
Mercury,. - - -	850	5·9	0·491	666	4·63	0·385	445	0·257
Stone, common, -	156	1·08	0·09	122	0·85	0·071	82	0·047
Clay, - - - - -	135	0·936	0·078	106	0·735	0·061	70	0·04
Earth, compact, -	127	0·88	0·0733	99	0·692	0·058	66	0·038
Earth, loose, - -	95	0·66	0·055	74	0·517	0·043	50	0·029
Oak, dry, - - -	58	0·4	0·033	44	0·316	0·026	30	0·017
Pine, - - - - -	30	0·208	0·017	24	0·163	0·014	16	0·009
Mahogany, - - -	66	0·457	0·038	52	0·36	0·03	34	0·02
Coal, stone, - - -	54	0·375	0·031	42	0·294	0·024	28·2	0·016
Charcoal, - - - -	27·5	0·19	0·016	21	0·15	0·012	14·4	0·008

To Find the Weight and Capacity by this Table

RULE. The product of the dimensions in feet or in inches, as noted in the columns, multiplied by the tabular coefficient, is the capacity of the solid, or weight in pounds avoirdupois.

Example 1. A cistern is 6 feet long, 27 inches wide, and 20 inches deep. How many gallons of liquid can be contained in it?

$$6\times27\times20\times0{\cdot}052 = 168\ 48 \text{ gallons.}$$

Example 2. A cast-iron cylinder is 4·5 feet long, and 7·5 inches diameter. Required the weight of it?

$$4{\cdot}5+7{\cdot}5^2\times2{\cdot}45 = 620 \text{ pounds.}$$

Angle. W	*Ordinates.* 1. 7.	2. 6.	3. 5.	4. h.	*Angle.* W	*Ordinates.* 1. 7.	2. 6.	3. 5.	4. h.
1°	·00084	·00164	·00193	·00218	53°	·05313	·08932	·11063	·11773
2	·00191	·00327	·00409	·00436	54	·05422	·09130	·11318	·12003
3	·00299	·00522	·00561	·00659	55	·05531	·09308	·11510	·12235
4	·00382	·00654	·00818	·00872	56	·05646	·09487	·11731	·12466
5	·00437	·00818	·01023	·01091	57	·05760	·09673	·11950	·12698
6	·00573	·00928	·01228	·01309	58	·05875	·09853	·12170	·12932
7	·00675	·01173	·01432	·01527	59	·05989	·10037	·12393	·13162
8	·00764	·01309	·01639	·01746	60	·06094	·10220	·12612	·13397
9	·00845	·01474	·01842	·01964	61	·06261	·10427	·12840	·13631
10	·00955	·01637	·02047	·02183	62	·06331	·10593	·13054	·13866
11	·01053	·01801	·02250	·02402	63	·06451	·10781	·13281	·14101
12	·01146	·01965	·02456	·02620	64	·06570	·10964	·13505	·14337
13	·01245	·02129	·02662	·02839	65	·06681	·11101	·13765	·14573
14	·01284	·02271	·02861	·03058	66	·06805	·11342	·13956	·14810
15	·01438	·02461	·03081	·03282	67	·06914	·11532	·14181	·15048
16	·01535	·02625	·03277	·03496	68	·07040	·11721	·14409	·15286
17	·01630	·02789	·03484	·03715	69	·07168	·11912	·14637	·15526
18	·01730	·02956	·03693	·03935	70	·07284	·12103	·14864	·15765
19	·01858	·03125	·03996	·04154	71	·07407	·12294	·15087	·16005
20	·01922	·03286	·04103	·04374	72	·07535	·12485	·15323	·16245
21	·02022	·03453	·04309	·04594	73	·07656	·12685	·15555	·16487
22	·02119	·03619	·04522	·04814	74	·07784	·12877	·15785	·16729
23	·02215	·03787	·04720	·05034	75	·07912	·13078	·16016	·16972
24	·02311	·03934	·04930	·05255	76	·08040	·13292	·16247	·17216
25	·02413	·04117	·05138	·05475	77	·08168	·13472	·16482	·17460
26	·02508	·04283	·05346	·05696	78	·08297	·13670	·16716	·17706
27	·02610	·04457	·05552	·05917	79	·08426	·13868	·16951	·17951
28	·02708	·04621	·05761	·06139	80	·08560	·14070	·17187	·18198
29	·02813	·04793	·05970	·06361	81	·08695	·14274	·17423	·18445
30	·02911	·04970	·06188	·06582	82	·08829	·14477	·17660	·18694
31	·03005	·05125	·06386	·06804	83	·08944	·14681	·17901	·18943
32	·03107	·05298	·06596	·07027	84	·09105	·14888	·18140	·19193
33	·03191	·05464	·06806	·07250	85	·09235	·15120	·18379	·19444
34	·03310	·05637	·07016	·07477	86	·09377	·15304	·18622	·19695
35	·03412	·05804	·07424	·07695	87	·09518	·15509	·18865	·19946
36	·03515	·05992	·07452	·07919	88	·09660	·15756	·19108	·20201
37	·03616	·06147	·07646	·08143	89	·09780	·15931	·19350	·20555
38	·03718	·06327	·07858	·08367	90	·09944	·16144	·19597	·20710
39	·03821	·06492	·08069	·08591	91	·10098	·16359	·19842	·20966
40	·03905	·06631	·08243	·08816	92	·10240	·16575	·20092	·21223
41	·04030	·06836	·08494	·09041	93	·10384	·16787	·20338	·21481
42	·04133	·07012	·08707	·09266	94	·10537	·17005	·20589	·21740
43	·04241	·07182	·08920	·09492	95	·10692	·17224	·20837	·22000
44	·04363	·07353	·09130	·09719	96	·10851	·17444	·21091	·22262
45	·04522	·07531	·09346	·09945	97	·10997	·17666	·21342	·22523
46	·04556	·07706	·09562	·10172	98	·11150	·17888	·21596	·22786
47	·04682	·07894	·09790	·10400	99	·11310	·18111	·22800	·23050
48	·04833	·08059	·09991	·10627	100	·11468	·18354	·22107	·23315
49	·04879	·08236	·00207	·10856	101	·11626	·18500	·22364	·23596
50	·04982	·08413	·00422	·11085	102	·11791	·18793	·22623	·23848
51	·05096	·08593	·10639	·11314	103	·11959	·19021	·22876	·24107
52	·05204	·08768	·10855	·11543	104	·12116	·19256	·23147	·24386

RAIL ROAD CURVES.

WHEN Railroads are to be connected by curves, we commonly have given the *distance* (chord c,) between the two ends *o o* of the tracks, and the tangential angle v. By these the curve is to be constructed.

Example 1. Fig. 128. The chord C = 168 feet, and the tangential angle $v = 19° 30'$. Required the centre angle w =, and the radius R = ?

$$w = 2(19° 30') = 39°. \quad R = {}^{39}k\,c = 1{\cdot}4979 \times 168 = 251{\cdot}647 \text{ feet.}$$

k = See Table for Segments, &c., of a circle.

By Tangential Angles.

The curve to be laid out by the three tangential angles *r o r*, *r o n*, and *n o o*, each angle = $\frac{1}{3}v = 6° 30'$. Required the chord r = ?

The centre angle for the chord r is

$$2 \times (6° 30') = 13°, \text{ and } r = {}^{13}k\,R = 0{\cdot}2264 \times 251{\cdot}647 = 56{\cdot}974 \text{ feet.}$$

By Angles of Deflexion.

Divide the centre angle w into an even number of parts = z. Set off at o the angle z = *r o n*, and bisect it into *r o r* and *r o n*,—find the chord r, and sub-chord a, and continue as shown by Figure.

Example 2. Fig. 128. The tangential angle $v = 78°$, and the chord C = 638 feet. Required the centre-angle w = ? Radius R = ? Chord r = ? and the sub-chord a = ?

$$w = 2 \times 78° = 156°. \quad R = {}^{156}k\,c = 0{\cdot}51117 \times 638 = 326{\cdot}126 \text{ feet.}$$

Let the curve be laid out by 6 angles of deflexion, and $z = \frac{1}{6} \times 156° = 26°$, and

$$r = {}^{26}k\,R = 0{\cdot}4499 \times 326{\cdot}126 = 146{\cdot}73 \text{ feet.}$$
$$a = {}^{26}k\,r = 0{\cdot}4495 \times 146{\cdot}73 = 66{\cdot}012 \text{ feet.}$$

By Ordinates.

Example 3. Fig. 129. The chord C = 368 feet, and $v = 36°$. Required the height h = ?

$$h = \tfrac{1}{2}C(\text{cosec.}v - \text{cot.}v).$$

From - - - - - - - - cosec.36° = 1·70130
Subtract - - - - - - - cot.36° = 1·37638
The height $h = \overline{0{\cdot}32492} \times 184 = 59{\cdot}785$ feet.

At x = 92 feet from h. Required the ordinate y?

$$\sin.z = \frac{2 \times 92 \sin.36°}{368} = 0{\cdot}2938926 = \sin.17° 6'.$$

$$y = \tfrac{1}{2} \times 368 \left(\frac{\cos.17° 6'}{\sin.36°} - \cot.36° \right) = 45{\cdot}9448 \text{ feet.}$$

By Sub-Chords.

Example 4. Fig. 130. The ends o and o of the tracks form different angles w and W to the chord C, and therefore must be connected by two curves of different radii, R and r. The chord C = 869 feet, $w = 38°$, and $W = 86°$. Required the distance from o to the height h, n = ? sub-chord b = ? sub-chord a = ? radii R and r = ?

$$v = \tfrac{1}{2} \times 38° = 19°, \quad \text{and } V = \tfrac{1}{2} \times 86° = 43°.$$

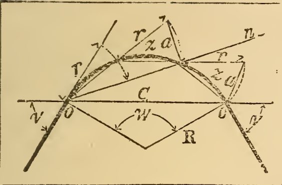
$$n = \frac{869 \tan.19°}{\tan.19° + \tan.43°} = 234{\cdot}35 \text{ feet.}$$

$b = 234{\cdot}35 \sec.43° = 320{\cdot}42$ feet. | $R = {}^{38}k\,a = 1{\cdot}5358 \times 671{\cdot}21 = 1030{\cdot}2$ ft.
$a = \sec.19°(869 - 234{\cdot}35) = 671{\cdot}21$ ft. | $r = {}^{86}k\,b = 0{\cdot}73314 \times 320{\cdot}42 = 234{\cdot}91$ ft.

By Eight Ordinates.

Example 5. Fig. 133. Required 8 ordinates for a curve of chord C = 710 feet and the centre angle $w = 69°$? (See Table on the preceding page.)

1st and 7th Ordinates	0·07168×710 =	50·8928 feet.
2nd " 6th "	0·11912×710 =	84·5752 "
3rd " 5th "	0·14637×710 =	103·9227 "
4th or height h	0·15526×710 =	110·2346 "

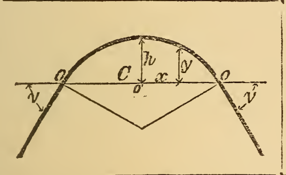


128.

By angles of deflexion.

$w = 2v, \quad R = {}^{w}k\,C = \frac{1}{2}C \text{ cosec.}v.$

$r = {}^{z}k\,R, \quad a = {}^{z}k\,r = 2r \text{ sin.}\frac{1}{2}z.$



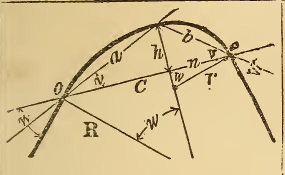
129.

By Ordinates.

$h = \frac{1}{2}C(\text{cosec.}v - \text{cot.}v).$

$y = \frac{1}{2}C\left(\frac{\text{cos.}z}{\text{sin.}v} - \text{cot.}v\right),$

$\text{sin.}z = \frac{2x \text{ sin.}v}{C}.$



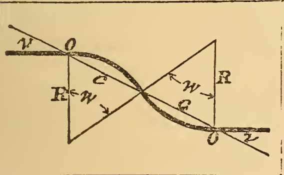
130. *By Sub-chords.*

$n = \frac{C \text{ tan.}v}{\text{tan.}v + \text{tan.}V}, \quad h = n \text{ tan.}V,$

$b = n \text{ sec.}V, \quad w = 2v$

$W = 2V$

$a = \text{sec.}v(C - n),$

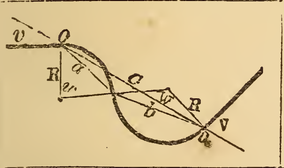


131.

Parallel tracks by a reverse curve.

Formulas same as above.

The length $o\,o = 2c$, length of a circle arc $l = 0{\cdot}035v\,R$.



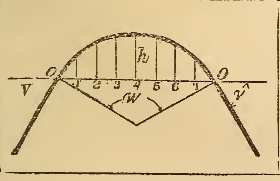
132.

The greatest radius in a reverse curve.

$w = \frac{1}{2}(V+3v), \quad W = w+V - v,$

$a = {}^{w}k\,R, \quad b = {}^{w}k\,R,$

$R = C \text{ sec.}w(\text{sin.}V - \sqrt{\text{sin.}^2V - \text{cos.}^2w}).$



133.

Curve by 8 Ordinates.

The ordinates are calculated in the accompanying Table, the chord $C = 1$ or the unit.

If the angle w is large, or there be some obstacle on the chord C, find the height h and lay out the curve by two or more sets of 8 ordinates.

By Ordinates and Subchords.

Example 6. Fig. 134. The tangents t being prolonged to where they meet at a, divide that angle into two equal parts, say $W=75°$. Required the tangents $t=?$ external secant $S=?$ chords $C=?$ and the angle $w=?$ Radius of the curve $R=1500$ feet.

$t=R$ cot.$75°=1500\times0{\cdot}26794=401{\cdot}91$ feet.
Centre angle $w=90-75°=15°$ for half the curve.
$S=R$ (sec.$15°-1$) $=1500$ $(1{\cdot}0352-1)=52{\cdot}8$ feet.
The chords $C=k$ $R=0{\cdot}26104\times1500=391{\cdot}56$ feet.

Measure off from a the tangents and the external secant.

Draw the chords C C, and divide them each into eight equal parts. In the table of ordinates under $w=15°$ will be found the

1st. 7th. $0{\cdot}01438\times391{\cdot}56=5{\cdot}631$,	3rd. 5th. $0{\cdot}03081\times391{\cdot}56=12{\cdot}063$,
2nd. 6th. $0{\cdot}02461\times391{\cdot}56=9{\cdot}636$,	4th. $0{\cdot}03282\times39{\cdot}56=12{\cdot}851$,

Thus by only four multiplications, 16 ordinates in the curve is obtained.

Should there be any obstacles for the chords C. C. as is often the case in excavations and on embankments, a line can be drawn further in on the track parallel to the chord and the ordinates obtained by subtraction, readily understood by the Engineer.

Ellipse by Ordinates.

By this arrangement ellipses can be constructed of any proportions. One of the two axes is divided into 16 equal parts. The ordinates drawn and calculated as shown by the figure 135.

Parallel Tracks by a Semi-Ellipse,

Example 7. Fig. 136. The instrument placed at b and b', divide the angles W and w each into two equal parts, prolong the chords which will meet at a, a point in the curve. Divide the chords each into eight equal parts, and draw the ordinates parallel to the tracks as shown in the figure. The grand chord C is the unit for calculating the ordinates, which latter are alike on both the chords c', c''.

1st	2nd.	3rd.	4th.	5th.	6th.	7th.
$0{\cdot}1795C$	$0{\cdot}2058C$	$0{\cdot}2029C$	$0{\cdot}1830C$	$0{\cdot}1477C$	$0{\cdot}1091C$	$0{\cdot}0586C$.

Suppose the grand chord to be $C=2050$ feet.

Required the length of the 6th ordinate? $0{\cdot}1091\times2050=223{\cdot}655$ feet.

Tracks not Parallel by Elliptic, arc,

Example 8. Fig. 137. Divide the angles W and w each into two equal parts, prolong the subchords until they intersect one another at a, which is a point in the curve. Divide the chord C into eight equal parts, join a with the 4th division and draw the other ordinates parallel thereto.

Suppose the angles are $W=18°$ and $w=12°$, the centre angle will be 30° for which the ordinates are to be calculated from the table. The chord $C=125$ feet. Required the 3rd and 5th ordinates? $0{\cdot}06188\times125=7{\cdot}335$ feet.

Springing of Rails.

Example 9. Fig. 138. A rail of $L=21$ feet is to be curved to a radius of $R=1250$ feet. Required the spring $S=?$ in sixteenths of an inch.

$$S=\frac{24\times21^2}{1250}=8{\cdot}47 \text{ sixteenths.}$$

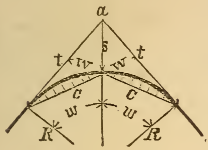
Super Elevation of the External Rail.

Example 10. Fig. 139. A train running $M=30$ miles per hour on a curve of $R=1550$ feet radii, the gauge of the track is $G=5$ feet. Required the angle of inclination $v=?$ and the super elevation of the external rail $h=?$

$$tan.v=\frac{30^2}{15\times1550}=0{\cdot}0237=\text{tan. } 1°\ 21'.$$

$h=G$ sin.$1°\ 21'=5\times0{\cdot}02356=0{\cdot}1178$ feet, or nearly $1\frac{1}{8}$ inches.

It is practically impossible to lay the super elevation to suit the different speeds of trains. If a mean speed is taken, the faster passenger trains will wear the outer rail, and the slow or freight train will wear the inner rail.



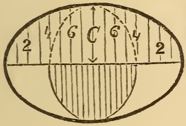
134.

By ordinates and subchords.

$$t = R\cot.W = R\tan.w, \quad W = 90 - w,$$

$$S = R(\sec.w - 1) = R(\operatorname{cosec}.W - 1)$$

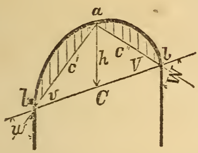
$C = k\,R$. For k, see table of segments.



135.

Ellipse by ordinates.

$1 = 0{\cdot}4840\,C$	$5 = 0{\cdot}9204\,C$
$2 = 0{\cdot}6616\,C$	$6 = 0{\cdot}9682\,C$
$3 = 0{\cdot}7808\,C$	$7 = 0{\cdot}9922\,C$
$4 = 0{\cdot}8660\,C$	$8 = C$ the unit.



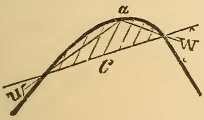
136.

Parallel tracks by elliptic curve.

$$h = \tfrac{1}{2}\,C. \quad w = 2\,v. \quad W = 2\,V,$$

$$c' = \frac{C\sin.W}{2\sin.v}, \qquad c'' = \frac{C\sin.w}{2\sin.V},$$

See example for ordinates.



137.

Tracks not parallel by elliptic arc.

Angle of the arc $= W + w$.

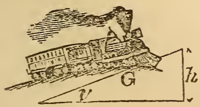
Ordinates to be calculated from the table.

138.

Spring of Rails.

$$S = \frac{1{\cdot}5\,L^2}{R} = \text{spring in inches.}$$

$$S = \frac{2\,4\,L^2}{R} = \text{16ths of an inch.}$$



139.

Inclination of tracks in curves.

$$\tan.v = \frac{M^2}{15\,R}. \qquad h = G\sin.v.$$

Meaning of letters, see example.

EXCAVATION AND EMBANKMENT.

Example 1. The Road-way of an excavated channel is $r = 15$ feet, the depth $D = 9$ feet, and the breadth at the top $b = 46\frac{1}{2}$ feet. Require the slope $S = ?$

Formula 6. $$S = \frac{46{\cdot}5 - 15}{2 \times 9} = 1{\cdot}75 \text{ or } 1\tfrac{3}{4} \text{ to } 1{\cdot}$$

Example 2. The Road way is to be $r = 15$, $D = 18$, and the slope $S = 1\frac{1}{4}$, Require the breadth $b = ?$ and the cross-section $\mathbf{A} = ?$

Formula 4. $$b = 2 \times 18 \times 1{\cdot}25 + 15 = 60 \text{ feet.}$$

Formula 7. $$\mathbf{A} = \frac{18}{2}\left(60 + 15\right) = 675 \text{ square feet.}$$

Example 3. The Road-way is to be $r = 16$ feet, the slope $S = 1\frac{1}{2}$, and the depth $D = 11$ feet. Required the area of Cross-section $\mathbf{A} = ?$

Formula 9. $$\mathbf{A} = 11\,(11 \times 1\tfrac{3}{4} + r) = 357{\cdot}5 \text{ square feet.}$$

Example 4. The Road-way $r = 18$ feet, slope $S = 1\frac{1}{4}$, $d = 14$ feet 6 inches, and the length from o is $l = 55$ feet. Required the cubic contents $\mathbf{c} = ?$

Formula 11. $\mathbf{c} = 55 \times 14{\cdot}5\left(\dfrac{14{\cdot}5 \times 1{\cdot}25}{3} + \dfrac{18}{2}\right) = 11995{\cdot}676$ cubic feet, divided by $27 = 444.28$ cubic yards.

Example 5. The Road-way is $r = 16$ feet, slope $S = 1\frac{1}{2}$ feet, $D = 17{\cdot}5$, $d = 7{\cdot}4$ and the length $L = 100$ feet. Required the cubic content $\mathbf{C} = ?$

Formula 12. $$\mathbf{C} = 100\left[1\tfrac{1}{2}\left(\frac{17.5^2 + 7{\cdot}4^2 + 17{\cdot}5 \times 7{\cdot}4}{3}\right) + \frac{16}{2}\left(17{\cdot}5 + 7{\cdot}4\right)\right]$$
$= 44445$ cubic feet, or $1645{\cdot}4$ cubic yards.

The computation is executed thus.

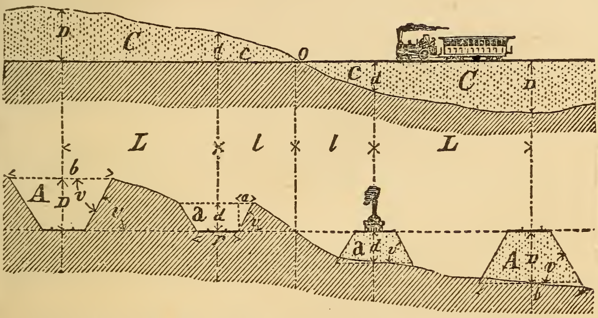
17·5	17·5
7·4	7.4
700	24·9
1225	8
129·50	199·2

$$\begin{array}{rl} & 129{\cdot}50 \\ 17{\cdot}5^2 = & 306{\cdot}25 \\ 7{\cdot}4^2 = & 54{\cdot}76 \end{array}\Big\}\text{ From table of Squares.}$$

$$3)\ 490{\cdot}51\ (163{\cdot}5 \quad \Big\}\ \text{Slope, add } \tfrac{1}{2}$$
$$81{\cdot}75$$
$$199{\cdot}2$$

$$\times\ 100 = 44445. \text{ cubic feet.}$$

134



Letters in the Formulas correspond with the Figure.

$$S = cot.\, v, \qquad 1.$$

$$a = D\,S, \qquad 2.$$

$$a = D\, cot.\, v, \qquad 3.$$

$$b = 2\,D\,S + r, \qquad 4.$$

$$S = \frac{a}{D}, \qquad 5.$$

$$S = \frac{b - r}{2\,D}, \qquad 6.$$

$$\mathbf{A} = \frac{D}{2}(b + r), \qquad 7.$$

$$\mathbf{a} = \frac{d}{2}(b + r), \qquad 8.$$

$$\mathbf{A} = D(D\,S + r), \qquad 9.$$

$$\mathbf{a} = d(d\,S + r), \qquad 10.$$

$$\mathbf{c} = l\,d\left(\frac{d\,S}{3} + \frac{r}{2}\right), \qquad 11.$$

$$\mathbf{C} = L\left[S\left(\frac{D^2 + d^2 + Dd}{3}\right) + \frac{r}{2}(D + d)\right]. \qquad 12.$$

Letters Denote,

A and **a** = Cross-Sections in square feet, of the excavated channel or embankment.

D and d = depth in feet, of the Sections.

r = width in feet of the Road-Way.

b = Base in feet of the embankment, or top breadth of the channel.

L = length in feet, between the two Sections **A** and **a**.

l = length in feet, from the Section **a** to the point **o** where the ground is level with the road.

C = cubic contents in feet, between **A** and **a**.

c = cubic contents in feet, between **a** and **o**.

S = slope of the sides. The slope is commonly given in proportions, thus: "Slope = $1\frac{1}{2}$ to 1," which means, that the side slopes $1\frac{1}{2}$ feet horizontally for 1 foot vertical.

v = angle of the slope.

TRACTION ON ROADS.

Letters denote.

F = tractive force in pound avoir., necessary to overcome the rolling friction, and ascending inclined plains.
M = miles per hour of the train or force F.
T = weight of the load in tons, including the weight of the carriages. On rail-roads T includes the weight of the locomotive and tender.
t = weight of the locomotive resting on the driving wheels in tons.
h = vertical rise in feet per 100 of inclined roads.
b = base in feet per 100 of the inclined road or plain.
k = tractive coefficient in pound per ton of the load T, as noted in the accompanying Table, under the different conditions of the road.
A = area of *one* of the two cylinder pistons in a locomotive, in sq. in.
P = mean pressure of steam in lbs. per sq. in. on cylinder pistons.
S = stroke of pistons in feet.
D = diameter of driving wheel in feet.
H = actual horse power of a locomotive or the power necessary for the load. About 25 per cent. is allowed for friction and working pumps.
f = adherence coefficient of the driving wheels to the rails, in pounds per ton of the weight t.
n = revolutions per minute of driving wheels.
d = continued working hours of a horse.
v = velocity in feet per second. t' = weight of a horse in pounds.

Example 11. Fig. 140. The area of one of the two cylinder pistons in a locomotive is A=314 square inches, stroke of piston P=2 feet, mean-pressure P=80 lbs. per square inch. Driving wheels D=4 feet diameter. Required the tractive force F=? of a locomotive.

$$F = \frac{314 \times 2 \times 80}{4} = 12560 \text{ lbs. the answer.}$$

The adhesive force of the driving wheels to the rails, $f\,t$, must always be greater than the retractive force of the locomotive, otherwise the wheels will slip on the track.

Example 12. Fig. 141. A locomotive of t=15 tons on an inclined plain rising h=10 feet, and the base b=99·5 feet per 100. f=560, other dimensions being the same as in the preceding example. Required the tractive, retractive and adhesive forces?

Tractive, $$F = \frac{314 \times 2 \times 80}{4} - 22{\cdot}4 \times 15 \times 10 = 9200 \text{ lbs.}$$

Retractive, $$F = 22{\cdot}4 \times 15 \times 10 = 3360 \text{ lbs.}$$

Adhesive, $$F = \frac{560 \times 15 \times 99{\cdot}5}{100} = 8358 \text{ lbs.}$$

Consequently the locomotive can ascend the inclined plain with a tractive force of 8358 – 3360 = 4998 lbs., without slip in the driving wheels.

Example 13. Fig. 142. A train of T=200 tons is to be drawn M=20 miles per hour on a horizontal track in good condition, k=4. Required retractive force F=?

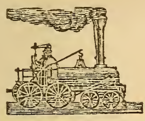
$$F = 200\,(4 + \sqrt{20}) = 1694{\cdot}4 \text{ lbs. the answer.}$$

Example 14. Fig. 143. A train of T=150 tons is to be drawn up an inclined plain of h=9 feet in 100, with a speed of M=16 miles per hour, k=4. Required the necessary horse power of the locomotive H=?

$$H = \frac{16 \times 150}{375}\,(22{\cdot}4 \times 9 + 4 + \sqrt{16}) = 1342{\cdot}144 \text{ horses.}$$

Example 15. Fig. 144. Required the tractive ability F=? of a horse, running M=7 miles per hour, in d=4 continued hours.

$$F = \frac{375}{7\sqrt{4}} = 26{\cdot}8 \text{ lbs. the answer.}$$

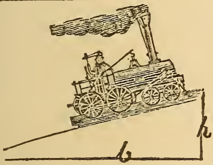


140.

$$F=\frac{A\,S\,P}{D}.\quad n=\frac{28\,M}{D},$$

$$H=\frac{A\,S\,P\,n}{11000}=\frac{A\,S\,P\,M}{376\,D}.$$

Adhesive force $=f\,t$.



141

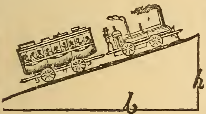
$$F=\frac{A\,S\,P}{D}-22{\cdot}4th.\quad M=\frac{D\,n}{28},$$

Adhesive, $\frac{f\,t\,b}{100}>22{\cdot}4th.$ retractive.

142

$$F=T\,(k+\sqrt{M}).\quad <f\,t=\text{Adhesive.}$$

$$H=\frac{M\,T}{375}\,(k+\sqrt{M}),$$



143.

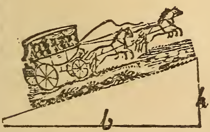
$$F=T\,(22{\cdot}4\,h+k+\sqrt{M}).\quad <\frac{f\,t\,b}{100}=\text{Ad.}$$

$$H=\frac{M\,T}{375}\,(22{\cdot}4\,h+k+\sqrt{M}).$$

144.

$$F=T\,(k+\sqrt{M}).\qquad v=1{\cdot}466\,M.$$

$F=\frac{550}{v\sqrt{d}}=\frac{375}{M\sqrt{d}}$. ability of a horse.



145.

$$F=T\,(22.4\,h+k+\sqrt{M}).$$

$$F=\frac{550}{v\sqrt{d}}-\frac{t'h}{100}\quad M=0{\cdot}6821\,v.$$

Example 16. Fig. 145. Required the tractive force $F=?$ of a load $T=5{\cdot}25$ tons, to be drawn $M=2$ miles per hour up a turnpike of $h=9$ feet in 100, the road being newly laid with coarse gravel $k=50$.

$$F = 5{\cdot}25\,(22{\cdot}4 \times 9 \times 50 \times \sqrt{2}) = 1328{\cdot}25 \text{ lbs.}$$

Suppose a horse to weigh $t' = 1000$ lbs., working continually in $d = 1$ hour up the turnpike. Required the tractive ability $F=?$ per horse.

$$F = \frac{375}{2\sqrt{1}} - \frac{1000 \times 9}{100} = 97{\cdot}5 \text{ lbs.}$$

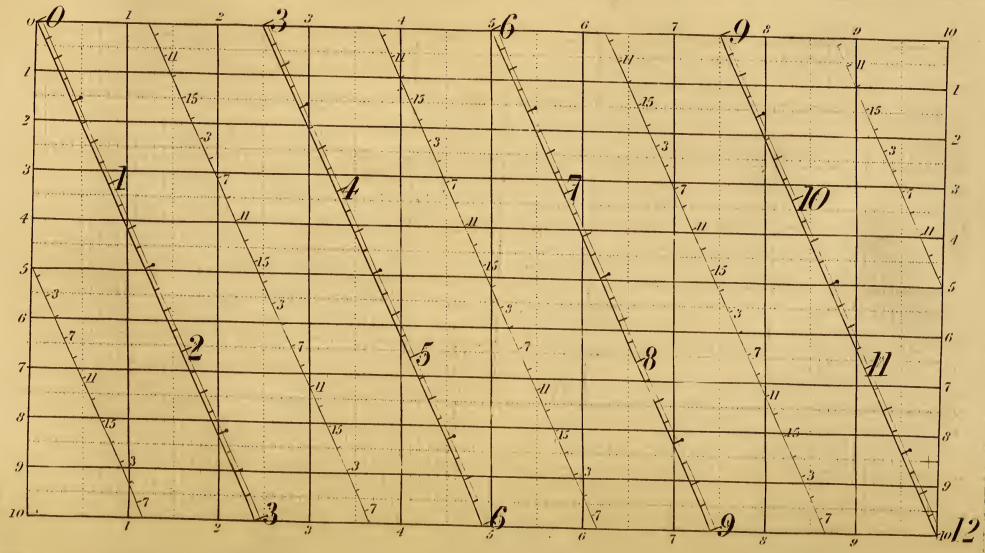
Number of horses $= \dfrac{1328{\cdot}25}{97{\cdot}5} = 13{\cdot}6$ say 14 horses which will be necessary for the load under the mentioned circumstances. In these examples it is necessary to take $M>1$. and $d>1$.

Traction Coefficient at very slow Speed.	k
On railroads in good condition, carriage axels well lubricated.	4
On railroads under ordinary, not very good condition.	8
On very smooth stone pavement,	12
On ordinary street pavements in good condition.	20
On street pavements and turnpikes.	30
On turnpikes newlaid with coarse gravel and broken stones,	50
On common roads in bad condition.	150
On natural loose ground or sand.	560
Adherence Coefficient.	f
On rails of maximum dryness.	672
" " very dry.	560
" " under ordinary circumstances.	450
" " in wet weather.	315
" " with snow or frost.	224

In railway curves, the retractive force is augmented so many per cent., as the whole train occupies degrees in the curve.

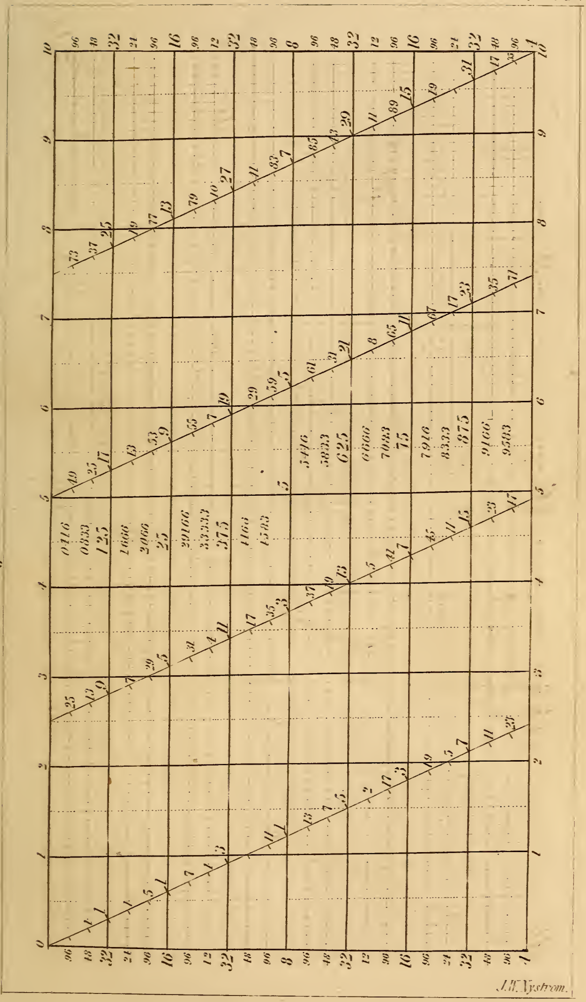
Plate I.

To reduce inches to feet.



J. W. Nystrom.

To reduce vulgar fractions to decimals.



J.W. Nystrom.

To Reduce Inches and Fractions thereof to Decimals of a Foot, and vice versa.

PLATE I.

This is a common decimal scale on which the 12 inches of a foot are laid out. Any length of a foot expressed by inches and the common fractions, intersects its own value in decimals, and are read off as on a common diagonal scale with 10 to the base.

Example 1. How much is $8\frac{1}{4}$ inches in decimals of a foot? Find $8\frac{1}{4}$ inches on the *rule*, which will be found to intersect 6875 on the decimal scale; cr $8\frac{1}{4}$ in. = 0·6875 feet.

The first figure 6 is marked at the top and bottom of the scale, the second figure 8 is the eighth vertical (or nearly so) line from 6; the third figure 7 is the seventh horizontal line marked at the ends of the scale, and the fourth figure 5 is the horizontal dotted line between 7 and 8.

For sixteenths see the fine single *rule*.

Example 2. How much is 0·526 feet in inches? $6\,{}^{5}/_{16}$ the answer.

To Reduce Vulgar Fractions into Decimals and vice versa.

PLATE II.

This is a similar arrangement to the preceding one. The vulgar fractions are laid out so that the denominators are marked at the ends of the scale and the nominators on the line that joins the given denominators; by this arrangement the nominator of the vulgar fraction intersects its own value in decimals on the scale. To facilitate the operations it is best to imagine in which quarter the vulgar fraction is; as ${}^{3}/_{16}$ is in the first, $\frac{3}{8}$ in the second; $\frac{5}{8}$ in the third, and $\frac{13}{16}$ in the fourth quarter, &c., &c.; each *rule* occupies a quarter of the scale, on which the vulgar fraction is to be found accordingly.

Example 1. How much is ${}^{9}/_{16}$ in decimals?

On the third rule it will be found at 5625 of decimals, or ${}^{9}/_{16}$ = 0·5625. The first and second figures 56 are marked as described for Plate I, and the third, fourth, &c., &c., figures are written down on the horizontal line of the given fraction.

Example 2. What nearest vulgar fraction answers to the decimals 0·39583?

$\frac{19}{48}$, the answer.

These two diagrams are exceedingly useful in practice. By Plate II, Vulgar Fractions can be added and subtracted.

TRIGONOMETRY.

TRIGONOMETRY is that part of Geometry which treats of Triangles. It is divided into two parts, viz.: *plane* and *spherical*.

Plane Trigonometry treats of triangles which are drawn (or imagined to be) on a plane. Spherical Trigonometry treats of the triangles which are drawn (or imagined to be) on a sphere.

A triangle contains seven quantities, namely, three sides, three angles, and the surface; when any three of these quantities are given, the four remaining ones can by them be ascertained, (one side or the area must be one of the given quantities) and the operation is called *solving the triangle*, which is only an application of arithmetic on Geometrical objects.

For the foundation of the above mentioned solution, there are assumed eight help quantities which are called *Trigonometrical functions*, and are here denoted with their names and number, corresponding with Figure 1? In the accompanying Tables, the functions are calculated at every 10 minutes per degree in the quadrant of the circle represented by Fig. 1? The angle for which the functions are mentioned, is the opening between the two lines 7 and 2, 3, this angle is denoted by the letter *C*, and the expression *sin.C.* means the line 1 compared with the radius *r* as a unit.

Example 2. Fig. 136. An inclined plane $a = 150$ feet long, and $c = 27$ feet, the height over its base. What is the angle of inclination $C = ?$

Formula 14. $$sin.C = \frac{c}{a} = \frac{27}{150} = 0{\cdot}18000.$$

Find 0·18000 in the table of *sines*, which will be found at 10° 30′ which is the angle C nearly.

Example 3. Fig. 137 An oblique angled triangle has the sides $c = 27{\cdot}6$ feet, the angle $C = 34° 10'$, and the angle $A = 47° 40'$. How long is the side $a = ?$

Formula 1. $$a = \frac{c\ sin.A}{sin.C} = \frac{27{\cdot}6 \times sin.47° 40'}{sin.34° 10'} = 36{\cdot}33 \text{ feet, the answer.}$$

By Logarithms.

$$log.a = log.c + log.sin.A - log.sin.C.$$

$$\begin{array}{lll} c & + & \log.27{\cdot}6 = 1{:}44090 \\ A & + & \log.\sin.47° 40' = \underline{1{:}86878} \\ & & 1{\cdot}30968 \\ C & - & \log.\sin.34° 10' = \underline{1{:}74942} \\ & & \log.\ 36{\cdot}4 = 1{\cdot}56026, \text{ or } a = 36{\cdot}4 \text{ feet.} \end{array}$$

Example. 1. Two ships of war notice a strong firing from a castle; in order to be safe, they keep themselves at a distance beyond the reach of the balls from the castle. To measure the distance from the castle, they place the vessels 800 yards from each other, and observe the angles between the castle and the vessels to be A = 63° 45′, B = 75° 50′. What will be the two distances from the castle?

$$C = 180 - 63° 45' - 75° 50' = 40° 25'.$$

To A the distance will be,

$$b = \frac{c \sin.B}{\sin.C} = \frac{800 \times \sin.\ 75° 50'}{\sin.\ 40° 25'} = 1195{\cdot}75 \text{ yards.}$$

To *B* the distance will be,

$$a = \frac{c \sin.\ A}{\sin.C} = \frac{800 \times \sin.\ 63° 45'}{\sin.\ 40° 25'} = 1106{\cdot}6 \text{ yards.}$$

Example 2. From a window in the lower floor of a house which lays level with the foot of a tower, is observed an angle = 40° to the top of the tower. From another window in the upper story, in the same perpendicular as the lower window, the altitude of the tower is observed to be = 37° 30′, which is 18 feet above the lower window.

Then we have $A = 90 - 40° = 50°$. $C = 90 + 37° 30' = 127° 30'$.
$B = 180 - 50 - 127° 39' = 2° 30'$. $b = 18$ feet.

What will be the height of the tower and the distance from it?

The distance from the lower window to the top of the tower $= c$.

$$c = \frac{b \sin.C}{\sin.B} = \frac{18 \times \sin.127° 35'}{\sin.2° 30'} = 327{\cdot}3 \text{ feet.}$$

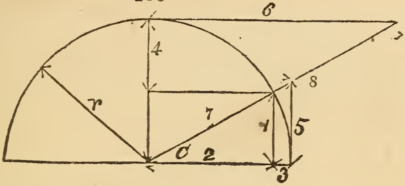
The height of the tower $= h$.

$$h = c.\ \sin.A = 327{\cdot}3 \times \sin.40° = 210 \text{ feet.}$$

The distance to the tower $= d$.

$$d = c.\ \cos.A = 327{\cdot}3 \times \cos.40° = 250{\cdot}8 \text{ feet.}$$

135



1 *Sinus*	abbreviated	*sin.C.*
2 *Cosinus*	"	*cos.C.*
3 *Sinus-versus*	"	*sinv.C.*
4 *Cosinus-versus*	"	*cosv.C.*
5 *Tangent*	"	*tan.C.*
6 *Cotangent*	"	*cot.C.*
7 *Secant*	"	*sec.C.*
8 *Cosecant*	"	*cosec.C.*

r = *Radius* of the circle, which is the unit by which the functions are measured.

$$r^2 = \sin.^2C+\cos.^2C.$$

$$\tan.C = \frac{\sin.C}{\cos.C},$$

$$\tan.C = \frac{1}{\cot.C},$$

$$\cot.C = \frac{\cos.C.}{\sin.C},$$

$$\cot.C = \frac{1}{\tan.C},$$

$$\sec.C = \frac{1}{\cos.C},$$

$$\text{cosec}.C = \frac{1}{\sin.C},$$

$$\text{sinv}.C = 1 -- \cos.C,$$

$$\text{cosv}.C = 1 \sin.C,$$

$$\sin.2C = 2\sin.C\cos.C,$$

$$\sin.\tfrac{1}{2}C = \tfrac{1}{2}\sqrt{(\sin.^2C+\text{sinv}.^2C)},$$

$$\sin.(C\pm B) = \sin.C\cos.B \pm \sin.B\cos.C.$$

Positive and Negative Signs.

Angles.	sin.	cos.	sinv.	cosv.	tan.	cot.	sec.	cosec.
$+0°$	$+0$	$+1$	$+0$	$+1$	$+0$	$+\infty$	$+1$	$+\infty$
$+90°$	$+1$	± 0	$+1$	$+0$	$\pm\infty$	± 0	$\pm\infty$	$+1$
$+180°$	± 0	-1	$+2$	$+1$	∓ 0	$\mp\infty$	-1	$\pm\infty$
$+270°$	-1	∓ 0	$+1$	$+2$	$\pm\infty$	$\pm$[illegible]	$\mp\infty$	-1
$+360°$	∓ 0	$+1$	$+0$	$+1$	∓ 0	$-\infty$	$+1$	$-\infty$

When a quantity has reached 0 or ∞, it has ceased to exist, because it can not be increased or diminished.

Example. What is the length of the secant for an angle of 74° 18′?

$$\text{Secant C} = \frac{1}{\cos.\ 74°\ 18'} = 3{\cdot}695.$$

Natural Sine.

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	·00000	·00291	·00581	·00872	·01163	·01454	·01745	89
1	·01745	·02036	·02326	·02617	·02908	·03199	·03489	88
2	·03489	·03780	·04071	·04361	·04652	·04943	·05233	87
3	·05233	·05524	·05814	·06104	·06395	·06685	·06975	86
4	·06975	·07265	·07555	·07845	·08135	·08425	·08715	85
5	·08715	·09005	·09294	·09584	·09874	·10163	·10452	84
6	·10452	·10742	·11031	·11320	·11609	·11898	·12186	83
7	·12186	·12475	·12764	·13052	·13340	·13629	·13917	82
8	·13917	·14205	·14493	·14780	·15068	·15356	·15643	81
9	·15643	·15930	·16217	·16504	·16791	·17078	·17364	80
10	·17364	·17651	·17937	·18223	·18509	·18795	·19080	79
11	·19080	·19366	·19651	·19936	·20221	·20506	·20791	78
12	·20791	·21075	·21359	·21643	·21927	·22211	·22495	77
13	·22495	·22778	·23061	·23344	·23627	·23909	·24192	76
14	·24192	·24474	·24756	·25038	·25319	·25600	·25881	75
15	·25881	·26162	·26443	·26723	·27004	·27284	·27563	74
16	·27563	·27843	·28122	·28401	·28680	·28958	·29237	73
17	·29237	·29515	·29793	·30070	·30347	·30624	·30901	72
18	·30901	·31178	·31454	·31730	·32006	·32281	·32556	71
19	·32556	·32831	·33106	·33380	·33654	·33928	·34202	70
20	·34202	·34475	·34748	·35020	·35293	·35565	·35836	69
21	·35836	·36108	·36379	·36650	·36920	·37190	·37460	68
22	·37460	·37730	·37999	·38268	·38536	·38805	·39073	67
23	·39073	·39340	·39607	·39874	·40141	·40407	·40673	66
24	·40673	·40939	·41204	·41469	·41733	·41998	·42261	65
25	·42261	·42525	·42788	·43051	·43313	·43575	·43837	64
26	·43837	·44098	·44359	·44619	·44879	·45139	·45399	63
27	·45399	·45658	·45916	·46174	·46432	·46690	·46947	62
28	·46947	·47203	·47460	·47715	·47971	·48226	·48480	61
29	·48480	·48735	·48988	·49242	·49495	·49747	·50000	60
30	·50000	·50251	·50502	·50753	·51004	·51254	·51503	59
31	·51503	·51752	·52001	·52249	·52497	·52745	·52991	58
32	·52991	·53238	·53484	·53729	·53975	·54219	·54463	57
33	·54463	·54707	·54950	·55193	·55436	·55677	·55919	56
34	·55919	·56160	·56400	·56640	·56880	·57119	·57357	55
35	·57357	·57595	·57833	·58070	·58306	·58542	·58778	54
36	·58778	·59013	·59248	·59482	·59715	·59948	·60181	53
37	·60181	·60413	·60645	·60876	·61106	·61336	·61566	52
38	·61566	·61795	·62023	·62251	·62478	·62705	·62932	51
39	·62932	·63157	·63383	·63607	·63832	·64055	·64278	50
40	·64278	·64501	·64723	·64944	·65165	·65386	·65605	49
41	·65605	·65825	·66043	·66262	·66479	·66696	·66913	48
42	·66913	·67128	·67344	·67559	·67773	·67986	·68199	47
43	·68199	·68412	.68642	·68835	·69046	·69256	·69465	46
44	·69465	·69674	·69883	·70090	·70298	·70504	·70710	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

Natural Cosine.

Natural Sine.

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	·70710	·70916	·71120	·71325	·71528	·71731	·71933	44
46	·71933	·72135	·72336	·72537	·72737	·72936	·73135	43
47	·73135	·73333	·73530	·73727	·73923	·74119	·74314	42
48	·74314	·74508	·74702	·74895	·75088	·75279	·75470	41
49	·75470	·75661	·75851	·76040	·76229	·76417	·76604	40
50	·76604	·76791	·76977	·77162	·77347	·77531	·77714	39
51	·77714	·77897	·78079	·78260	·78441	·78621	·78801	38
52	·78801	·78979	·79157	·79335	·79512	·79688	·79863	37
53	·79863	·80038	·80212	·80385	·80558	·80730	·80901	36
54	·80901	·81072	·81242	·81411	·81580	·81748	·81915	35
55	·81915	·82081	·82247	·82412	·82577	·82740	·82903	34
56	·82903	·83066	·83227	·83388	·83548	·83708	·83867	33
57	·83867	·84025	·84182	·84339	·84495	·84650	·84804	32
58	·84804	·84958	·85111	·85264	·85415	·85566	·85716	31
59	·85716	·85866	·86014	·86162	·86310	·86456	·86602	30
60	·86602	·86747	·86891	·87035	·87178	·87320	·87461	29
61	·87461	·87602	·87742	·87881	·88020	·88157	·88294	28
62	·88294	·88430	·88566	·88701	·88835	·88968	·89100	27
63	·89100	·89232	·89363	·89493	·89622	·89751	·89879	26
64	·89879	·90006	·90132	·90258	·90383	·90507	·90630	25
65	·90630	·90753	·90875	·90996	·91116	·91235	·91354	24
66	·91354	·91472	·91589	·91706	·91811	·91936	·92050	23
67	·92050	·92163	·92276	·92387	·92498	·92609	·92718	22
68	·92718	·92826	·92934	·93041	·93147	·93253	·93358	21
69	·93358	·93461	·93564	·93667	·93768	·93869	·93969	20
70	·93969	·94068	·94166	·94264	·94360	·94456	·94551	19
71	·94551	·94646	·94739	·94832	·94924	·95015	·95105	18
72	·95105	·95195	·95283	·95371	·95458	·95545	·95630	17
73	·95630	·95715	·95798	·95881	·95964	·96045	·96126	16
74	·96126	·96205	·96284	·96363	·96440	·96516	·96592	15
75	·96592	·96667	·96741	·96814	·96887	·96958	·97029	14
76	·97029	·97099	·97168	·97236	·97304	·97371	·97437	13
77	·97437	·97402	·97566	·97629	·97692	·97753	·97814	12
78	·97814	·97874	·97934	·97992	·98050	·98106	·98162	11
79	·98162	·98217	·98272	·98325	·98378	·98429	·98480	10
80	·98480	·98530	·98580	·98628	·98676	·98722	·98768	9
81	·98768	·98813	·98858	·98901	·98944	·98985	·99026	8
82	·99026	·99066	·99106	·99144	·99182	·99218	·99254	7
83	·99254	·99289	·99323	·99357	·99389	·99421	·99452	6
84	·99452	·99482	·99511	·99539	·99567	·99593	·99619	5
85	·99619	·99644	·99668	·99691	·99714	·99735	·99756	4
86	·99756	·99776	·99795	·99813	·99830	·99847	·99862	3
87	·99862	·99877	·99891	·99904	·99917	·99928	·99939	2
88	·99939	·99948	·99957	·99965	·99972	·99979	·99984	1
89	·99984	·99989	·99993	·99996	·99998	·99999	1·0000	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

Natural Cosine.

Natural Tangent.

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	·00000	·00290	·00581	·00872	·01163	·01454	·01745	89
1	·01745	·02036	·02327	·02618	·02909	·03200	·03492	88
2	·03492	·03783	·04074	·04366	·04657	·04949	·65240	87
3	·05240	·05532	·05824	·06116	·06408	·06700	·06992	86
4	·06992	·07285	·07577	·07870	·08162	·08455	·08748	85
5	·08748	·09042	·09335	·09628	·09922	·10216	·10510	84
6	·10510	·10804	·11098	·11393	·11688	·11983	·12278	83
7	·12278	·12573	·12869	·13165	·13461	·13757	·14054	82
8	·14054	·14350	·14647	·14945	·15242	·15540	·15838	81
9	·15838	·16136	·16435	·16734	·17033	·17332	·17632	80
10	·17632	·17932	·18233	·18533	·18834	·19136	·19438	79
11	·19438	·19740	·20042	·20345	·20648	·20951	·21255	78
12	·21255	·21559	·21864	·22169	·22474	·22780	·23086	77
13	·23086	·23393	·23700	·24207	·24315	·24624	·24932	76
14	·24932	·25242	·25551	·25861	·26172	·26483	·26794	75
15	·26794	·27106	·27419	·27732	·28045	·28359	·28674	74
16	·28674	·28989	·29305	·29621	·29938	·30255	·30573	73
17	·30573	·30891	·31210	·31529	·31849	·32170	·32491	72
18	·32491	·32813	·33136	·33459	·33783	·34107	·34432	71
19	·34432	·34758	·35084	·35411	·35739	·36067	·36397	70
20	·36397	·36726	·37057	·37388	·37720	·38053	·38386	69
21	·38386	·38720	·39055	·39391	·39727	·40064	·40402	68
22	·40402	·40741	·41080	·41421	·41762	·42104	·42447	67
23	·42447	·42791	·43135	·43481	·43827	·44174	·44522	66
24	·44522	·44871	·45221	·45572	·45924	·46277	·46630	65
25	·46630	·46985	·47340	·47697	·48055	·48413	·48773	64
26	·48773	·49133	·49495	·49858	·50221	·50586	·50952	63
27	·50952	·51319	·51687	·52056	·52426	·52798	·53170	62
28	·53170	·53544	·53919	·54295	·54672	·55051	·55430	61
29	·55430	·55811	·56193	·56577	·56961	·57347	·57735	60
30	·57735	·58123	·58513	·58904	·59296	·59690	·60086	59
31	·60086	·60482	·60880	·61280	·61680	·62083	·62486	58
32	·62486	·62892	·63298	·63707	·64116	·64527	·64940	57
33	·64940	·65355	·65771	·66188	·66607	·67028	·67450	56
34	·67450	·67874	·68300	·68728	·69157	·69588	·70020	55
35	·70020	·70455	·70891	·71329	·71769	·72210	·72654	54
36	·72654	·73099	·73546	·73996	·74447	·74900	·75355	53
37	·75355	·75812	·76271	·76732	·77195	·77661	·78128	52
38	·78128	·78598	·79069	·79543	·80019	·80497	·80978	51
39	·80978	·81461	·81946	·82433	·82923	·83415	·83909	50
40	·83909	·84406	·84906	·85408	·85912	·86414	·86928	49
41	·86928	·87440	·87955	·88472	·88992	·89515	·90040	48
42	·90040	·90568	·91099	·91633	·92169	·92704	·93251	47
43	·93251	·93796	·94345	·94896	·95450	·96008	·96568	46
44	·96568	·97532	·97699	·98269	·98843	·99419	1·0000	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

Natural Cotangent.

Natural Tangent.

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	1·0000	1·0058	1·0117	1·0176	1·0235	1·0295	1·0355	44
46	1·0355	1·0415	1·0476	1·0537	1·0599	1·0661	1·0723	43
47	1·0723	1·0786	1·0849	1·0913	1·0977	1·1041	1·1106	42
48	1·1106	1·1171	1·1236	1·1302	1·1369	1·1436	1·1503	41
49	1·1503	1·1571	1·1639	1·1708	1·1777	1·1847	1·1917	40
50	1·1917	1·1988	1·2059	1·2130	1·2203	1·2275	1·2348	39
51	1·2348	1·2422	1·2496	1·2571	1·2647	1·2722	1·2799	38
52	1·2799	1·2876	1·2954	1·3032	1·3111	1·3190	1·3270	37
53	1·3270	1·3351	1·3432	1·3514	1·3596	1·3679	1·3763	36
54	1·3763	1·3848	1·3933	1·4019	1·4106	1·4193	1·4281	35
55	1·4281	1·4370	1·4459	1·4550	1·4641	1·4732	1·4825	34
56	1·4825	1·4919	1·5013	1·5108	1·5204	1·5301	1·5398	33
57	1·5398	1·5497	1·5596	1·5696	1·5798	1·5900	1·6003	32
58	1·6003	1·6107	1·6212	1·6318	1·6425	1·6533	1·6642	31
59	1 6642	1·6752	1·6864	1·6976	1·7090	1·7204	1·7320	30
60	1·7320	1·7437	1·7555	1·7674	1·7795	1·7917	1·8040	29
61	1·8040	1·8164	1·8290	1·8417	1·8546	1·8676	1·8807	28
62	1·8807	1·8939	1·9074	1·9209	1·9347	1·9485	1·9626	27
63	1·9626	1·9768	1·9911	2·0056	2·0203	2·0352	2·0503	26
64	2·0503	2·0655	2·0809	2·0965	2·1123	2·1283	2·1445	25
65	2·1445	2·1608	2·1774	2·1942	2·2113	2·2285	2·2460	24
66	2·2460	2·2637	2·2816	2·2998	2·3182	2·3369	2·3558	23
67	2·3558	2·3750	2·3944	2·4142	2·4342	2·4545	2·4750	22
68	2·4750	2·4959	2·5171	2·5386	2·5604	2·5826	2·6050	21
69	2·6050	2·6279	2·6510	2·6746	2·6985	2·7228	2·7474	20
70	2·7474	2·7725	2·7980	2·8239	2·8502	2·8769	2·9042	19
71	2·9042	2·9318	2·9600	2·9886	3·0178	3·0474	3·0776	18
72	3·0776	3·1084	3·1397	3·1715	3·2040	3·2371	3·2708	17
73	3·2708	3·3052	3·3402	3·3759	3·4123	3·4495	3·4874	16
74	3·4874	3·5260	3·5655	3·6058	3·6470	3·6890	3·7320	15
75	3·7320	3·7759	3·8208	3·8667	3·9136	3·9616	4·0107	14
76	4·0107	4·0610	4·1125	4·1652	4·2193	4·2747	4·3314	13
77	4·3314	4·3896	4·4494	4·5107	4·5736	4·6382	4·7046	12
78	4·7046	4·7728	4·8430	4·9151	4·9894	5·0658	5·1445	11
79	5·1445	5·2256	5·3092	5·3955	5·4845	5·5763	5·6712	10
80	5·6712	5·7693	5·8708	5·9757	6·0844	6·1970	6·3137	9
81	6·3137	6·4348	6·5605	6·6011	6·8269	6·9682	7·1153	8
82	7·1153	7·2687	7·4287	7·5957	7·7703	7·9530	8·1443	7
83	8·1443	8·3449	8·5555	8·7768	9·0098	9·2553	9·5143	6
84	9·5143	9·7881	10·078	10·385	10·711	11·059	11·430	5
85	11·430	11·826	12·250	12·760	13·196	13·726	14·300	4
86	14·300	14·924	15·604	16·349	17·169	18·074	19·081	3
87	19·081	20·205	21·470	22·003	24·541	26·431	28·636	2
88	28·636	31·241	34·367	38·188	42·964	49·103	57·289	1
89	57·289	68·750	85·939	114·58	171·38	343·77	∞	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

Natural Cotangent.

Natural Secant.

Deg.	0′	10′	20′	30′	40′	50′	60′	
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0001	1.0001	89
1	1.0001	1.0002	1.0002	1.0003	1.0004	1.0005	1.0006	88
2	1.0006	1.0007	1.0008	1.0009	1.0010	1.0012	1.0013	87
3	1.0013	1.0015	1.0016	1.0018	1.0020	1.0022	1.0024	86
4	1.0024	1.0026	1.0028	1.0031	1.0033	1.0035	1.0038	85
5	1.0038	1.0040	1.0043	1.0046	1.0049	1.0052	1.0055	84
6	1.0055	1.0058	1.0061	1.0064	1.0068	1.0071	1.0075	83
7	1.0075	1.0078	1.0082	1.0086	1.0090	1.0094	1.0098	82
8	1.0098	1.0102	1.0106	1.0111	1.0115	1.0120	1.0124	81
9	1.0124	1.0129	1.0134	1.0139	1.0144	1.0149	1.0154	80
10	1.0154	1.0159	1.0164	1.0170	1.0175	1.0181	1.0187	79
11	1.0187	1.0192	1.0198	1.0204	1.0210	1.0217	1.0223	78
12	1.0223	1.0229	1.0236	1.0242	1.0249	1.0256	1.0263	77
13	1.0263	1.0269	1.0277	1 0284	1.0291	1.0298	1.0306	76
14	1.0306	1.0313	1.0321	1.0329	1.0336	1.0344	1.0352	75
15	1.0352	1.0360	1.0369	1.0377	1.0385	1.0394	1.0403	74
16	1.0403	1.0411	1.0420	1.0429	1.0438	1.0447	1.0456	73
17	1.0456	1.0466	1.0475	1.0485	1.0494	1.0504	1.0514	72
18	1.0514	1.0524	1.0534	1.0544	1.0555	1.0565	1.0576	71
19	1.0576	1.0586	1.0597	1.0608	1.0619	1.0630	1.0641	70
20	1.0641	1.0653	1.0664	1.0676	1.0687	1.0699	1.0711	69
21	1.0711	1.0723	1.0735	1.0747	1.0760	1.0772	1.0785	68
22	1.0785	1.0798	1.0810	1.0823	1.0837	1.0850	1.0863	67
23	1.0863	1.0877	1.0890	1.0904	1.0918	1.0932	1.0946	66
24	1.0946	1.0960	1.0974	1.0989	1.1004	1.1018	1.1033	65
25	1.1033	1.1048	1.1063	1.1079	1.1094	1.1110	1.1126	64
26	1.1126	1.1141	1.1157	1.1174	1.1190	1.1206	1.1223	63
27	1.1223	1.1239	1.1256	1.1273	1.1290	1.1308	1.1325	62
28	1.1325	1.1343	1.1361	1.1378	1.1396	1.1415	1.1433	61
29	1.1433	1.1452	1.1470	1.1489	1.1508	1.1527	1.1547	60
30	1.1547	1.1566	1.1586	1.1605	1.1625	1.1646	1.1666	59
31	1.1666	1.1686	1.1707	1.1728	1.1749	1.1770	1.1791	58
32	1.1791	1.1833	1.1835	1.1856	1.1878	1.1901	1.1923	57
33	1.1923	1.1946	1.1969	1.1992	1.2015	1.2038	1.2062	56
34	1.2062	1.2085	1.2109	1.2134	1.2158	1.2182	1.2207	55
35	1.2207	1.2232	1.2257	1.2283	1.2308	1.2334	1.2360	54
36	1.2360	1.2386	1.2413	1.2440	1.2466	1.2494	1.2521	53
37	1.2521	1.2548	1.2576	1.2604	1.2632	1.2661	1.2690	52
38	1.2690	1.2719	1.2748	1.2777	1.2807	1.2837	1.2867	51
39	1.2867	1.2898	1.2928	1.2959	1.2990	1.3022	1.3054	50
40	1.3054	1.3086	1.3118	1.3150	1.3183	1.3216	1.3250	49
41	1.3250	1.3283	1.3317	1.3351	1.3386	1.3421	1.3456	48
42	1.3456	1.3491	1.3527	1.3563	1.3599	1.3636	1.3673	47
43	1.3673	1.3710	1.3748	1.3785	1.3824	1.3862	1.3901	46
44	1.3901	1.3940	1.3980	1.4020	1.4060	1.4101	1.4142	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

Natural Cosecant.

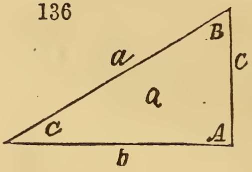
Natural Secant.

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	1·4142	1·4183	1·4225	1·4267	1·4309	1·4352	1·4395	44
46	1·4395	1·4439	1·4483	1·4527	1·4572	1·4617	1·4662	43
47	1·4662	1·4708	1·4755	1·4801	1·4849	1·4896	1·4944	42
48	1·4944	1·4993	1·5042	1·5091	1·5141	1·5191	1·5242	41
49	1·5242	1·5293	1·5345	1·5397	1·5450	1·5503	1·5557	40
50	1·5557	1·5611	1·5666	1·5721	1·5777	1·5833	1·5890	39
51	1·5890	1·5947	1·6005	1·6063	1·6122	1·6182	1·6242	38
52	1·6242	1·6303	1·6364	1·6426	1·6489	1·6552	1·6616	37
53	1·6616	1·6680	1·6745	1·6811	1·6878	1·6945	1·7013	36
54	1·7013	1·7081	1·7150	1·7220	1·7291	1·7362	1·7434	35
55	1·7434	1·7507	1·7580	1·7655	1·7730	1·7806	1·7882	34
56	1·7882	1·7960	1·8038	1·8118	1·8198	1·8278	1·8360	33
57	1·8360	1·8443	1·8527	1·8611	1·8697	1·8783	1·8870	32
58	1·8870	1·8959	1·9048	1·9138	1·9230	1·9322	1·9416	31
59	1·9416	1·9510	1·9606	1·9702	1·9800	1·9899	2·0000	30
60	2·0000	2·0101	2·0203	2·0307	2·0412	2·0519	2·0626	29
61	2·0626	2·0735	2·0845	2·0957	2·1070	2·1184	2·1300	28
62	2·1300	2·1417	2·1536	2·1656	2·1778	2·1901	2·2026	27
63	2·2026	2·2153	2·2281	2·2411	2·2543	2·2676	2·2811	26
64	2 2811	2·2948	2·3087	2·3228	2·3370	2·3515	2·3662	25
65	2.3662	2·3810	2·3961	2·4114	2·4269	2·4426	2·4585	24
66	2·4585	2·4747	2·4911	2·5078	2·5247	2·5418	2·5593	23
67	2·5593	2·5769	2·5949	2·6131	2·6316	2·6503	2·6694	22
68	2·6694	2·6888	2·7085	2·7285	2·7488	2·7694	2·7904	21
69	2·7904	2·8117	2·8334	2·8554	2·8778	2·9006	2·9238	20
70	2·9238	2·9473	2·9713	2·9957	3·0205	3·0458	3·0715	19
71	3·0715	3·0977	3·1243	3·1515	3·1791	3 2073	3·2360	18
72	3·2360	3·2653	3·2951	3·3255	3·3564	3·3880	3·4203	17
73	3·4203	3·4531	3·4867	3·5209	3·5558	3·5915	3·6279	16
74	3·6279	3·6651	3·7031	3·7419	3·7816	3·8222	3·8637	15
75	3·8637	3·9061	3·9495	3·9939	4·0393	4·0859	4·1335	14
76	4·1335	4·1823	4·2323	4·2836	4·3362	4·3900	4·4454	13
77	4·4454	4·5021	4·5604	4·6202	4·6816	4·7448	4·8097	12
78	4·8097	4·8764	4·9451	5·0158	5.0886	5·1635	5·2408	11
79	5·2408	5·3204	5·4026	5·4874	5.5749	5·6653	5·7587	10
80	5·7587	5·8553	5·9553	6·0588	6·1660	6·2771	6·3924	9
81	6·3924	6·5120	6·6363	6·7654	6·8997	7·0396	7·1852	8
82	7·1852	7·3371	7·4957	7·6612	7·8344	8·0156	8·2055	7
83	8·2055	8·4046	8·6137	8·8336	9·0651	9·3091	9·5667	6
84	9·5667	9·8391	10·127	10.437	10·758	11·104	11·473	5
85	11·473	11·868	12·291	12.745	13·234	13·763	14·335	4
86	14·335	14·957	15·636	16.380	17·198	18·102	19·107	3
87	19·107	20·230	21·493	22.925	24·562	26·450	28·653	2
88	28·653	31·257	34·382	38.201	42·975	49·114	57·298	1
89	57·298	68·757	85·945	114.59	171.83	343·77	∞	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

Natural Cosecant.

FORMULA FOR RIGHT-ANGLED TRIANGLES.

136



$$a = \sqrt{b^2+c^2}, \qquad 1,$$

$$a = \frac{c}{\sin. C}, \qquad 2,$$

$$a = \frac{b}{\cos. C}, \qquad 3,$$

$$a = 2\sqrt{\frac{Q}{\sin.2C}}, \qquad 4,$$

$$b = a\cos.C, \qquad 5,$$

$$b = c\cot.C, \qquad 6,$$

$$b = a\sin.B, \qquad 7,$$

$$b = c\tan.B, \qquad 8,$$

$$b = \sqrt{\frac{2Q}{\tan.C}}, \qquad 9,$$

$$Q = \frac{a^2\sin.2C}{4}, \qquad 10,$$

$$Q = \tfrac{1}{2}b^2\tan.C, \qquad 11,$$

$$Q = \tfrac{1}{2}c^2\cot.C, \qquad 12,$$

$$Q = \tfrac{1}{2}c\sqrt{(a+c)(a-c)} \qquad 13,$$

$$\sin. C = \frac{c}{a}, \qquad 14,$$

$$\cos. C = \frac{b}{a}, \qquad 15,$$

$$\tan. C = \frac{c}{b}, \qquad 16,$$

$$\sin.2C = \frac{4Q}{a^2}, \qquad 17,$$

$$\tan. C = \frac{2Q}{b^2}, \qquad 18,$$

Say the angle to be $C = 60°$. In the first column of the table of *sines*, 60° corresponds with 0·86602 in the next column, which is the length of *sin.* 60°, when the radius of the circle is one, or the unit, and the expression *sin.* $60° \times 36$ means $0·86602 \times 36 = 31·17672$, and likewise with all the other Trigonometrical expressions.

In a triangle the functions for an angle have a certain relation to the opposite side; it is this relationship which enables us to solve the triangle by the application of Simple Arithmetic.

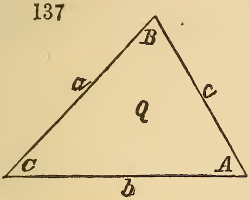
In triangles the sides are denoted by the letters a, b, and c; their respective opposite angles are denoted by A, B, and C, and the area by Q.

Example 1. Fig. 136 The side c in a right angled Triangle being 365 feet, and the angle $C = 39° 20'$. How long is the side $a = ?$

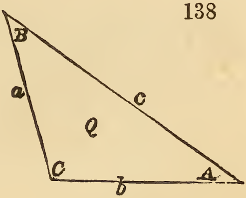
Formula 2. $a = \frac{c}{sin.C} = \frac{365}{sin.\ 39°.20'} = \frac{365}{0·63383} = 575·86$ feet, the answer.

FORMULA FOR OBLIQUE-ANGLED TRIANGLES.

137



138



$a : b = \sin.A : \sin.B$, and $b : c = \sin.B : \sin.C$.
$a : c = \sin.A : \sin.C$, and $Q : ab = \sin.C : 2$.

$$a = \frac{c \sin.A}{\sin.C}, \quad 1,$$

$$a = \frac{c \sin.A}{\sin.(A+B)}, \quad 2,$$

$$a = \frac{2Q}{b \sin.C}, \quad 3,$$

$$b = \frac{c \sin.B}{\sin.C}, \quad 4,$$

$$b = \frac{2Q}{c \sin.A}, \quad 5,$$

$$\sin.C = \frac{c \sin.B}{b}, \quad 6,$$

$$\sin.C = \frac{c \sin.A}{a}, \quad 7,$$

$$\sin.A = \frac{2Q}{bc}, \quad 8,$$

$$\sin.A = \frac{a \sin.C}{c}, \quad 9,$$

$$a = \sqrt{b^2+c^2-2bc \cos.A}, \quad 10,$$

$$a = \sqrt{\frac{2Q \sin.A.}{\sin.B \sin.(A+B)}} \quad 11,$$

$$S = \tfrac{1}{2}(a+b+c) \quad 12,$$

$$\sin.\tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}}, \quad 13,$$

$$\sin.\tfrac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{ac}}, \quad 14,$$

$$\cos.\tfrac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}, \quad 15,$$

$$\cos.\tfrac{1}{2}B = \sqrt{\frac{s(s-b)}{ac}}, \quad 16,$$

$$Q = \frac{bc \sin.A}{2}, \quad 17,$$

$$Q = \frac{ab \sin.C}{2}, \quad 18,$$

$$Q = \frac{c^2 \sin.A \sin.B}{2 \sin.(A+B)}, \quad 19,$$

$$Q = \sqrt{S-a)(S-b)(S-c)} \quad 20,$$

$$b = \sqrt{\frac{2Q \sin.(A+C)}{\sin.A \sin.C}}, \quad 21,$$

$$c = \sqrt{\frac{2Q \sin.C}{\sin.A \sin.(A+C)}}, \quad 22,$$

To Solve Triangles Mechanically.

PLATE III.

THE accompanying diagram is so constructed that the moveable arm represents the hypothenuse, the square lines the two sides, and the circular scale the angles in a right-angled triangle. The scale numbered from the centre towards 0 will be called b, and the one at right angle to be called c.

Example 1. Let the lines that form the right-angle be given as $b = 12$ and $c = 4$ inches. Required the hypothenuse a, and the angles B and C?

Find where the two lines 12 and 4 crosses each other, move the arm to this crossing-point, which then indicates the length of the hypothenuse = 12·65 on the arm a; the two angles will be found at a on the scale. $B = 71° 40'$ and $C = 18° 20'$.

If one angle and a side is given, set the arm on the given angle, and the intersection of the given side with the arm shows the length of the hypothenuse and the other side.

An oblique angled triangle can be two right angled triangles by drawing a line from the largest angle perpendicular to the opposite side, and can be solved by this diagram.

Example 2. An oblique angled triangle being $a = 65$ feet, $C = 34° 30'$ and $B = 68° 20'$.

Required the two sides b and c?

Set the arm on the given angle 34° 30′, and at 65 feet on the arm will be found the height of the triangle = 37 feet on scale c, and one part of the side b is 54 feet on the scale b.

From the given angle	$B = 68° 20'$
Subtract the complement of 34° 30′	$= 55° 30'$
Set the arm on the angle,	$12° 50'$

Now at the height 37 on the scale b will be found 38 feet on the arm, which is the length of the side c, and the other part of the side b is 9 feet on the scale c, then $b = 54 + 9 = 63$ feet.

In a similar manner any plane triangle can be so solved. By a little practice, this Table is very useful for approximating triangles.

SPHERICAL TRIGONOMETRY.

Spherical Trigonometry treats of triangles which are drawn (or imagined to be) on the surface of a sphere; their sides are arcs of the great circle of the sphere, and measures by the angle of the arc. Therefore the trigonometrical functions bear quite a different relation to the sides.

Every section of a sphere cut by a plane is a *circle*. A line drawn through the centre and at right angles to the sectional circle is called an *axis*, and the two points where the axis meets the surface of the sphere are called the *poles* of the sectional circle.

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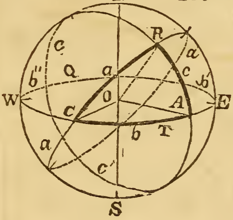
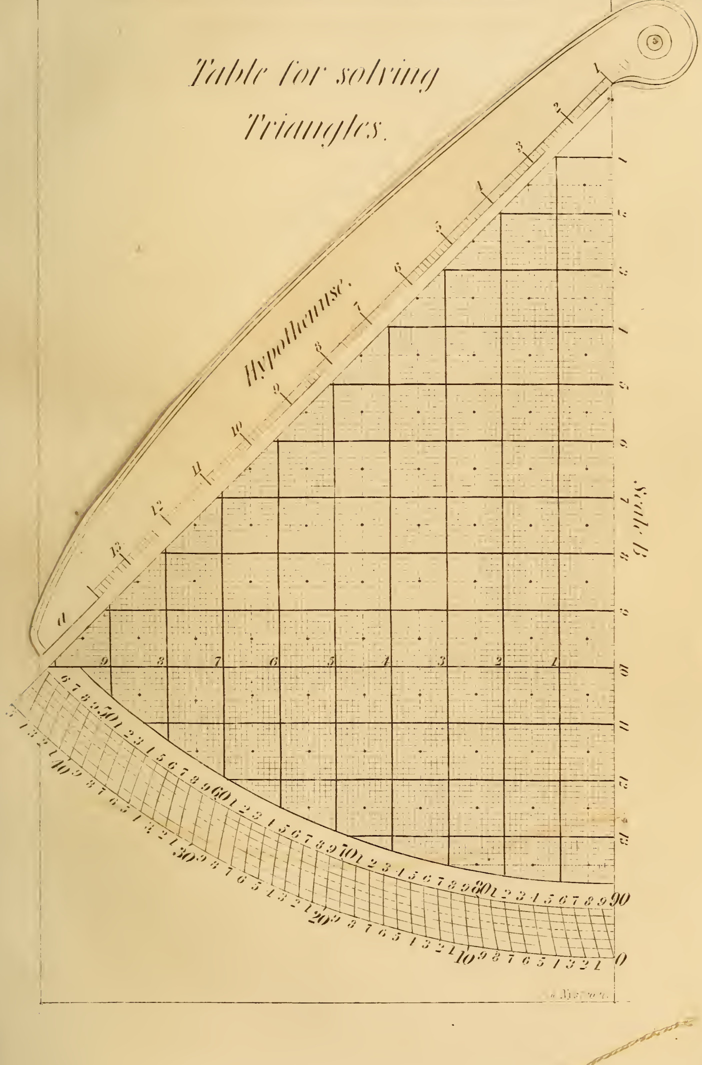


When the cutting plane goes through the centre of the sphere, it will pass through the great circle, and is then called the *Equator* for the poles. Axis = N.S. Equator = *G.E.T.W.*

Three great circle-planes, $aa'a''a'''$, $bb'b''$, and $cc'c''$, cutting a sphere, *NESW*, will form a solid angle at the centre *O*, and a triangle *ABC* on the surface of the sphere, in which the arcs a, b, c, are the sides. The angles formed by each two planes are congruent to each of the appertinent angles *A*, *B*, and *C*.

Table for solving Triangles.



RIGHT-ANGLED SPHERICAL TRIANGLE.

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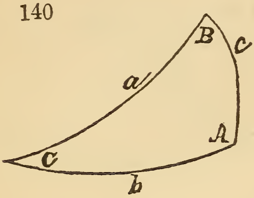

$\sin.b = \sin.a \sin.B,$ 1,

$\tan.c = \tan.a \cos.B,$ 2,

$\cot.C = \cos.a \tan.B,$ 3,

$\tan.c = \sin.b \tan.C,$ 4,

$\cos.a = \cos.b \cos.c,$ 5,

$\cos.B = \cos.b \sin.C,$ 6,

$$\tan.a = \frac{\tan.b}{\tan.C}, \quad 7,$$

$$\sin.c = \frac{\tan.b}{\tan.B}, \quad 8,$$

$$\sin.a = \frac{\sin.b}{\tan.B}, \quad 9,$$

$$\sin.C = \frac{\cos.B}{\cos.b}, \quad 10,$$

$$\cos.c = \frac{\cos.a}{\cos.b}, \quad 11,$$

$$\sin.B = \frac{\sin.b}{\sin.a}, \quad 12,$$

$$\sin.C = \frac{\tan.b}{\tan.a} \quad 13,$$

$$\tan.C = \frac{\tan.c}{\sin.b}, \quad 14,$$

$$\tan.B = \frac{\tan.b}{\sin.c}, \quad 15,$$

$$\cos.c = \frac{\cos.C}{\sin.B}, \quad 16,$$

$$\cos.b = \frac{\cos.B}{\sin.C}, \quad 17,$$

$$\cos.a = \frac{\cot.C}{\tan.B}, \quad 18.$$

The sum of the three angles in a spherical triangle is greater than two right angles, and less than six right angles.

By Spherical Trigonometry we ascertain distances and courses on the surface of the earth; positions and motions of the heavenly bodies, &c., &c. Examples will be furnished in Geography and Astronomy.

Example 1. Fig. 140 In a right-angled spherical triangle the side or hypothenuse $a = 36° 20'$, the angle $B = 68° 50'$. How long is the side $b = ?$

Formula 1. $\sin.b = \sin.a.\sin.B = \sin.36°20' \times \sin.68°60'$.

a	log.sin. 36° 20′ = 1:77267	
B	log.sin. 68° 50′ = 1:96966	
The answer,	log.sin. 33° 32′ = 1:74233	or $b = 33° 32'$.

OBLIQUE-ANGLED SPHERICAL TRIANGLE.

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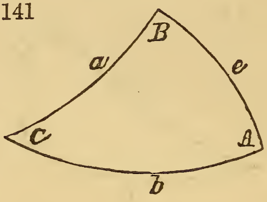

$\sin.a : \sin.b = \sin.A : \sin.B,$ $\qquad \sin.a = \dfrac{\sin.b \sin.A}{\sin.B},$ 19

$\sin.b : \sin.c = \sin.B : \sin.C,$ $\qquad \sin.b = \dfrac{\sin.c \sin.B}{\sin.C},$ 20,

$$\tan.(a+b) = \tan.\tfrac{1}{2}c \, \frac{\cos.\frac{1}{2}(A-B)}{\cos.\frac{1}{2}(A+B)}, \quad - \quad - \quad - \quad 21,$$

$$\tan.(a-b) = \tan.c \, \frac{\sin.\frac{1}{2}(A-B)}{\sin.\frac{1}{2}(A+B)}, \quad - \quad - \quad 22,$$

$$\tan.\tfrac{1}{2}(A+B) = \cot.\tfrac{1}{2}A \, \frac{\cos.\frac{1}{2}(b-c)}{\cos.\frac{1}{2}(b+c)}, \quad - \quad - \quad 23,$$

$$\tan.\tfrac{1}{2}(A-B) = \cot.\tfrac{1}{2}A \, \frac{\sin.\frac{1}{2}(b-c)}{\sin.\frac{1}{2}(b+c)}, \quad - \quad - \quad 24,$$

$$\cot.\tfrac{1}{2}A = \tan.\tfrac{1}{2}(B-C) \, \frac{\sin.\frac{1}{2}(b+c)}{\sin.\frac{1}{2}(b-c)} \quad - \quad - \quad 25,$$

$$\tan.\tfrac{1}{2}c = \tan.\tfrac{1}{2}(a-b) \, \frac{\sin.\frac{1}{2}(A+B)}{\sin.\frac{1}{2}(A-B)}, \quad - \quad 26,$$

Example 2. Fig. 141 Oblique angled spherical triangle. $c = 72° 30'$. $B = 17° 30'$. $C = 79° 50'$.
How long is the side $b = ?$

Formula 20. $\qquad \sin.b = \dfrac{\sin.c \sin.B}{\sin.C} = \dfrac{\sin.72° 30' \times \sin.17° 30'}{\sin.79° 50'}$

c	+ log.sin. 72° 30′ =	1:97942
B	+ log.sin. 17° 30′ =	1:47812
	+ =	11:45754
C	+ log. sin. 79° 50′ =	1:99312
The answer	log.sin. 16° 56′ =	1:46442

or $b = 16° 56.'$

OBLIQUE-ANGLED SPHERICAL TRIANGLE.

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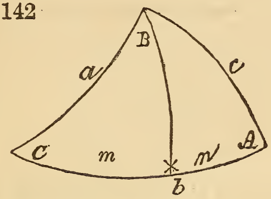

$$\tan.\tfrac{1}{2}(m+n)\tan\tfrac{1}{2}(m-n = \tan.\tfrac{1}{2}(a+c)\tan\tfrac{1}{2}(a-c)$$

$$\tan.m = \tan.c\,\cos.A, \quad - \quad - \quad - \quad - \quad 27,$$

$$\tan.C = \frac{\sin.m\,\tan.A}{\sin.(b-m)}, \quad - \quad - \quad - \quad - \quad - \quad 28,$$

$$\cos.a = \frac{\cos.c\,\cos.(b-m)}{\cos.m}, \quad - \quad - \quad - \quad - \quad 29,$$

$$\cos.n = \frac{\cos.a\,\cos.m}{\cos.c}, \quad - \quad - \quad - \quad - \quad - \quad 30,$$

$$b = m \pm n.$$

$$\cot.m = \frac{\cos.c\,\tan.A}{\tan.a}, \quad - \quad - \quad - \quad - \quad - \quad - \quad 31,$$

$$s = a+b+c \qquad S = A+B+C,$$

$$\sin.\tfrac{1}{2}A = \sqrt{\frac{\sin.(s-c)\,\sin.(s-b)}{\sin.b\,\sin.c}}, \quad - \quad - \quad 32,$$

$$\sin.\tfrac{1}{2}a = \sqrt{\frac{\cos.S\,\cos.(S-A)}{\sin.B\,\sin.C}}, \quad - \quad - \quad 33,$$

To Find the Area of a Spherical Triangle.

Let Q be the area of the triangle in square degrees; if R = radius of the sphere, the length of one degree will be,

$$= \frac{2\pi R}{360}, \text{ or one square degree} = \frac{R^2}{3285{\cdot}58}.$$

$$\cot.\tfrac{1}{2}Q = \frac{\cot.\tfrac{1}{2}c\,\cot.\tfrac{1}{2}a+\cos.B}{\sin.B}, \quad - \quad - \quad - \quad - \quad 1,$$

$$\sin.\tfrac{1}{2}Q = \frac{\sin.\tfrac{1}{2}c\,\sin.\tfrac{1}{2}a\,\sin.B}{\cos.\tfrac{1}{2}b}, \quad - \quad - \quad - \quad - \quad 2,$$

CONIC SECTIONS.

A Conic Section is the section obtained when a plane cuts a cone.

The conic sections are of five different kinds, namely.

1st. *Triangle.* When the plane cuts the cone through its axis.

2d. *Circle.* When the plane cuts the cone at right angles to its axis.

3d. *Ellipse.* When the plane cuts the cone obliquely passing through the *two sides.*

4th. *Parabola.* When the plane cuts the cone parallel to one side.

5th. *Hyperbola.* When the plane cuts the cone at an angle to the axis less than the angle of the axis and the side of the cone.

The position of a point in a plane surface is determined by its course and distance from a given point on a straight line, or by its distances from two lines inclined to one another

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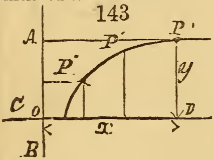


Let AB and CD be two infinite lines inclined to one another, and P being a point in the same plane as the lines. It is evident that the position of this point P is determined by the distances x and y from the lines AB and CD. Those lines and distances are called, AB the axis of ordinate, and CD the axis of abscissa, y the ordinate, x the abscissa, and o the origin.

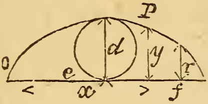
The abscissa x is commonly taken on the absciss' axis. Now take different values of the abscissa x, and by some formula or rule calculate the ordinate y; then a number of points $P, P'P''$, &c., may be obtained; join those points by a line, then the rule or formula is called an equation for that line. Equations of this kind will here be furnished for the curve in the conic sections.

Transverse-axis is the longest line that can be drawn in an Ellipse.

Conjugate-axis is a line drawn through the centre, at right angles to the transverse axis.

Parameter of any diameter is a third proportional to that diameter, and its conjugate.

Focus is the point in the axis where the ordinate is equal to half the diameter.

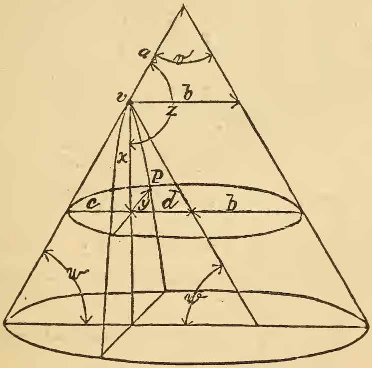


144. *Cycloid.*

$$y = 0{\cdot}637\ \sqrt{x(\pi d - x)},$$

$$e = 1{\cdot}211d, \quad p = 0{\cdot}637d.$$

CONIC SECTIONS.



145.

$a : b = \sin.w : \sin.v,$	$b = \dfrac{a \sin.v}{\sin.w},$	1,
$x : c = \sin.w : \sin.z,$	$c = \dfrac{x \sin.z}{\sin.w},$	2,
$x : d = \sin.w : \sin.(z+v),$	$d = \dfrac{x \sin.(z+v)}{\sin.w},$	3,

$y^2 = c(d+b)$. See Fig. 64, page. 91, 4,

$$y^2 = \frac{x.\sin.z}{\sin.w}.\left(\frac{a \sin.(z+v)}{\sin.w} + \frac{a \sin.v}{\sin.w}\right), \quad 5,$$

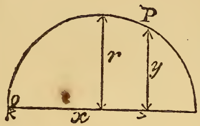
$$y^2 = \frac{x.\sin.z}{\sin.^2 w}(a \sin.[z+v] + a \sin.v). \quad 6,$$

This is the general formula for all conic sections.

In any conic section, a point P can be calculated by this formula 6, but for the different sections, it will be found greatly simplified on the next pages.

For a Parabola $z+v = 180$, therefore $\sin.z = \sin.v$, and

$$y^2 = \frac{x \sin.^2 v}{\sin.^2 w}.$$

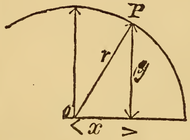


146. *Circle.*

$$y = \sqrt{2rx - x^2},$$

$$r = \frac{y^2+x^2}{2x},$$

$$x = r + \sqrt{r^2 - y^2}.$$

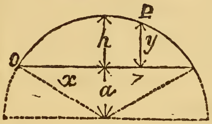


147. *Circle.*

$$y = \sqrt{r^2 - x^2},$$

$$r = \sqrt{y^2 + x^2},$$

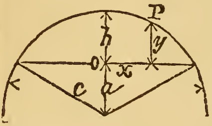
$$x = \sqrt{r^2 - y^2}.$$



148. *Circle Arc.*

$$y = \sqrt{a^2 + cx - x^2} - a,$$

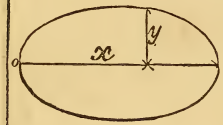
$$a = \frac{c^2 - 4h^2}{8h}.$$



149. *Circle Arc.*

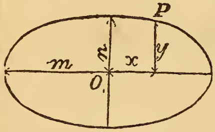
$$y = \sqrt{\left(\frac{c^2+h^2}{8h}\right)^2 - x^2} - a.$$

$$a = \frac{c^2 - 4h^2}{8h}.$$



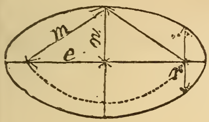
150. *Ellipse.*

$$y = \frac{n}{m}\sqrt{2mx - x^2}.$$



151. *Ellipse.*

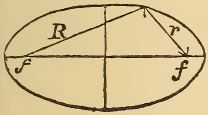
$$y = \sqrt{n^2 - \left(\frac{nx}{m}\right)^2}$$



152. *Ellipse.*

$$e = \sqrt{m^2 - n^2},$$

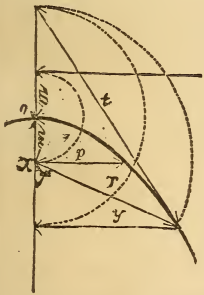
$$p = \frac{2n^2}{m}.$$



153. *Ellipse.*

$$R = 2m - r,$$

$$r = 2m - R.$$



154. *Parabola.*

$$y = \sqrt{px}, \quad p = 4m,$$

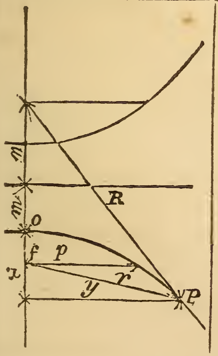
$$r = \sqrt{y^2 + (x - m)^2} = x + m,$$

$$y = \sqrt{t^2 - 4x^2}.$$

$$y = 2\sqrt{x(x+m) - x^2},$$

$$t = \sqrt{4x^2 + y^2},$$

$$t = 2\sqrt{x(x+m)}.$$



155. *Hyperbola.*

$$R - r = 2m, \quad e = \frac{n}{m}.$$

$$y = \frac{1}{m}\sqrt{(e^2 - 1)(x^2 - m^2)},$$

$$b^2 = \frac{e^2 - 1}{m^2},$$

$$y = b\sqrt{\frac{(x+m)^2}{m^2} - 1}.$$

MECHANICS.

Mechanics is that branch of Natural Philosophy which treats of the action of force, motion, and power. Mechanics is divided into four parts, namely,

Statics the science of forces in equilibrium.

Dynamics, the science of forces in motion, it produces power or effect.

Hydrostatics, the science of fluids in equilibrium

Hydrodynamics the science of fluids in motion, its causes, power or effects.

Statics. Lever. Momentum.

Lever is an inflexible bar, supported in one point called the *Fulcrum*, or, centre of motion. The length of a *lever* is measured from the *fulcrum* to where the *force* or *resistance* acts, (when the force acts at right angles to the lever) or, the length of a lever is measured from the fulcrum at right angles to the direction of the *force*.

$$\left.\begin{array}{ll} W = \textit{Weight}, & \text{and } l = \textit{lever} \text{ for } W \\ F = \textit{Force}, & \text{and } L = \textit{lever} \text{ for } F \end{array}\right\} \text{See Fig. 156.}$$

Momentum is the product of *force* or *weight*, multiplied by the length of the *lever* it acts upon.

The products Wl and FL are called *Statics Momentums;* when these momentums are equal there will be no motion, and the weight W will balance the force F. When one momentum is greater than the other, there will be a motion, and the velocity of that motion is measured by the difference of the momentums.

Levers are of three distinct kinds, with reference to the relative positions of the *Force* F, *Weight* W, and *Fulcrum* C.

1st. *Fulcrum* C, is between the *force* F, and the *weight* W.

2d. *Weight* W, is between the *fulcrum* C, and the *force* F.

3d. *Force* F, is between the *fulcrum* C, and the *weight* W.

Example 1. Figure 156. The weight $W = 68$ pounds, the lever $l = 3{\cdot}86$ feet, and $L = 10$ feet 6 inches.

Required the force $F = ?$

$$\textit{Formula } 1. \quad F = \frac{Wl}{L} = \frac{68 \times 3{\cdot}86}{10{\cdot}5} = 25 \text{ pounds nearly.}$$

$a =$ distance between the force F and the weight W.

The formula 3, 4, 7, 8, 11, 12, are for finding the fulcrum C, when the *force* F, *weight* W, and the distance a, are given.

Example 2. Fig. 157. The force $F = 360$ pounds, $W = 1870$, and $a = 8$ feet, 4 inches.

Required the position of the fulcrum c?

$$\textit{Formula } 7. \quad l = \frac{Fa}{W - F} = \frac{360 \times 8{\cdot}333}{1870 - 360} = \frac{2999{\cdot}988}{1510} = 19{\cdot}86 \text{ feet.}$$

$$L = 8{\cdot}333 + 19{\cdot}86 = 28{\cdot}193 \text{ feet, the answer.}$$

Example 3. Fig. 161. The weight of the lever is $Q = 18$ pounds. The centre of gravity is $x = 2{\cdot}25$ feet from the fulcrum. $W = 299$ pounds, $l = 5{\cdot}5$ feet, and $L = 11{\cdot}95$.

Required the force $F = ?$ in pounds.

$$F = \frac{Wl - Qx}{L} = \frac{299 \times 5{\cdot}5 - 18 \times 2{\cdot}25}{11{\cdot}95} = 134{\cdot}25 \text{ pounds.}$$

Inclined Plane.

Example 4. Fig. 180. A load $W = 3466$ pounds, is to be drawn up an inclined plane, $l = 638$ feet long, and $h = 86$ feet high.

What force is required to keep the load on the inclined plane?

$$F = \frac{hW}{l} = \frac{86 \times 3466}{638} = 467{\cdot}2 \text{ pounds.}$$

Example 4. Fig. 184. A Cylinder of cast iron, weighing $W = 5245$ pounds, is to be rolled up an inclined plane; the angles $v = 18^\circ\ 20'$ and $v' = 8^\circ\ 10'$ What force is required to keep the cylinder on the plane?

$$F = W.\ \sin.(v+v') = 5245 \times \sin.(18^\circ\ 20' + 8^\circ\ 10') = 2340 \text{ pounds.}$$

Example 5. Fig. 185. An iron ball which weighs 398 pounds, is tied to an inclined plane with a rope; the angle of the rope and the inclined plane is $v' = 16^\circ\ 40'$, and $v = 14^\circ\ 30'$. What force is acting on the rope?

$$F = \frac{W \sin.v}{\cos.v'} = \frac{398 \times \sin.14^\circ\ 30'}{\cos.16^\circ\ 40'} = 10{\cdot}4 \text{ pounds.}$$

Example 6. Fig. 170. What force F is required to raise a weight $W = 8469$ pounds, by a double moveable pulley?

$$F = \tfrac{1}{4} W = \tfrac{1}{4} \times 8469 = 2117{\cdot}25 \text{ pounds.}$$

Example 7. Fig. 173. How much weight can a force $F = 269$ pounds lift by three compound moveable pulleys?

$$W = 2^u F = 2^3 \times 269 = 2152 \text{ pounds, the answer.}$$

Screw.

Example 8. Fig. 189. What force is required to lift a weight $W = 16785$ pounds, by a screw, with a pitch $P = 0{\cdot}125$ feet, the lever being $r = 5$ feet. 4 inches?

$$F = \frac{WP}{2\pi r} = \frac{16785 \times 0{\cdot}125}{2 \times 3{\cdot}14 \times 5{\cdot}333} = 62{\cdot}62 \text{ pounds, the answer.}$$

Including friction the force F will be

$$F = \frac{W(P + f d \pi)}{2 \pi r}.$$

Find the friction f on page 155. d diameter of the screw.

Wedge.

Example 9. Fig. 186. The head of the wedge $a = 3$ inches, and length $l = 16\frac{1}{2}$ inches; the resistance to be separated is $R = 4846$ pounds. Required the force $F = ?$ (Friction omitted.)

$$F = \frac{4846 \times 3}{16{\cdot}5} = 881 \text{ pounds.}$$

Including friction the force F will be, 146

$$F = R\left[\frac{a}{l} + f\left(2 + \frac{a^2}{l\,s}\right)\right]$$

in which the friction f is to be found on page 155.

Catenaria.

Example 9. An iron chain 256 feet long, weighing 1560 pounds, is to be suspended between two points in the same horizontal line, but 196 feet apart.

How *deep* will the chain hang under the line of suspension, and with what *force* will the chain act at the points of suspension?

Figure and *Formula* 178. we have given,

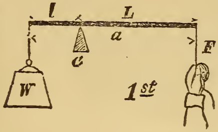
$W = \frac{1}{2} \times 1560 = 780$ pounds, $l = \frac{1}{2} \times 256 = 128$ feet, and $a = \frac{1}{2} \times 196 = 98$ feet.

$h = 0{\cdot}6525\sqrt{128^2 - 98^2} = 53{\cdot}73$ feet, the required depth under the horizontal line.

$$\cot.v = \frac{2 \times 53{\cdot}73}{98} = 1{\cdot}096, \quad \text{or } v = 44^\circ\ 44', \text{ and } 2v = 89^\circ\ 28'.$$

The required force will be,

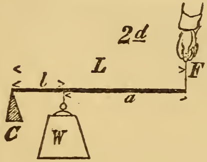
$$F = \frac{780 \times \sin.44^\circ\ 44'}{\sin.89^\circ\ 28'} = 549 \text{ pounds.}$$



156. $F : W = l : L, \quad F L = W l,$

$$F = \frac{W l}{L}, \qquad W = \frac{F L}{l}, \qquad 1, 2,$$

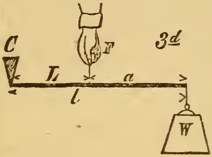
$$l = \frac{F a}{W+F}, \qquad L = \frac{W a}{W+F}, \qquad 3, 4,$$



157. $F : W = l : L, \quad F L = W l,$

$$F = \frac{W l}{L}, \qquad W = \frac{F L}{l}, \qquad 5, 6,$$

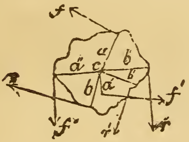
$$L = \frac{W a}{W-F}, \qquad l = \frac{F a}{W-F} \qquad 7, 8,$$



158. $F : W = l : L, \quad FL = W l,$

$$F = \frac{W l}{L}, \qquad W = \frac{F L}{l}, \qquad 9, 10,$$

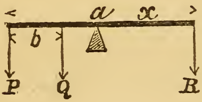
$$L = \frac{W a}{F-W}, \qquad L = \frac{F a}{F-W}. \qquad 11, 12.$$



159.

$$af + a'f' + a''f'' = br + b'r' + b''r''.$$

If the sum of the momentums that act to move the body in one direction are equal to the sum of the momentums that act opposite, the acting forces will be in equilibrium; c being the centre or fulcrum.

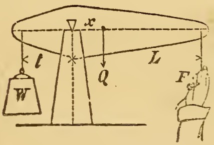


160.

To find the fulcrum c when three forces act on one lever

$$R x = Q(a - b - x) + P(a - x),$$

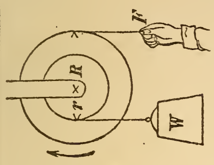
$$x = \frac{Q a + P a - Q b}{R + Q + P}.$$



161.

Q = weight of the lever. x = distance from the centre of gravity of the lever to the fulcrum. Balance the lever over a sharp edge, and the centre of gravity is found.

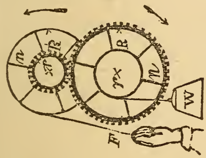
$$F = \frac{W l - Q x}{L}, \qquad W = \frac{F L + Q x}{l}.$$



162. $F:W=r:R.\ FR=Wr,$

$$F=\frac{Wr}{R},\quad R=\frac{Wr}{F},$$

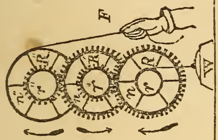
$$W=\frac{RF}{r},\quad r=\frac{RF}{W}.$$



163.

$$F=\frac{W\,r\,r'}{R\,R'},\quad W=\frac{F\,R\,R'}{r\,r'},$$

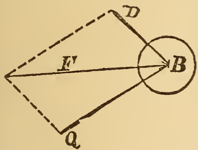
n = number of revolutions of the wheels,
$n:n'=r':R,\quad v:v'=rr':RR',$
v = velocity of W, v' = velocity of F.



164.
$$F=\frac{W\,r\,r'r''}{R\,R'R''},\quad W=\frac{F\,R\,R'R''}{r\,r'r''},$$

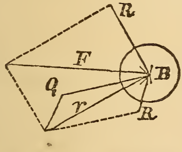
$n:n''=r'r'':R\,R',\ v:v'=r\,r'r'':R\,R'R''.$

$r\,r'r''$ &c. = radii of the pinions.
$R\,R'R''$&c. = radii of the wheels.



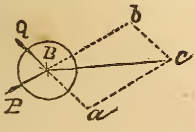
165.

Let P and Q represent the magnitudes and directions of two forces which act to move the body B. By completing the parallelogram, there will be obtained a diagonal force F, whose magnitude and direction is equal to the sum P and Q. F is called the resultant of P and Q.



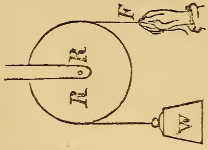
166.

If three or more forces act in different directions to move a body B, find the resultant of any two of them, and consider it as a single force. Between this and the next force find a second resultant, thus: P, Q, and R are magnitudes and directions of the forces. $P+Q=r,\ r+R=F=P+Q+R$, or F is the magnitude and direction of the three forces, P, Q, and R.



167.

A force Q acting (alone) on the body B, can move it to a in a unit of time, another force P is able to move it to b in the same time; now if the two forces act at the same time, they will move the body to c. c is the resultant of a and b.

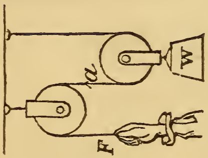


168.

PULLEYS.—*A single fixed Pulley.*

$$F : W = R : R, \text{ or } F = W,$$

$$v = v'.$$



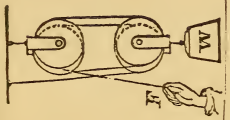
169.

A single moveable Pulley.

$$F : W = R : 2R, \text{ or } F = \tfrac{1}{2}W,$$

If the force F being applied at a and act upwards, the result will be the same.

$$v' = 2v.$$

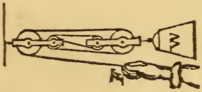


170.

A double moveable Pulley.

$$F : W = R : 4R, \text{ or } F = \tfrac{1}{4}W,$$

$$v' = 4v.$$

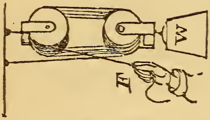


171.

A double moveable Pulley.

$$F : W = R : 4R, \text{ or } F = \tfrac{1}{4}W,$$

$$F = \frac{W}{2u}, \qquad v : v' = 1 : 2u.$$



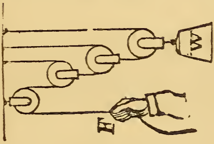
172.

Quadruple moveable Pulley.

$$F = \tfrac{1}{8}W. \qquad F : W = R : 8R.$$

Let u = any number of moveable pulleys, then

$$F = \frac{W}{2u}, \qquad v : v' = 1 : 2u.$$

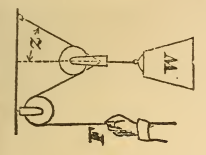


173. *Compound Pulleys.*

u = number of moveable Pulleys.

$$F = \frac{W}{2^u}, \quad W = 2^u F,$$

$$v : v' = 1 : 2^u.$$

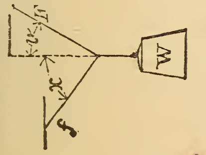


174.

An oblique fixed Pulley.

$$F : W = 2\cos.z : 1,$$

$$W = \frac{F}{2\cos.z}, \quad F = 2W\cos.z.$$

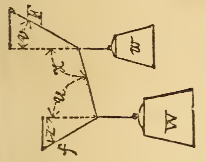


175.

$$W : F = \sin.(x+v) : \sin.x,$$

$$F = \frac{W\sin.x}{\sin.(x+v)}, \quad f = \frac{W\sin.v}{\sin.(x+v)},$$

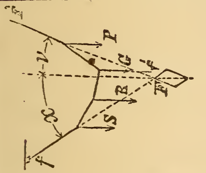
$$W : f = \sin.(x+v) : \sin.v.$$



176.

$$F = \frac{w\sin.x}{\sin.(x+v)},$$

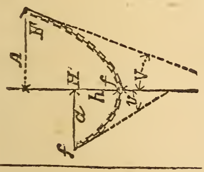
$$f = \frac{W\sin.u}{\sin.(u+z)}.$$



177.

$$W = P+Q+R+S, \quad F = F', f = f',$$

$$F = \frac{W\sin.x}{\sin.(x+v)}, \quad f = \frac{W\sin.v}{\sin.(x+v)}.$$

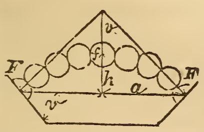


178.

$$l = \sqrt{\frac{4h^2}{3}+a^2}, \quad h = 0{\cdot}8660\sqrt{l^2-a^2},$$

l = length, and W = weight of half the chain, ff',

$$\cot.v = \frac{2h}{a}, \quad F = \frac{W.\sin.V}{\sin.2V}, \quad f = W\tan.v.$$

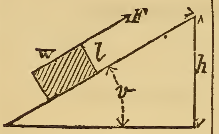


179.

$$F = \frac{W.\sin.v}{\sin.2v}, \quad f = W\tan.v,$$

$$\cot.v = \frac{2h}{a}.$$

W = weight of half the number of balls.

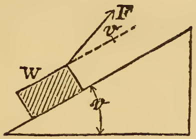


180.

$$F = \frac{W\,h}{l} = W \sin.v,$$

$$W = \frac{F\,l}{h} = \frac{F}{\sin.v},$$

$$w = \frac{W\,l}{b} = W \cos.v.$$

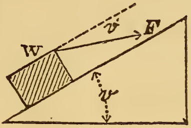


181.

$$F = W \sin.(v+v'),$$

$$W = \frac{F}{\sin.(v+v')},$$

$$W = W \cos.(v+v').$$

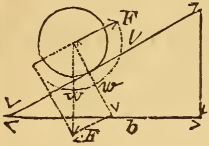


182.

$$F = \frac{W \sin.v}{\cos.v},$$

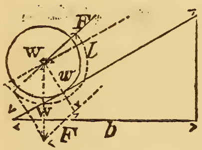
$$W = \frac{F \cos.v'}{\sin.v},$$

$$w = W(\cos.v + \sin.v.\ \tan v').$$

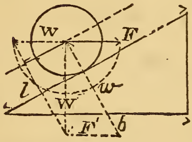


183.
To solve an Inclined Plane by diagrams.

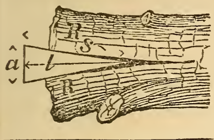
F = magnitude and direction of the force, which is obtained by completing the parallelogram.
By calculation see Formula, Fig. 180.



184.
W = weight of the body, and direction of the force of gravity; to be drawn at right-angles to the base b, and F parallel to F.
By calculation see Formula, Fig. 181.



185.
w = the force with which the body presses against the plane, to be drawn at right-angles to the plane l; then the parallelogram is completed.
By calculation see Formula, Fig. 182.

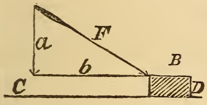


186

Wedge.

$$F = \frac{R\,a}{l}, \qquad R = \frac{F\,l}{a}.$$

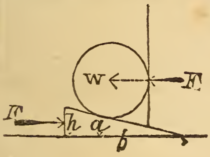
F = force acquired to drive the wedge.



187.

Let the line F represent the magnitude and direction of a force acting to move the body B on the line CD; then the line a represents a part of F which presses the body B against CD, and the line b represents the magnitude of the force which actually moves the body B.

$$b = \sqrt{F^2 - a^2}, \qquad b = F \cos.v.$$

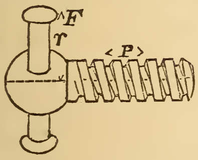


188.

$$F : W = h : b = \sin.v : \cos.v = \tan.v.$$

$$F = \frac{Wh}{b} = W \tan.v. \qquad F' = F.$$

$$W = \frac{Fb}{h} = \frac{F}{\tan.v} = F \cot.v.$$



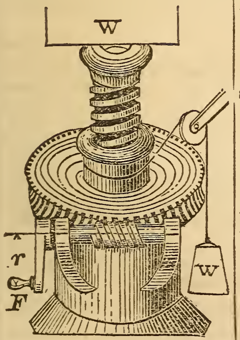
189. *Force by a Screw.*

P = Pitch of the screw,

r = radius on which the force F acts.

$$F : W = P : 2\pi\,r.$$

$$F = \frac{WP}{2\pi r}, \qquad W = \frac{F 2\pi r}{P}.$$



190. *Force by Compound Screws.*

P = Pitch of the large screw,

p = Pitch of the endless screw.

R = radius of spur-wheel for the endless screw.

$$W : F = 4\pi^2\,R\,r : P\,p.$$

$$F = \frac{W\,P\,r}{4\pi^2\,R\,r}, \qquad W = \frac{F 4^2 \pi\,R r}{P\,p}.$$

On the spur-wheel is a cylinder by which the weight W' is wound up, the formula will be, r' = radius of the cylinder, and

$$F : W' = p\ r' : 2\pi\ R\,r.$$

$$F = \frac{W'\ p\ r'}{2\pi\ R\ r}, \qquad W = \frac{F 2\pi\ R\,r}{p\,r'}.$$

DYNAMICS.

Velocity is a space passed through in a unit of time.

In machinery velocities are measured in feet per second; for Steamboats and Railroads in miles per hour.

Circular or *angular* velocity is the number of revolutions a revolving body makes per unit of time (minute.)

Velocity of Men.

A foot-soldier travels about 28 inches per step.

In common time 90 steps per minute	= 3·5 feet per second	= 2·4 miles per hour.
In quick time 110 "	= 4·3 "	= 3 "
Double quick time 140 "	= 5·5 "	= 3·75 "

A soldier occupies in the rank, a front of 20 inches, and 13 inches deep without knapsack; the interval between the ranks is 13 inches.

Average weight of men, 150 pounds each.

Five men can stand in a space of 1 square yard.

Example 1. A man walked 450 feet in 75 seconds. With what velocity did he walk?

Formula 1. $$v = \frac{s}{t} = \frac{450}{75} = 6 \text{ feet per second.}$$

Example 2. A body moves 368 feet in $t = 8$ seconds. What velocity has it?

Formula 1. $$v = \frac{s}{t} = \frac{368}{8} = 46 \text{ feet per second.}$$

Example 3. The radius of a wheel is 4 feet and 4 inches; it makes 131 revolutions per minute. What is the velocity of a point in the circumference?

Formula 18. $$v = \frac{2\,\pi\, r\, n}{60} = \frac{2 \times 3{\cdot}14 \times 4{\cdot}33 \times 131}{60} = 59{\cdot}5 \text{ feet per second.}$$

Power is the product of force and velocity: that is, a *force* multiplied by the *velocity* with which it moves, is the power of the force. *Force* without velocity is no power.

Force and *Power* are two distinct quantities. Power can *not* be increased or diminished by mechanical means; but *Force* can be increased and diminished *ad libitum.*

When a *force* is in motion, and increased or diminished by mechanical means, the alteration will be at the expense of the velocity, so that the power will remain the same.

Horse Power.

THE unit for the measurement of mechanical *Powers* is assumed to be the power of a horse; or a force of 33000 pounds, moved through a space of one foot in one minute, or 550 pounds moved 1 foot in 1 second. Another unit for measuring small powers is, *one pound moved through a space of one foot in one second;* this unit is called *effect.*

One horse power is = 550 effects.
One man's power is = 50 effects.
One horse power is = 11 men's power.

Example 4. A man draws up a bucket of water which weighs 52 pounds, from the bottom of a well 83 feet deep, which space the bucket passes in 43 seconds. With what effect is that man working?

Formula 5. $$P = \frac{Fs}{t} = \frac{52 \times 83}{43} = 100 \text{ Effects.}$$

Dynamical Formula.

Velocity	$v = \frac{s}{t}$, - -	1,	*Horse Power*	$H = \frac{F\,v}{550}$,	11,
Space	$s = v\,t$, - -	2,	*Horse Power*	$H = \frac{F\,s}{550\,t}$, -	12,
Time	$t = \frac{s}{v}$, - -	3,	*Force*	$F = \frac{550\,H}{v}$,	13,
Power	$P = v\,F$, - -	4,	*Force*	$F = \frac{550\,H\,t}{s}$,	14,
Power	$P = \frac{F\,s}{t}$, - -	5,	*Velocity*	$v = \frac{550\,H}{F}$,	15,
Force	$F = \frac{P}{v}$, - -	6,	*Space*	$s = \frac{550\,H\,t}{F}$,	16,
Force	$F = \frac{P\,t}{s}$, - -	7,	*Time*	$t = \frac{F\,s}{550\,H}$,	17,
Velocity	$v = \frac{P}{F}$, - -	8,	*Velocity*	$v = \frac{2\,\pi\,r\,n}{60}$, -	18,
Space	$s = \frac{P\,t}{F}$, - -	9,	*Effect*	$P = \frac{F\,2\,\pi\,r\,n}{60}$,	19,
Time	$t = \frac{F\,s}{P}$, -	10,		$H = \frac{F\,2\,\pi\,r\,n}{550 \times 60} = \frac{F\,r\,n}{5250}$,	20.

Letters denote.

P = *Power* in *Effects.*
H = *Horse-power*, number of.
v = *Velocity* of the force F in feet per second.
t = *Time* in seconds.
s = the *space* in feet which the force F passes through in the time t.

Circular Motion.

r = *radius* of the circle in feet.
n = *number of revolutions per minute.*

OBSERVED RESULTS OF POWER.

Description of works.	*Work hours per day.*	*Force.*	*Velocity* V	*Effects.* P	*Horses.* H
A man can raise a weight by a single fixed pulley.	6	50	0·8	40	0·072
A man working a crank,	8	20	2·5	50	0·090
A man on a tread-wheel, (horizontal,)	8	144	0·5	72	0·130
A man in a tread-wheel, (axis 24° from vertical,)	8	30	2·3	69	0·125
A man draws or pushes in a horizontal direction.	8	30	2	60	0·109
A man pulls up or down,	8	12	3·7	44·4	0·080
A man can bear on his back,	7	95	2·5	237·5	
A horse in a horse-mill, walking moderately,	8	106	3	318	0·577
do. do. do. running fast.	5	72	9	648	0·165
An ox do. do. walking moderately,	8	154	2	308	0·558
A mule do. do. do.	8	71	3	213	0·308
An ass do. do. do.	8	33	2·65	87·4	0·160
Flour Mills.					
For every 100 pounds of fine flour ground per hour, requires				550	1·000
One pair of mill-stones of 4 feet diameter making 120 rev. per minute, can grind 5 bushels of wheat to fine flour per hour.				2400	4·36
Do. rye to coarse flour,				1600	2·91
Saw Mills, alterative.					
For every 20 square feet sawed per hour, in dry oak, there requires,				550	1·000
Dry pine 30 square feet per hour.				550	1·000
Circular Saw.					
A saw 2·5 feet in diameter; and making 270 revolutions per minute will saw 40 square feet in oak per hour, with				550	1·000
In dry spruce, 70 square feet per hour.				550	1·000
Threshing Machine.					
Velocity of the feed rollers at the circumference 0·55 feet per second. Diameter of threshing-cylinder 3·5 feet and 4½ feet long making 300 revolutions per minute, can thresh from 30 to 40 bushels of oats, and from 25 to 35 bushels of wheat, per hour.				2200	4·000
One man by a flail can thresh half a bushel per hour, (wheat.)				70	0·127
Rolling Mills.					
Bar iron-mills. Two pair of rough rollers, two pair of finishing rollers, six puddle furnaces, two welding furnaces, making 10 tons of bar iron per 24 hours, rollers making 70 revolutions per minute, requires				29000	80
Plate-mill requires about five horses per square foot of plates rolled. Largest size plate rollers should not make over 30 revolutions per minute.					

DREDGING MACHINES.

Letters denote.

T = tons of materials excavated and raised per hour.
h = hight in feet in which the materials are raised above the bottom of the excavated channel.
k = 0·1 for hard clay with gravel.
k = 0·07 for hard pure clay.
k = 0·05 for common clay or sand.
k = 0·04 for soft clay or loose sand.
k = 0·03 for very loose materials.
H = horse power required for excavating and raising the materials.
F = force in pounds required to feet the Dredge ahead.
v = velocity of the buckets in feet per second.

Formulas.

$$H=T\left(\frac{h}{700}+k\right) \quad - \quad - \quad - \quad 1$$

$$T=\frac{700\,H}{h+700\,k} \quad - \quad - \quad - \quad 2$$

$$F=\frac{550\,H}{v} \quad - \quad - \quad - \quad - \quad 3$$

$$F=\frac{550\,Tk}{v} \quad - \quad - \quad - \quad - \quad 4$$

$$k=\frac{H}{T}+\frac{h}{700} \quad - \quad - \quad - \quad - \quad 5$$

Example 1. What power is required to excavate T=160 tons of hard pure clay per hour, and raise it up h=25 feet above the bottom of the channel? For hard clay k=0·07.

$$H=160\left(\frac{25}{700}+0{\cdot}07\right)=16{\cdot}9, \text{ or } 17 \text{ horses.}$$

Example 2. What force F=? is required to feet the Dredge ahead for the above example when the buckets move v=1 foot per second.

$$F=\frac{550\times16{\cdot}9}{1}=9295 \text{ pounds.}$$

LEATHER BELTS.

Letters denote.

b = breadth of leather belt in inches,
H = horse power transmitted by the belt.
v = velocity of the belt in feet per second.
d = diameter in inches } of the smallest belt pulley.
n = revolutions per mintute } of the smallest belt pulley.
F = force in pounds transmitted by the belt.
a = number of degrees occupied by the belt on the small pulley.

$$v=\frac{d\,n}{230}, \quad - \quad - \quad 1$$

$$H=\frac{v\,F}{550}, \quad - \quad - \quad 2$$

$$H=\frac{d\,n\,F}{126500}, \quad - \quad - \quad 3$$

$$F=\frac{126500H}{d\,n}, \quad - \quad - \quad 4$$

$$b=\frac{7500H}{d\,a}, \quad - \quad - \quad - \quad 5$$

$$b=\frac{13{\cdot}5\,v\,F}{d\,a}, \quad - \quad - \quad - \quad 6$$

$$b=\frac{n\,F}{18{\cdot}8}, \quad - \quad - \quad - \quad 7$$

$$b=\frac{29\,n\,H}{v\,a}. \quad - \quad - \quad - \quad 8$$

Example. Required the breadth of a leather belt to transmit a power of H=75 horses over a pulley of 72 inches diameter, angle a=170°.

Formulæ 5. $b=\dfrac{7400\times75}{72\times170}$ =45·4 inches, the answer.

COLLISION OF BODIES IN MOTION.

When bodies in motion come in collision with each other, the sum of their concentrated momentum will be the same after the collision as before, but their velocities and sometimes their directions will differ.

On the accompanying page the bodies are supposed to move in the same straight line, and the formula illustrates the consequences after collision.

Letters denote.

M and m = weight of the bodies in pounds.
V and v = their respective velocities in feet per second.
V' and v' = respective velocties of the bodies after impact.
K and k = coefficient of elasticity, which for perfectly hard bodies $k=0$ and for perfect elastic bodies $k=1$, therefore the elastic coefficient will always be between 0 and 1. When the bodies are perfectly hard their velocities after impact will be common.

$$\text{For } M,\quad K=\frac{MV}{M(V-V')},\qquad \text{For } m,\quad k=\frac{mv}{m(v-V')}.$$

Example 1. Fig 191. The non-elastic body weighs $M=25$ pounds, and moves at a velocity $V=12$ feet per second; $m=16$ pounds, and $v=9$. Required the bodies' common velocities, $v'=?$ after impact.

$$v'=\frac{MV+mv}{M+m}=\frac{25\times12+16\times9}{25+16}=10{\cdot}83 \text{ feet per second.}$$

Example 2. Fig. 195. The perfect elastic body $M=84$ pounds, $V=18$, $m=48$, and $v=27$. Required the velocity $V'=?$ after impact with the body m.

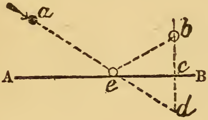
$$V=\frac{18\,(84-48)-2\times48\times27}{84+48}=-23{\cdot}64.$$

the negative sign denotes that the body will return after the collision with a velocity of 23·63 feet per second.

Example 3. Fig. 196. The partly elastic body $M=38$ pounds and $V=79$ feet per second, will strike the body in rest $m=24$ pounds; what will be the velocity $v'=?$ of the body m, its elasticity being $k'=0{\cdot}6$.

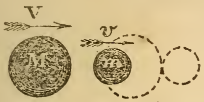
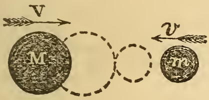
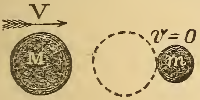
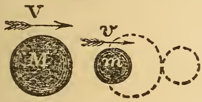
$$v'=\frac{79\times38\,(1+0{\cdot}6)}{38+24}=70{\cdot}6 \text{ feet per second.}$$

When a moving body strikes a stationary elastic plane, its course of departure from the plane will be equal to its course of incident.



Problem. A body in a is to strike the plane AB so that it will depart to the given point b; required its course of incident from a?

Draw bd, at right angles through AB, make $cd=bc$ join a and d; then ad is the course of incident, and eb, the course of departure, and the body will strike in e.

	191. *The bodies move in the same direction.* $v'(M+m)=MV+mv,$ $v'=\frac{MV+mv}{M+m}.$	The bodies perfectly hard.
	192. *The bodies move in opposite directions.* $v'(M+m)=MV-mv,$ $v'=\frac{MV-mv}{M+m}.$	
	193. *Only one body in motion.* $v'(M+m)=MV,$ $v'=\frac{MV}{M+m}.$	
	194. *The bodies move in the same direction.* $V'=\frac{V(M-Km)+vm(1+K)}{M+m},$ $v'=\frac{MV(1+k)+v(m-kM)}{M+m}.$	The bodies elastic.
	195. *The bodies move in opposite directions.* $V'=\frac{V(M-Km)-vm(1+K)}{M+m},$ $v'=\frac{MV(1+k)-v(m-kM)}{M+m.}$	
	196. *Only one body in motion.* $V'=\frac{V(M-Km)}{M+m},$ $v'=\frac{VM(1+k)}{M+m}.$	

PILE DRIVING.

Letters denote.

M = weight of the ram in pounds.
S = fall of the ram in inches.
m = weight of the pile in pounds.
s = space in inches which the pile sinks by the blow.
r = resistance of the ground in pounds.
a = section area in sq. in. of the pile, sharpened to a point not more than 45°.
k = coefficient for the hardness of the ground.
h = depth to which the pile is driven.
W = weight in pounds which a driven pile can bear with safety after the last blow when the pile sunk s inches.
V = velocity in feet per second by which the ram strikes the pile.
Ram and pilehead considered non-elastic and perfectly hard.

$$V = 8\sqrt{S} \quad \text{- -} \quad 1$$

$$s = \frac{M^3 S}{r\,(M+m)^3} \quad \text{- -} \quad 2$$

$$W = \frac{M^3 S}{6s\,(M+m)^2} \quad \text{- -} \quad 3$$

$$v = \frac{8\,M\sqrt{S}}{M+m} \quad \text{- - - -} \quad 4$$

$$r = \frac{M^3 S}{s\,(M+m)^3} \quad \text{- - -} \quad 5$$

$$r = ak\sqrt{h} \quad \text{- - - -} \quad 6$$

Example 1. A wooden pile 18 feet long by 12 inches square, driven $h=12$ feet into common natural ground imbedded with tenacious clay for which may be assumed the coefficient $k=50$. Required how much the pile will set $s=?$ into the ground at a blow with a ram of $M=3500$ lbs. falling $S=42$ inches.

The weight of the wooden pile will be about $m=18\times40=720$ lbs.
Area of the pile $a=144$ square inches.

Resistance $r=144\times50\sqrt{12}=23840$ lbs.

The resistance sought from this formula 6, cannot be depended upon for calculating the weight the pile can bear with safety.

$$\text{The set } s = \frac{3500^3\times42}{23840\,(3500+720)^3} = 4{\cdot}23 \text{ inches.}$$

Suppose the set to be $s=4{\cdot}23$ inches at the last blow, required what weight the pile can bear with safety?

$$W = \frac{3500^3\times42}{6\times4{\cdot}23\,(3500+720)^3} = 3984 \text{ lbs.}$$

This can be depended on with safety, if calculated from the actual set of the pile at the last blow.

For ordinary pile driving a heavy ram and short fall is the most effective, but in some cases when the ground itself is elastic, or when driving piles in pure sand it is found more advantageous to use a high fall of the ram.

The Author has written to several places for information on pile driving, in order to establish a table for the coefficient k; have received answers, but with no information. Hope to be able to give such a table in the next edition.

STEAM HAMMER.

A heavy steam hammer with short fall produces a better forging than a light hammer with a high fall, although the dynamical momentum may be the same in both cases. This is accounted for by the inertia of the ingot forged.

The effect of blows of a heavy hammer and short fall will penetrate through the metal and nearly with the same effect on the anvil side, while a light hammer and high fall will effect the metal on or near the surface of the blow, because most of the momentum is in the latter case discharged in the inertia of the ingot forged. In forging large shaft it is generally piled up with iron bars sometimes rolled into a segment form to suit the pile, when placed under the hammer in a welding heat, very light and gentle blows are first given, then, the momentum of a light hammer may be discharged in the bars nearest to the blow, while a heavy hammer will squeeze the whole mass together throughout and a sound welding produced.

The additional expense of a heavy hammer is fully compensated by the waste of labour and materials under a small one. I have often seen in broken shafts the bars in the centre as clear and unwelded as when first piled, which is a pure indication that the shaft had been forged by a too light hammer. Crank shafts for propeller engines forged under a light hammer, when brought to the Machineshop the best part of the metal is worked away by plaining and turning, and the poorest left for the engine, but if forged under a heavy hammer, the difference of quality of metal will not be so great.

Cases of this kind are well known in the U. S. Navy.

Weight of Steam Hammers.

The weight of a steam hammer in pounds should be at least 80 times the square of the diameter of the shaft in inches.

CEMENT, CONCRETE AND MORTAR.

Roman Cement. *Parker's analysis.*

One part of common clay to 2½ parts of Chalk, set very quick.

Concrete. Eight parts of pebble or pieces of brick about the size of an egg, to 4 parts of scrap river sand, and 1 part of good lime, mixed with water and grouted in, makes a good concrete.

Lime Mortar. One part of river sand to two parts of powdered lime, mixed with fresh water.

Hydralic Mortar. One part of pounded brick powder to two parts of powdered lime mixed with fresh water. This mortar must be laid very thick between the bricks, and the latter well soaked in water before laid.

Hydralic Concrete, by Treussart.

30 parts of hydraulic lime, measured in bulk before slacked.
30 " Sand.
20 " Gravel.
40 " Broken stone, a hard lime stone.

This concrete diminishes about one fifth in volume after manipulation.

Asphalt Composition for street pavement by Colonel Emy.

2½ pints (wine measure,) of pure mineral pitch.
11 lbs of Gaugeac bitumen.
17 pints of powdered stonedust, woodashes or minion.

FRICTION.

THE resistance occasioned by Friction is independent of the velocity of motion; but the re-effect of friction is proportional to the velocity. Friction is independent of the extent of surface in contact when the pressure remains the same, but proportional to the pressure. This law was established from experiments by Arthur Morin in the years 1831-32 and 1833, from which a summary is contained in the accompanying Table.

Letters denote.

a = Fibres of the woods are parallel to themselves, and to the direction of motion.
b = Fibres at right-angles to fibres.
c = Fibres vertical on the fibres which are parallel to the motion.
d = Fibres parallel to themselves, but at right-angles to the motion, length by length.
e = Fibres vertical, end to end.

Example. A vessel of 800 tons is to be hauled up an inclined plane, which inclines 9° 40′ from the horizon; the plane is of oak, and greased with tallow. What power is required to haul her up?

The coefficient for oak on oak with continued motion is $f = 0{\cdot}097$, say 0·1, then,

$$800 \times \sin.9^\circ\ 40' = 800 \times 0{\cdot}16791 = 124{\cdot}328 \text{ tons,}$$

the force required if there were no friction, and

$$800 \times \cos.9^\circ\ 40' \times 0{\cdot}1 = 800 \times 0{\cdot}9858 \times 0{\cdot}1 = 78{\cdot}864 \text{ tons,}$$

the force required for the friction only, and

$$\begin{array}{r} 134{\cdot}328 \\ \underline{78{\cdot}864} \\ 213{\cdot}192 \end{array}$$ tons, the force required to haul her up.

The effect lost by friction in axle and bearings is expressed simply by the formula

$$P = \frac{\pi d\, W n f}{12{\cdot}60} = \frac{W d n f}{230},$$

in which W = the weight of pressure in the bearing, d = diameter on which the friction acts in inches, n = number of revolutions per minute, and f = coefficient of friction from the Table. In common machinery kept in good order the coefficient of friction can be assumed to $f = 0{\cdot}065$, then

$$P = \frac{W d n}{3530}, \qquad H = \frac{W d n}{1941500}$$

Example. The pressure on a steam-piston is 20000 pounds, and makes $n = 40$ double strokes per minute. Required the friction in the shaft of $d = 8$ inches?

$$H = \frac{20000 \times 8 \times 40}{1941500} = 3{\cdot}3 \text{ horses, the loss by friction.}$$

Friction in Guides.

W = pressure on the steam piston in pounds.
S = stroke of piston in feet.
l = length of connecting rod in feet.
H = horse power of the friction.

$$H = \frac{W S n}{350000 \sqrt{5l^2 - S^2}}$$

Example. The pressure on a steam piston being W = 30,000 pounds, stroke $S = 4$ feet, length of connecting rod $l = 7$ feet, and making 50 revolutions per minute. Required the horse power of the friction $H = ?$

$$H = \frac{30000 \times 4 \times 50}{350000 \sqrt{5 \times 7^2 - 4^2}} = 1{\cdot}13 \text{ horses.}$$

TABLE OF FRICTION FOR PLANE SURFACES IN CONTACT.

Kind of Materials in contact.		*Lubricated with.*	*Coefficient in Motion.*	*Coefficient in Starting.*
Oak on Oak, - - - - -	a	o	0·478	0·625
" " - - - - -	"	tallow	0·097	0·160
" " - - - - -	"	lard	0·067	....
" " - - - - -	b	o	0·324	0·540
" " - - - - -	"	unctuous	0·143	0·314
" " - - - - -	"	tallow	0·083	0·254
" " - - - - -	"	water,	0·25	....
" " - - - - -	d	o	0·336	....
" " - - - - -	c	o	0·192	0·271
" " - - - - -	e	o	....	0·43
C t-iron on Oak, - -	a	o	0·400	0·570
" " - - - -	"	soap	0·214	....
" " - - - -	"	tallow	0·078	0·108
Wrought-iron on Oak, - -	"	o	0·252	....
" " - -	"	tallow	0·078	....
Wrought iron, together, - -	a	o	0·138	0·137
" " - -	a	unctuous	0·177	....
" " - -	"	tallow	0·082	....
" " - -	"	olive oil	0·070	0·115
Wrought on cast-iron, - -	a	o	0·194	0·194
" " - -	"	unctuous	0·18	0·118
" " - -	"	tallow	0·103	0·10
" " - -	"	olive oil	0·066	0·100
Cast-iron on cast-iron, - -	a	water	0·314	0·314
" " - -	"	soap	0·197	....
" " - -	"	tallow	0·100	0·100
" " - -	"	olive oil	0·064	....
Wrought-iron on brass, - -	a	o	0·172	....
" " - -	"	unctuous	0·160	....
" " - -	"	tallow	0·103	....
" " - -	"	lard	0·075	....
" " - -	"	olive oil,	0·078	....
Cast-iron on brass, - -	a	o	0·147	...
" " - - -	"	unctuous	0·132	....
" " - - -	"	tallow	0·103	....
" " - - -	"	lard	0·075	....
" " - - -	"	olive oil	0·078	....
Brass on brass, - - - -	a	o	0·201	....
" " - - - - -	"	unctuous	0·134	....
" " - - - - -	"	olive oil	0·053	....
Steel on cast-iron, - - -	"	o	0·202	....
" " - - -	"	tallow	0·105	....
" " - - -	"	lard	0 081	....
" " - -	a	olive oil	0·079	. .

FRICTION OF AXLES IN MOTION.

Designation of surface in contact.	*Dry or slightly greasy, or wet.*	*Oil, Tallow, or Hog's Lard.* *Supplied in the ordinary manner.*	*Oil, Tallow, or Hog's Lard.* *The grease continually running.*
Brass on Brass, - - -	.....	0·079	.....
" on cast-iron, -	.....	0·072	0·049
Iron on Brass, - - -	0·251	0·075	0·054
" on cast-iron, - -	.....	0·075	0·054
Cast-iron on cast-iron, -	0·137	0·075	0·054
" on Brass, - -	0·194	0·075	0·054
Iron on lignum-vitæ, -	0·188	0·125	.....
Cast-iron on " -	0.185	0·100	0·092
Lignum-vitæ on cast-iron,	.....	0·116	0·170

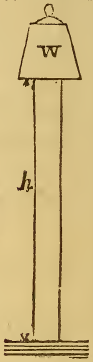
STRENGTH OF MATERIALS.

Table I., shows the weight a column can bear with *safety;* when the weight presses through the length of the column. The tabular number is the weight in pounds or tons per square inch on the transverse section of a column of a length less than 12 times its smallest thickness.

Table I.

RESISTANCE FOR COMPRESSION. 197

Kind of Materials.	*Pounds.*	*Tons.*
Oak, of good quality, - - - -	432	0·1885
Oak, common, - - - - -	280	0·125
Spruce, red (Sapin rouge), - - -	540	0·241
" white, (Sapin blanc), - -	140	0·6256
Iron, wrought, - - - - -	14400	6·43
Iron, cast, - - - - - -	28750	12·85
Basalt, - - - - - - -	2875	1·285
Granite, hard, - - - - -	1000	0·446
" common, - - - - -	575	0·256
Marble, hard, - - - - - -	1435	0·640
" common, - - - - -	431	0·192
Sandstone, hard, - - - - -	1295	0·577
" loose, - - - - -	5·6	0·0025
Brick, good quality, - - - -	175	0·078
" common, - - - - -	58	0·0259
Lime-stone, of hardest kind, - - -	720	0·321
" common, - - - -	432	0·193
Plaster-Paris, - - - - - -	86	0·0384
Mortar, good quality, and 18 months old,	58	0·0259
Do. common, - - - - -	36	0·016



When the length or height of the column is more than 12 times its smallest thickness, divide the tabular weight by the corresponding number in this Table.

Length×thickness	12	18	24	30	36	42	48	54	60
Divide by	1·2	1·6	2	2·8	4	5	6	8	12

Example. A building which is to weigh 2000 tons is to be supported by piles of Sapin rouge Spruce 18 feet in length, and 12 inches diameter. How many piles are required to support the building?

$$\frac{12^2\times0{\cdot}785\times0{\cdot}241}{1{\cdot}6} = 17 \text{ tons, the weight which each pile can bear,}$$

and $$\frac{2000}{17} = 118 \text{ piles.}$$

Professor Hodgkinson's Formulæ for Crushing Strength of Cast Iron Pillars.

The ends of the pillars should be perfectly flat and square, and the load to bear even on the whole surface.

T=crushing weight in tons.
D=outside and d inside diameters in inches.
l=length or height of pillar in feet.

$$T=46{\cdot}65\left(\frac{D^{3.55}-d^{3.55}}{l^{1.7}}\right)$$

Table showing the Weight in tons which Cast Iron Pillars or Tubes can bear with Safety.

Diameters in Inches. For Tubes subtract the weight due to the bore.

L	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	20	24
	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons	tons,
1	22	218	1156	3256	7272	14...	244..	395..	6.....	88...	124..	169 .	226..	296..	379..	478..	59....	73...	1.....	2.....
2	6·8	82·5	355	1000	2238	4315	7500	1215	85.1	2714	38...	523..	697..	910..	116..	147..	18....	22....	32....	63....
3	3·5	41·5	178	500	1123	2166	3744	6101	9322	136..	162..	262..	350..	457..	586..	739..	920..	113..	165..	318..
4	2·1	25·5	110	308	689	1328	2313	3741	5716	8354	1177	161..	2148	280..	359..	453..	564..	693..	101..	195..
5	1·4	17·4	75	211	471	908	1583	2560	3912	5716	8056	110..	1470	1919	246..	310..	386..	474..	693..	133..
6	1·0	12·8	55	155	346	666	1161	1878	2869	4192	5909	8082	1078	1407	1804	2277	283..	347..	508..	980..
7	0·81	9·8	42	119	266	513	894	1444	2207	3226	4547	6219	8296	1083	1388	1752	217..	267..	391..	754..
8	0·65	7·8	36	95	212	408	712	1151	1760	2571	3624	4956	6611	8633	1106	1396	173..	213..	311..	601..
9	0·53	6·4	28	78	174	335	583	942	1440	2104	2966	4057	5412	7066	9059	1142	142..	174..	255..	491..
10	0·44	5·3	23	65	145	280	487	788	1200	1760	2479	3391	4524	5907	7574	9554	118..	146..	213..	411..
11	0·38	4·6	20	55	123	238	414	670	1024	1496	2108	2884	3847	5024	6441	8125	101..	124..	181..	349..
12	0·32	3·9	17	48	106	205	357	578	883	1292	1818	2487	3318	4333	5555	7008	8717	107..	156..	301..
13	0·28	3·4	15	42	93	179	312	507	771	1126	1587	2171	2896	3782	4848	6116	7608	9346	136..	263..
14	0·25	3·0	13	37	82	158	275	445	679	993	1400	1914	2553	3333	4274	5392	6707	8240	120..	232..
15	0·22	2·7	11·5	33	73	140	250	396	604	883	1244	1702	2271	2965	3801	4795	5965	7328	102..	206..
16	0·20	2·4	10	23	65	126	218	354	541	792	1115	1526	2035	2657	3406	4297	5345	6551	9590	184..
17	0·18	2·3	9·4	26	58	113	198	320	489	714	1006	1376	1836	2397	3073	3876	4712	5923	8656	166..
18	0·16	2·0	8·5	24	53	103	174	290	443	648	913	1248	1665	2175	2788	3517	4375	5375	7854	151..
19	0·14	1·8	7·8	22	49	93	163	265	404	591	832	1139	1519	1984	2543	3208	3991	4903	7165	138..
20	0·13	1·6	7·1	20	45	86	150	243	371	541	763	1044	1392	1818	2331	2940	3658	4497	6066	126..

These values are about half of that by Prof. Hodgkinson's formula. The points after the numbers mean ciphers.

Table II.

COHESIVE STRENGTH PER SQ. INCH OF CROSS-SECTION.

Kind of Materials.	Just tear asunder. Pounds.	Tons.	With safty. Pounds.	Tons.
Cast Steel, - - -	134256	59·93	33600	14·98
Blistered Steel, - -	133152	59·43	33300	14·86
Steel, Shear, - - -	128632	56·97	32160	14·24
Iron, Swedish bar, - -	65000	29·2	16260	7·3
" Russian, - -	59470	26·7	14900	6·7
" English, - -	56000	25·0	14000	6·25
" common, over 2 in. sq.,	36000	16·00	9000	4·0
" sheet, parallel rolling,	40000	17·85	10000	4·46
" at right angles to roll,	34400	15·35	8600	3·84
Cast iron, good quality, -	45000	20·05	11250	5·00
" inferior, - -	18000	8·03	4500	2·0
Copper, cast, - - -	32500	14·37	8130	3·6
" rolled, - -	61200	27·2	15300	6·8
Tin, cast, - - - -	5000	2·23	12500	0·56
Lead, cast, - - -	880	0·356	220	0·09
" rolled, - - -	3320	1·48	830	0·37
Platinum, wire, - -	53000	23·6	13250	5·9
Brass, common, - -	45000	20·05	11250	5·0
Wood.				
Ash, - - - - -	16000	7·14	4000	1·87
Beach, - - - - -	11500	5·13	2875	1·28
Box, - - - - - -	20000	8·93	5000	2·23
Cedar, - - - - -	11400	5·09	2850	1·27
Mahogany, - - -	21000	9·38	5250	2·34
" Spanish, -	12000	5·36	3000	1·34
Oak, American white, -	11500	5·13	2875	1·28
" English " -	10000	4·46	2500	1·11
" seasoned, - - -	13600	6·07	3400	1·52
Pine, pitch, . - -	12000	5·35	3000	1·34
" Norway, - - -	13000	5·8	3250	1·45
Walnut, - - - -	7800	3·48	1950	0·87
Whalebone, - - -	7600	3·40	1900	0·85
Hemp ropes, good, - -	6400	2·86	2130	0·95
Manilla ropes, - -	3200	1·43	1100	0·49
Wire ropes, - - .	38000	17	12600	5·36
Iron chain, - - -	65000	29	21600	9·38
" with cross pieces, -	90000	40	30000	13·4

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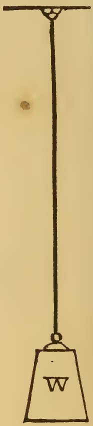


To Find the Cohesive Strength.

RULE.—Multiply the cross-section of the materials in square inches by the tabular number in Table II., and the product is the cohesive strength.

Example An iron-bar has a cross-section of 2·27 sq. in. How many tons are required to tear it asunder, and how many pounds can it bear with safety?

English iron $2·27 \times 25 = 56·75$ tons, which will tear it asunder, and it will bear with safety

$$2·27 \times 14000 = 31780 \text{ pounds.}$$

Safety proof.	Inches and 16ths.			Wht. per fathom.			Price per fathom.			Ultimate Strain.
	Chain.	Hemp.	Wire.	Chain.	Hemp.	Wire.	Chain.	Hemp.	Wire.	
Cwt.	Diam.	Circ'm.	Circ m.	Pounds	Pounds	Pounds	$ cts.	$ cts.	$ cts.	Cwt.
1.3	1	0·10	0·4	0·23	0·08	0·06	0·15	0·06	0·08	2·6
4·5	2	1.6	0·8	0·93	0·47	0·24	0·25	0·12	0·15	9
10	3	2·1	0·12	2·11	1·06	0·54	0·36	0·17	0·22	20
18	4	2·12	1·1	3·75	1·89	1·10	0·48	0·25	0·32	35
28	5	3·7	1·6	5·86	2·94	1·83	0·60	0·33	0·43	55
40	6	4·2	1·10	8·45	4·52	2·56	0·96	0·42	0·54	80
55	7	4·15	1·14	11·5	6.09	3·42	1·25	0·48	0·62	109
69	8	5·8	2·2	15·0	7·55	4·39	1·44	0·60	0·78	138
80	9	6·3	2·6	18·8	9·56	5·48	1·60	0·76	0·90	160
94	10	6·14	2·11	23·0	11·8	7·00	1·86	0·95	1·20	218
109	11	7·9	2·15	27·7	14·3	8·38	2·16	1·14	1·50	187
127	12	8·4	3·3	33·0	17·1	9.90	2·43	1·37	1·80	254
147	13	8·15	3·8	38·5	19·9	11·9	2·70	1·60	2·10	293
168	14	9·10	3·12	44·7	23·1	13·6	3·06	1·85	2·28	335
199	15	10·5	4·1	51·1	26·3	16·0	3·70	2·10	2·45	397
220	1 in.	11·	4·6	58·0	30·2	18·6	4·33	2·42	2·73	440
246	1·1	11·11	4·11	65·6	34·1	21·3	4·68	2·73	3·10	492
278	1·2	12·6	5 in.	73·7	38·2	24·2	5·58	3·06	3·50	545
302	1·3	13·1	5·5	82·1	42·6	27·4	5·86	3·40	3·91	604
332	1·4	13·12	5·10	91·0	47·1	30·7	6·42	3·77	4·35	663
365	1·5	14·7	6 in	100	52·0	35·	7.08	4·16	4·89	730
399	1·6	15·2	6·5	110	57·1	38·7	7·75	4·57	5·35	798
435	1·7	15·15	6·10	120	63·4	42·6	8·42	5·07	5·86	869
472	1·8	16·8	6·15	131	67·9	46·7	9·15	5·44	6·35	944
553	1·10	17·14	7·10	154	79·8	56·4	10·07	6·38	7·63	1105
638	1·12	19·4	8·4	178	92·6	66·0	12·38	7·40	8·83	1275
729	1·14	20·10	8·14	205	106	76·5	14·15	8·48	10·00	1457
825	2 in	22·	9.8	232	121	88·0	16·00	9·70	11·50	1650
1072	2·4	24·12	10·12	293	153	112	20·75	10·25	14·60	2141
1288	2·8	27·8	12 in.	363	189	140	25·	15·10	18·00	2575
1559	2·12	30·4	13·4	438	229	172	30·25	18·30	21·80	3117
1854	3 in.	33·	14·8	522	272	205	36·00	21·80	25·90	3708

The prices of the chains are taken from that in England and added 50 per cent. Price of hemp ropes from Weaver, Fitler & Co., Rope manufacturers, Philadelphia. The prices of Wire ropes are deduced from the price list of John A. Roebling, Patent Wire Rope Manufacturer, Trenton, N. J.

The Safety proof is here taken one half of the ultimate strength which may be trusted on for new ropes, but when much in use only one quarter or less should be trusted upon for safety.

CABLES AND ANCHORS.

TABLE *showing the size of Cables and Anchors proportioned to the Tonnage of Vessels.*

Tonnage of Vessels.	*Cables. Circumference in inches.*	*Chain Cables Diameter in inches.*	*Proof in tons.*	*Weight of Anchor in pounds.*	*Weight of fathom of Chain.*	*Weight of a fathom of Cables.*
5	3·	$\cdot\frac{5}{16}$	$\cdot\frac{3}{4}$	56	$5\cdot\frac{1}{2}$	2·1
8	4·	$\cdot\frac{3}{8}$	$1\cdot\frac{3}{4}$	84	8·	4·
10	$4\cdot\frac{1}{2}$	$\cdot\frac{7}{16}$	$2\cdot\frac{1}{2}$	112	11·	4·6
15	$5\cdot\frac{1}{2}$	$\cdot\frac{1}{2}$	4·	168	14·	6·5
25	6·	$\cdot\frac{9}{16}$	5.	224	17·	8·4
40	$6\cdot\frac{1}{2}$	$\cdot\frac{5}{8}$	6·	336	24·	9·8
60	7·	$\cdot\frac{11}{16}$	7·	392	27·	11·4
75	$7\cdot\frac{1}{2}$	$\cdot\frac{3}{4}$	9·	532	30·	13·
100	8·	$\cdot\frac{13}{16}$	10·	616	36·	15·
130	9·	$\cdot\frac{7}{8}$	12·	700	42·	18·9
150	$9\cdot\frac{1}{2}$	$\cdot\frac{15}{16}$	14·	840	50·	21·
180	$10\cdot\frac{1}{2}$	1·	16·	952	56·	25·7
200	11·	$1\cdot\frac{1}{16}$	18·	1176	60·	28·2
240	12·	$1\cdot\frac{1}{8}$	20·	1400	70·	33·6
270	$12\cdot\frac{1}{2}$	$1\cdot\frac{3}{16}$	21·	1456	78·	36·4
320	$13\cdot\frac{1}{2}$	$1\cdot\frac{1}{4}$	$22\cdot\frac{1}{2}$	1680	86·	42·5
360	14·	$1\cdot\frac{5}{16}$	25·	1904	96·	45·7
400	$14\cdot\frac{1}{2}$	$1\cdot\frac{3}{8}$	27·	2072	104·	49·
440	$15\cdot\frac{1}{2}$	$1\cdot\frac{7}{16}$	30·	2240	115·	56·
480	16·	$1\cdot\frac{1}{2}$	33·	2408	125·	59·5
520	$16\cdot\frac{1}{2}$	$1\cdot\frac{9}{16}$	36·	2800	136·	63·4
570	17·	$1\cdot\frac{5}{8}$	39·	3360	144·	67·2
620	$17\cdot\frac{1}{2}$	$1\cdot\frac{11}{16}$	42·	3920	152·	71·1
680	18·	$1\cdot\frac{3}{4}$	45·	4200	161.	75·6
740	19·	$1\cdot\frac{13}{16}$	49·	4480	172·	84·2
820	20·	$1\cdot\frac{7}{8}$	52·	5600	184·	93·3
900	22·	$1\cdot\frac{15}{16}$	56·	6720	196·	112·9
1000	24·	2·	60·	7168	208·	134·6

The proof in the U. S. Naval Service is about $12\frac{1}{2}$ per cent. less than the above for the larger sizes, and from 25 to 30 per cent. for the smaller.

The results of experiments at the U. S. Navy Yard, Washington, D. C., give for the cohesive force of chain iron, per square inch, as follows:

Mean of experiments with good iron, ... 41000 lbs.
Mean of experiments with best iron, ... 46000 lbs.

To find the weight of Castings, by the weight of Pine Patterns.

RULE.

Multiply the weight of the Pattern by			and the product is the weight of the Castings.
	12	for Cast Iron,	
	13	" Brass,	
	19	" Lead,	
	12·2	" Tin,	
	11·4	" Zinc,	

Reductions for Round Cores and Core-prints.

Rule. Multiply the square of the diameter by the length of the Core in inches, and the product by 0·017, is the weight of the pine core, to be deducted from the weight of the pattern.

Shrinking of Castings.

Pattern Makers' Rule should be for		of an inch	longer per Linear Foot.
	Cast Iron,	$\frac{1}{8}$	
	Brass,	$\frac{3}{16}$	
	Lead,	$\frac{1}{8}$	
	Tin,	$\frac{1}{12}$	
	Zinc,	$\frac{3}{16}$	

Weight and Capacity of Balls.

Diameter in inches.	Capacity in cubic inches.	CAST IRON. Pounds.	LEAD. Pounds.
1·	·5235	·1365	·2147
1·½	1·7671	·4607	·7248
2·	4·1887	1·0920	1·7180
2·½	8·1812	2·1328	3·3554
3·	14·1371	3·6855	5·7982
3 ½	22·4492	5·8525	9·2073
4·	33·5103	8·7361	13·744
4·½	47·7129	12·4387	19·569
5·	65·4498	17·0628	26·843
5½·	87·1137	22·7206	35·729
6·	113·0973	29·4845	46·385
6·½	143·7932	37·4528	58·976
7·	179·5943	46 8203	73·659
7·½	220·8932	57·5870	90·598
8·	268·0825	69·8892	109·952
8·½	321·5550	83·8396	131·383
9·	381·7034	99·5103	156·553
9·½	448·9204	117·0338	184·121
10·	523·5987	136·5025	214·749
11·	696·9098	181·7648	285·832
12·	904·7784	235·8763	371·096
13·	1150·346	299·6230	471·806
14·	1436·754	374·5629	589·273
15·	1767·145	460·6959	724·781
16·	2144·660	559·1142	879·616
17·	2572·440	670·7168	1055·066
18·	3053·627	796·0825	1252·422
19·	3591·363	936·2708	1472·970
20·	4188·790	1092·0200	1717·995

LATERAL STRENGTH.

For wrought iron beams, letters denote.

W = weight in tons with safety, uniformly distributed on a beam resting on two supports.

S = compressive strain in tons per square inch of top $0{\cdot}5\left(a+\frac{a'}{3}.\right)$

a = section area in square inches of top and bottom flanges of the beam. Top and bottom flanges to be alike.
a' = section area in square inches of web or stem.
h = height of beam in inches.
l = length in feet between supports.
f = deflection in inches of the beam in the centre, when the weight is uniformly distributed.
P = weight in pound per square foot of flooring to be supported by the beams, which in ordinary cases is estimated to P=140 lbs.
B = distance in feet between the beams.
w = weight of the whole beam in pounds.

$$W=\frac{3h}{l}\left(a+\frac{a'}{3},\right) \quad \text{- - 1}$$

$$W=\frac{Sh}{3l}\left(a+\frac{a'}{3},\right) \quad \text{- 2}$$

$$S=\frac{3Wl}{h\left(a+\frac{a'}{3}\right)} \quad \text{- - 3}$$

$$w=3{\cdot}384\,l\,(a+a'.) \quad \text{- 4}$$

$$f=\frac{Wl^3}{46\,h^2\,(3\,a+a')}, \quad \text{- 5}$$

$$f=\frac{l^2}{46h}, \quad \text{- - - 6}$$

$$B=\frac{2240\,W}{Pl}, \quad \text{- - - 7}$$

$$P=\frac{22w\,W}{Bl}. \quad \text{- - - 8}$$

Formula 6 gives the safety deflection of a wrought iron beam, which should never be exceeded.

Example. A flooring of l=32 feet by 60 feet long to be constructed to support P=175 pounds per square foot. Required what kind of beams and how many are necessary? and what will be the cost of them?

In the table will be found the nearest star to 32 feet span is a 12 inch beam bearing W=8·71 tons, when the distance between the beams in the flooring will be,

$$\text{Formula 7.}\quad B=\frac{2240\times 8{\cdot}71}{175\times 32}=3{\cdot}5\text{ feet.}$$

$$\text{Number of beams}=\frac{60}{3{\cdot}5}-1=16\text{ about.}$$

Add one foot to each beam for the supports at the ends, and the cost will be, 33×16×1·90=1003·70 dollars.

The following Table contains sections of iron rolled by the Phœnix Iron Company. Office 410 Walnut Street, Philadelphia.

Rules.

The price per foot multiplied by 5280 gives the price per mile.

The weight in pounds per foot multiplied by 2·36 gives the weight in tons per mile.

The price per foot multiplied by 2240 and divided by the weight in pounds per foot gives the price per ton.

Strength of different Sections of Wrought Iron Beams

Made by the Phœnix Iron Company, for Sustaining with Safety a Load Uniformly Distributed.

Dis. bet. sup $=l$.	Compound Girders.			Solid Rolled Beams.					
	$W=\frac{800}{l}$ $h=18$ i.	$W=\frac{667}{l}$ $h=15$ i.	$W=\frac{553}{l}$ $h=12$ i.	$W=\frac{490}{l}$ $h=15$ i.	$W=\frac{296}{l}$ $h=12$ i.	$W=\frac{308}{l}$ $h=9$ i.	$W=\frac{168}{l}$ $h=9$ i.	$W=\frac{84}{l}$ $h=7$ i.	$W=\frac{48}{l}$ $h=6$ i.
feet.	tons.	tons.	tons.	tons.	tons.	tons.	tons.	tons.	tons.
10	80·00	66·67	55·33	49·00	29·60	30·80	16·80	8·40	4·80
12	66·66	55·56	44·44	40·83	24·66	25·69	14·00	7·00	4·00
14	57·14	47·61	38·09	35·00	21·14	22·05	12·00	6·00	3·43*
16	50·00	41·67	33·33	30·63	18·50	19·25	10·50	5·25	3·00
18	44·44	37·04	28·52	27·22	16·44	17·11	9·33	4·66	2·66
20	40·00	33·33	26·66	24·50	14·80	15·40	8·40	4·20	2·40
22	36·36	30·30	24·24	22·27	13·45	14·00	7·63	3·81*	2·18
24	33·33	27·77	22·22	20·42	12·33	12·85	7·00	3·50	2·00
26	30·77	25·64	20·05	18·85	11·38	11·87	6·46	3·23	1·84
28	28·57	23·80	19·05	17·50	10·57	11·00	6·00*	3·00	1·71
30	26·66	22·22	17·77	16·33	9·86	10·26	5·60	2·80	1·60
32	25·00	20·83	16·66	15·31	9·25	9·62	5·25	2·62	1·50
34	23·53	19·60	15·65	14·41	8·71*	9·06*	4·94	2·47	1·40
36	22·22	18·52	14·26	13·61	8·22	8·55	4·66	2·33	1·33
38	21·05	17·37	14·00	12·90*	7·80	8·11	4·42	2·21	1·26
40	20·00	16·66	13·33	12·25	7·40	7·70	4·20	2·10	1·20
42	19·05	15·87	12·70*	11·67	7·05	7·34	4·00	2·00	1·14
44	18·18	15·15	12·12	11·13	6·72	7·00	3·81	1·91	1·09
46	17·37	14·48*	11·44	10·66	6·43	6·70	3·65	1·83	1·04
48	16·66	13·88	11·11	10·21	6·16	6·42	3·50	1·75	1·00
50	16·00*	13·33	10·66	9·80	5·92	6·16	3·36	1·68	·96
Per Foot	78 *lbs.* $4·68	71 *lbs.* $4·26	59 5 *lbs* $3·57	47·8 *lbs.* $2·40	40 *lbs.* $1·90	50 *lbs.* $2·40	29 3 *lbs.* $1·50	20 *lbs.* $1·00	13 3 *lbs.* $0.66

The above Table gives the weight in tons, sustained by the several kinds of beams, uniformly distributed over them as in a floor. The weights given are what may be used in practice, being only 9 tons per square inch of that part of the metal subjected to a crushing force.

Under these weights the beams are within the limits of perfect elasticity, and the deflections are therefore in direct proportion to the load.

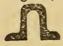
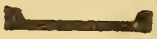
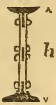
If it be intended to apply the entire weight at the centre, the figures in the Table must be divided by two ; if at any other point, the weight at the centre is to the weight at any other point, as the square of half the beam is to the rectangle of the two parts from where the weight is applied. The prices are subject to changes of the market and agreement.

* When the span of the flooring is given, the star in the Table gives an approximation to what beam ought to be employed ; for instance, $l=38$ feet span should have beams of $h=15$ inches high, able to bear $W=12{\cdot}9$ tons uniformly distributed.

Angle Iron.

Dimensions.	Per Foot. Weight.	Per Foot. Price.
Inches.	lbs.	cents.
$\frac{3}{16}(1\frac{1}{2}+1\frac{1}{2})$	1·77	5·13
$\frac{1}{4}(1\frac{1}{2}+1\frac{1}{2})$	2·32	6·74
$\frac{3}{16}(1\frac{3}{4}+1\frac{3}{4})$	2·09	6·07
$\frac{5}{16}(1\frac{3}{4}+1\frac{3}{4})$	3·49	10·1
$\frac{1}{4}(2+2)$	3·17	8·60
$\frac{3}{8}(2+2)$	4·59	13·3
$\frac{5}{16}(2\frac{1}{2}+2\frac{1}{2})$	4.97	14·4
$\frac{7}{16}(2\frac{1}{2}+2\frac{1}{2})$	6.84	19·9
$\frac{3}{8}(3+3)$	7·13	20·7
$\frac{1}{2}(3+3)$	9·32	27·1
$\frac{3}{8}(3\frac{1}{2}+3\frac{1}{2})$	8·40	24·4
$\frac{9}{16}(3\frac{1}{2}+3\frac{1}{2})$	12·2	34·9
$\frac{7}{16}(4+4)$	11·2	32·5
$\frac{5}{8}(4+4)$	15·5	45·0
ShipFrames.		
$\frac{1}{4}\cdot1\frac{1}{2}+\frac{3}{16}\cdot2\frac{1}{4}$	2.5	7·95
$\frac{5}{16}\cdot2+\frac{1}{4}\cdot3$	4·36	13·8
$\frac{3}{8}\cdot2\frac{1}{2}+\frac{5}{16}\cdot3\frac{3}{4}$	6.68	21·2
$\frac{1}{2}\cdot3\frac{1}{2}+\frac{5}{16}\cdot4\frac{1}{2}$	8.85	28·1
$\frac{7}{16}\cdot3+\frac{3}{8}\cdot5\frac{1}{4}$	11·0	35·0
$\frac{5}{8}\cdot4+\frac{7}{16}\cdot6$	16·4	51·0

Variety of Forms.

Section.		Area.	Wt.p.ft.	Wt. p. Mile.	Price Per Ft.
Figure.		Sq. In.	lbs.	Tons	$ cts.
1		7·4	25	117·86	0·56
2		5·9	20	92·7	0·45
3		7·1 *	24	113.14	0·54
4		1·95	6·6	31·45	0·16
5		5·41	18·3	86·43	0·45
6		4·44	15	70·71	0·37
7		4·22	14·3	67·75	0·35
8		7·00	23·6	111·57	0·58
9		5·32	18	Chair.	0·72
10		9·65	32.6	Channel.	1·16
11		5·41	18·3	Channel.	0·65
12		2·66	9	Purlin.	0·35
13		2·66	9	T iron.	0·32
14		0·65	2·2	Window-	12
15		0·50	1·7	Sashes.	12
16		0·89	3	Sash bar.	12
17		2·07	7	Shoe.	0·25
18		6·66	22·5	Girder.	0·80
20		This is the beam for which the formulas and table are set up. Top and bottom are alike.			
21		This compound Girder is made to order of any size, for about 6 cents per pound.			

* 20 intermediate sizes.

LATERAL STRENGTH OF MATERIALS.

The formulas for lateral strength are here reduced to the simplest possible form, and are in consequence subject to conditions which must be particularly attended to. In calculating the strength of beams of irregular sections as shown by the figures 210 to 217 on page 173, it is necessary to maintain the proportions marked on the figures and the calculation will be correct. For the sections 206 to 209 any proportion will answer in the formulas. The weight of the beam itself has not here been taken into consideration, for which allowance must be made if considerable.

Letters denote.

l = length of beam in feet. See figures.
h = height, b=breadth or thickness in inches of the beam, where the strain is acting.
k = coefficient for the different materials and sections of beams, to be found in the tables.
x = modulus of elasticity of materials. See Table.
f = elastic deflection in inches.
W= weight in pounds which the beam can bear with safety, being about one quarter of the ultimate strain at which the beam would break.

Example 1. Fig. 200. A rectangular beam of oak fastened in a wall projects out l=6 feet 4 inches, h=8 inches, and b=5 inches. Required what weight it can bear on the end W=?

$$W=\frac{30\times5\times8^2}{6\cdot333}=1509 \text{ pounds, with perfect safety.}$$

Example 2. Fig. 201. A beam of section fig. 211, with thickness b=1·25 inches, height h=22·5 inches, supported at the two ends in a length l=25 feet. Required what weight W=? it can bear in the middle. For cast iron coefficient k=260.

$$W=\frac{4\times260\times1\cdot25\times22\cdot5^2}{25}=2639\cdot5 \text{ lbs.}=11\cdot8 \text{ tons nearly.}$$

Example 3. Required the elastic reflection for the same beam and condition as in the foregoing example? See Table, modulus of elasticity x=2285 for cast iron. See page 176.

$$f=\frac{26325\times25^3}{16\times2285\times1\cdot25\times22\cdot5^3}=0\cdot80 \text{ inches, nearly.}$$

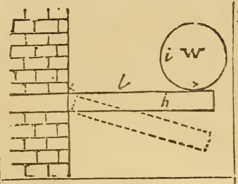
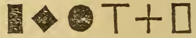
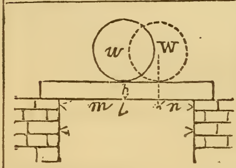
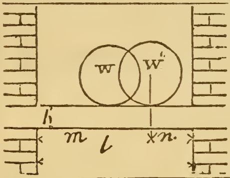
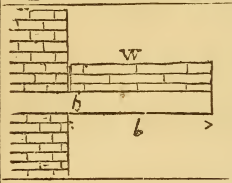
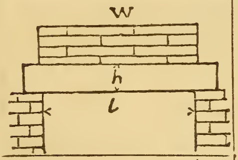
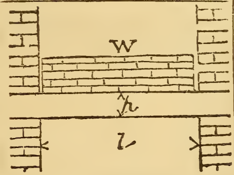
Example 4. Fig. 204. A wrought iron girder of section fig. 217, consisting of four angle irons of a=3·5×0·5×2×4=14 square inches, the plate being 0·5:1·35=0·37 inches thick, and h=18 inches deep by l=22 feet. Required how much weight evenly distributed the girder can bear with safety?

$$W=\frac{8\times800\times14\times18}{22}=73309 \text{ lbs.}=32\cdot75 \text{ tons.}$$

If plates being riveted to the angle iron at top and bottom, add that area to a.

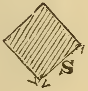
Example 5. Fig. 222. The crank R= 3·5 feet, force F=3860 lbs., length of the shaft l=64 feet, diameter D=5·25 inches. Required the twisting in degrees. The shaft being of wrought iron for which x=4110.

$$\text{Degrees}=\frac{425\times3860\times3\cdot5\times64}{4110\times5\cdot25^4}=11\cdot76^\circ.$$

	200. $W = \frac{k\,b\,h^2}{l}$,  $f = \frac{W\,l^3}{x\,b\,h^3}$. elastic set in inches.
	201. $W = \frac{4\,k\,b\,h^2}{l}$, $W' = \frac{W\,l^2}{4\,m\,n}$.  $f = \frac{W\,l^3}{16\,x\,b\,h^3}$. elastic set in inches.
	202. $W = \frac{8\,k\,b\,h^2}{l}$, $W' = \frac{W\,l^2}{4\,m\,n}$.  $f = \frac{W\,l^3}{32\,x\,b\,h^3}$. elastic set in inches.
	203. $W = \frac{2\,k\,b\,h^2}{l}$,  $f = \frac{5\,W\,l^3}{2\,x\,b\,h^3}$: elastic set in inches.
	204. $W = \frac{8\,k\,b\,h^2}{l}$,  $f = \frac{5\,W\,l^3}{32\,x\,b\,h^3}$. elastic set in inches.
	205. $W = \frac{16\,k\,b\,h^2}{l}$,  $f = \frac{5\,W\,l^3}{64\,x\,b\,h^3}$. elastic set in inches.

206.

Coefficient	k.
Cast iron,	150
Wro't iron,	120
Wood,	30



207.

Coefficient	k.
Cast iron,	150
Wro't iron,	120
Wood,	30

$b\ h^2 = S^3$.

208.

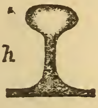
Coefficient	k.
Cast iron,	88
Wro't iron,	70
Wood,	18

$b\ h^2 = D^3$.

209.

Coefficient	k.
Cast iron,	88
Wro't iron,	70
Wood,	18

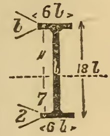
$b\ h^2 = D^3 - d^3$.



210.

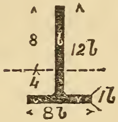
Coefficient	k.
Wro't iron,	700

$b\ h^2 = a\ h$.



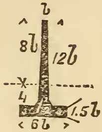
211.

Coefficient	k.
Cast iron,	260
Wro't iron,	208



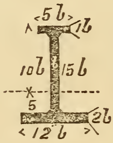
212.

Coefficient	k.
Cast iron,	236
Wro't iron,	189



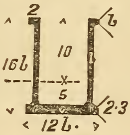
213.

Coefficient	k.
Cast iron,	250
Wro't iron,	200



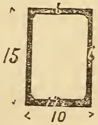
214.

Coefficient	k.
Cast iron,	700
Wro't iron,	560



215.

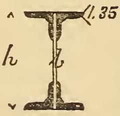
Coefficient	k.
Cast iron,	900



216.

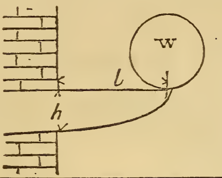
Cast iron tube,

$k = 800$.



217.

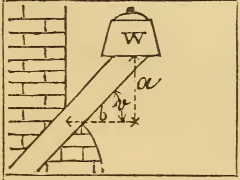
$k = 800$,
$b\ h^2 = a\ h$,
$a =$ area of all the four angle irons in square inches.



218.

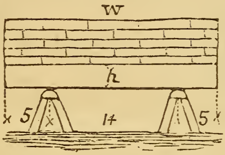
A beam fixed in one end and loaded at the other, should have the form of a Parabola, in which l=abscissa and h=ordinate. y=depth, x=length from W.

$$y=l\sqrt{\frac{x}{h}}.$$



219.

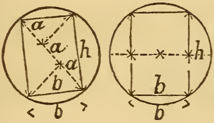
$$W=\frac{k\ b\ h^2}{l\ \cos.v}=\frac{k\ b\ h^2}{b'}.$$



220.

$$W=\frac{36\ k\ b\ h^2}{l}.$$

Divide the length into 24 equal parts, place 14 in the middle and 5 at each end.



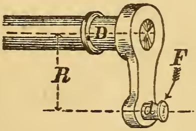
221. *To cut out the stoutest rectangular beam from a log.*

1st case, divide the diameter in 3 equal parts, and draw lines at right-angles as represented.

2d, divide the diameter in 4 equal parts.

1, b=1·414 b, non-elastic.

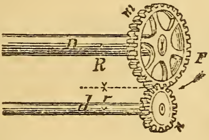
2, h=1·73 b, elastic beams.



222.

$$D=4\sqrt[3]{\frac{F\ R}{x}}=80\sqrt[3]{\frac{H}{x\ n}},$$

$$\text{Twisting in degrees}=\frac{425\ F\ R\ l}{x\ D^4}.$$



223.

$D : d=\sqrt[3]{R} : \sqrt[3]{r},$

$$D=80\sqrt[3]{\frac{H}{x\ n}},$$

$$\text{Twisting in degrees}=\frac{2233000\ H\ l}{x\ n\ D^4}.$$

Number of revolutions of wrought iron shafts per minute.

Horse Power	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	90	100	110	120	130	140	150	160
H	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
10	5	4·37	3·97	3·68	3·47	3·29	3·15	3·03	2·92	2·83	2·75	2·68	2·61	1·56	2·50	2·40	2·22	2·25	2·19	2·13	2·07	2·03	1.99
15	5·72	5·00	4·54	4·22	3·97	3·76	3·60	3·47	3·35	3·24	3·15	3·07	3·00	2·93	2·86	2·75	2·66	2·57	2·50	2·43	2·38	2·32	2·26
20	6·30	5·50	5·00	4·64	4·37	4·14	3·97	3·82	3·96	3·57	3·57	3·38	3·29	3·22	3·15	3·03	2·92	2·83	2·75	2·68	2·62	2·56	2·50
25	6·79	5·93	5·39	5·00	4·71	4·46	4·28	4·11	3·97	3·84	3·74	3·64	3·55	3·47	3·40	3·27	3·15	3·05	2·97	2·89	2·82	2·75	2·70
30	7·21	6·30	5·72	5·32	5·00	4·74	4·54	4·37	4·22	4·08	3·97	3·87	3·77	3·69	3·61	3·47	3.34	3·24	3·15	3·07	3·00	2·92	2·87
35	7·59	6·63	6·03	5·60	5·26	5·00	4·78	4·60	4·44	4·30	4.18	4·07	3·97	3·88	3·80	3·65	3·52	3·41	3·32	3·23	3·15	3·08	3·02
40	7·94	6·93	6·30	5·85	5·50	5·22	5·00	4·81	4·65	4·5	4·37	4·26	4·15	4·06	3·98	3·82	3·68	3·57	3·47	3·38	3·30	3·22	3·16
45	8·25	7·20	6·55	6·09	5·73	5·43	5·20	5·00	4·83	4·67	4·54	4·43	4·31	4·22	4·13	3·97	3·83	3·71	3·60	3·51	3·43	3·34	3·28
50	8·55	7·47	6·79	6·30	5·93	5·62	5·38	5·18	5·00	4·84	4·71	4·58	4·47	4·37	4·28	4·11	3·97	3·84	3·73	3.64	3·55	3·47	3·40
60	9.08	7.93	7·21	6·70	6·30	5·98	5·72	5·50	5·32	5·15	5·00	4·87	4·75	4·64	4·55	4·37	4·21	4·09	3·97	3·86	3·77	3·68	3·61
70	9.57	8·36	7·59	7·05	6·64	6·30	6·03	5·80	5·60	5·42	5·27	5·13	5·00	4 89	4·79	4·60	4·44	4 30	4·18	4·07	3·97	3·88	3·81
80	10·0	8·74	7·94	7·37	6·94	6·58	0·30	6·07	5·85	5·67	5·51	5·37	5·23	5·11	5·00	4·81	4·64	4·50	4·37	4·25	4·15	4·05	3·98
90	10·4	9·11	8 25	7·67	7·22	6·84	6·55	6·30	6·10	5·90	5·73	5·58	5·44	5·32	5·20	5·00	4·82	4·68	4·54	4·42	4·32	4·22	4·14
100	10·8	9·41	8·55	7·94	7·47	7·09	6·78	6·53	6·31	6·10	5·93	5·78	5·63	5·50	5·39	5·18	5·00	4·84	4·70	4·58	4·47	4·37	4·28
120	11·4	10·0	9·09	8·44	7·95	7·53	7·21	6·94	6·70	6·49	6·31	6·14	5·99	5·85	5·73	5·50	5·31	5·15	5·00	4·87	4·75	4·64	4·55
140	12·0	11·5	9·57	8·89	8·37	7·93	7·59	7·31	7·06	6·83	6·64	6·47	6·30	6·16	6·03	5·80	5·59	5·42	5·27	5·12	5 00	4·89	4·79
160	12·6	10·0	10·0	9·29	8·74	8·29	7·94	7·64	7·38	7·14	6·94	6·76	6·59	6·45	6·31	6·06	5·85	5·67	5·50	5·36	5·23	5·10	5·00
180	13·1	11·4	10·4	9·67	9·09	8·62	8·26	7·95	7·67	7·42	7·21	7·03	6·85	6·70	6·55	6·30	6·08	5·89	5·72	5·58	5·44	5·31	5·21
200	13·6	11·9	10·8	10·0	9·42	8·94	8·55	8 23	7·94	7·69	7.48	7·28	7·10	6·94	6·79	6·53	6·30	6·10	5·93	5·77	5·63	5·50	5·39
250	14·6	12·8	11·6	10·8	10·1	9·63	9·22	8·87	8·56	8·29	8·06	7·84	7·65	7·48	7·32	7·03	6·78	6·58	6·38	6·22	6·07	5·93	5·81
300	15·5	13·6	12·3	11·5	10·8	10·2	9·80	9·43	9·10	8·80	8·57	8·34	8·12	7·94	7·77	7·47	7·21	6·98	6·79	6·61	6·45	6·30	6·18
350	16·3	14·3	13·0	12·0	11·3	10·8	10·3	9·92	9·58	9·27	9·00	8·78	8·55	8·36	8·18	7·89	7·59	7·35	7·14	6·96	6·81	6·63	6·50
400	17·1	15·0	13·6	12·6	11·9	11·2	10·8	10·4	10·0	9·69	9·42	9·18	8·94	8·74	8·55	8·22	7·94	7·68	7·47	7·27	7·10	6·93	6·80
450	17·8	15·5	14·1	13·1	12·3	11·7	11·2	10·8	10·4	10·1	9·80	9·54	9·30	9·09	8·89	8·55	8·26	7·99	7·77	7·57	7·38	7.21	7·06
500	18·4	16·1	14·6	13·6	12·8	12·1	11·6	11·2	10·8	10·4	10·1	9·89	9·64	9.42	9·21	8·86	8·55	8·29	8·05	7.84	7·64	7·46	7·31
550	19·0	16·6	15·1	14·0	13·2	12·5	12·0	11·5	11·1	10·8	10·5	10·2	9·94	9·72	9·50	9·14	8·82	8·55	8·30	8·09	7·90	7·71	7·56
600	19·6	17·1	15·5	14·4	13·5	12·9	12·3	11·9	11·5	11·1	10·8	10·5	10·2	10·0	9·79	9·41	9·08	8·80	8·55	8·32	8·13	7·92	7·78

Absolute and Ultimate Strength of Materials.

Kind of Materials.	*Coefficient k.*				*Elasticity.*
	Safety.	*Inter.*	*Pr. cir.*	*Ultimate*	x
Wrought iron - - -	120	162	240	488	4110
Cast iron, - - -	150	200	300	600	2285
Cast steel, soft, - - -	385	519	170	1540	4300
Cast steel, hardened, -	1050	1400	2100	4200	6000
Blasted steel, soft, - -	175	233	350	700	4200
Brass, - - - - -	58	75	113	226	1280
Copper, - - - - -	53	71	106	212	2160
Zinc, - - - - -	15	20	30	61	2360
Tin, - - - - - -	17	23	34	69	
Lead, - - - - -	4	6	9	18	100
Ash, - - - - - -	45	56	85	170	221
Hickory, - - - - -	67	90	135	270	
Chestnut, sweet, - -	42	56	85	170	
Oak, white, - - -	50	66	100	200	300
Oak, English, - - -	25	33	50	100	248
Canadian Oak, - -	37	49	73	147	283
Pine, white, - - -	34	45	67	135	
Yellow, pine, - - -	38	50	75	150	268
Teak, - - - - - -	51	68	102	205	316

The absolute safety weight is here taken one quarter of the ultimate breaking weight, but when the weight is acting at short intervals one third might be relied upon, or in pressing circumstances one half, when the materials in the beams are known to be of good quality; but the latter never to be exceeded.

BRICKS.

Dimensions.

Common brick $8\times4\frac{1}{4}\times2\frac{1}{2}$ inches = 85 cubic inches.
Front brick $8\frac{1}{4}\times4\frac{1}{2}\times2\frac{1}{2}$ „ = 92·8 „ „

When laid in a wall with cement it occupies a space of

Common brick $8\frac{1}{4}\times4\frac{1}{2}\times2\frac{3}{4}$ inches = 102 cubic inches.
Front brick $8\frac{1}{2}\times4\frac{3}{4}\times2\frac{3}{4}$ „ = 111 „ „

Weight and Bulk of Bricks.

Tons.	Pounds.	Cub. feet.	Number of bricks, by itself.		in wall with cement	
			C. Brick.	F. Brick.	C. Brick.	F. Brick.
1	2240	22·4	448	416·6	381	347
0·04464	100	**1**	20	18·6	17	$15\frac{1}{2}$
2·23	5000	50·00	**1000**	930	850	772
2·4	5376	53·76	1075	**1000**	914	834
2·62	5872	58·72	1130	1100	**1000**	913
2·88	6451	64·51	1240	1200	1100	**1000**

WOOD.

A Cord of wood is 4 feet wide, 4 feet high, and 8 feet deep. 128 cubic feet.

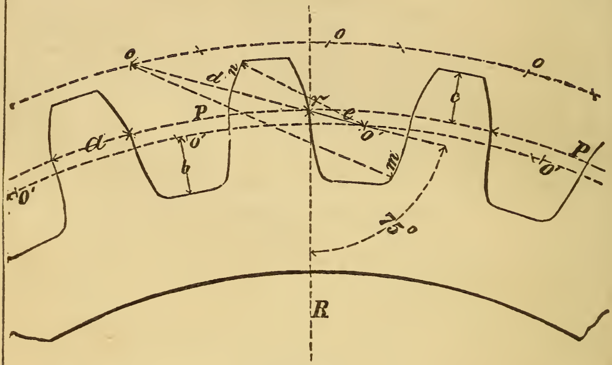
Inspector's Table for Steam-Boilers on Western Rivers.

Thickness iron. W. g. No.	inches. in,	*Diameter of boiler in inches.* 34	36	38	40	42	44	46
		lbs,	lbs,	lbs,	lbs,	lbs,	lbs.	lbs,
1	0·300	169·8	160·4	151·9	144·3	137·5	131·2	125·5
2	0·284	158·5	149·7	141·8	134·7	128·3	122·5	117·1
3	0·259	147·2	139·0	131·7	125·1	119·1	113·7	108·8
4	0·238	138·8	128·3	121·5	115·5	110·0	105·0	100·4
5	0·220	124·5	117·6	111·4	105·8	100·8	96·2	92·0
6	0·202	113·2	606·9	101·3	96·2	91·6	87·5	83·6
7	0·180	101·9	96·2	91·1	86·6	82·5	78·7	75·3
		Working steam pressure per square inch.						

The following table is given by Mr. Fairbairn, as exhibiting the strongest form and best proportions of Riveted joints, as deduced from experiments and actual practice.

Thickness of plate. in, 16ths.	Diameter of rivet. in.	Ratio.	Length of rivet fro. head, in.	Ratio.	Distance from centre to cent. in.	Ratio.	Quantity of lap in single riveted. in.	Ratio.	double riveted. in,	Ratio.
0·19= 3	0·38	2	0·88	4·5	1·25	6	1·25	6	2.10	10
0 25= 4	0·50	2	1·13	4·5	1·50	6	1·50	6	2·50	10
0·31= 5	0·63	2	1·38	4·5	1·63	5	1·88	6	3·15	10
0·38= 6	0·75	2	1·63	4·5	1·75	5	2·00	5·5	3·33	9·2
0·50= 8	0·81	1·5	2·25	4·5	2·00	4	2·25	4·5	3·75	7·5
0·63=10	0 94	1·5	2·75	4·5	2·50	4	2·75	4·5	4 58	7·5
0·75=12	1·13	1·5	3·25	4·5	3·00	4	3·25	4·5	5·42	7·5

GEARING.



Letters denote.

P = pitch,—the distances between the centres of two teeth in the pitch circle.

D = diameter
C = circumference
M = number of teeth
N = number of revolutions
} of the wheel.

d = diameter
c = circumference
m = number of teeth
n = number of revolutions
} of the pinion.

Pitch

$$P = \frac{C}{M} \quad \cdot \quad 1$$

$$P = \frac{\pi D}{M} \quad \cdot \quad 2$$

No. of teeth

$$M = \frac{C}{P} \quad \cdot \quad 3$$

$$M = \frac{\pi D}{P} \quad \cdot \quad 4$$

Circum.

$$C = P M \quad \cdot \quad 5$$

$$C = \pi D \quad \cdot \quad 6$$

Diameter

$$D = \frac{P M}{\pi} \quad \cdot \quad 7$$

$$D = \frac{C}{\pi} \quad \cdot \quad 8$$

$$D : d = C : c = M : m = n : N$$

Example 1. A wheel of $D = 40$ inches in diameter, is to have $M = 75$ teeth. Required the pitch $P = ?$

Formula 2. Pitch $P = \dfrac{3{\cdot}14 \times 40}{75} = 1{\cdot}66$ inches nearly.

Example 2. The pitch of teeth in a wheel, is to be $P = 1{\cdot}71$ inches, and having $M = 48$ teeth. Required the diameter $D = ?$ of the wheel.

Formula 7. Diam. $D - \dfrac{1{\cdot}71 \times 48}{3{\cdot}14} = 26{\cdot}14$ in. of the pitch circle

Construction of Teeth for Wheels.

Draw the radius $R\,r$, and pitch circle $P\,P$. Through r draw the line $o\,o'$ at an angle of 75° to the radius $R\,r$.

Half the angle between two teeth in the
- wheel, $v = \dfrac{180}{M}$. 1
- pinion, $V = \dfrac{180}{m}$. 2

$$D : d = sin.\ V : sin.\ v.$$

Diameter of the
- wheel, $D = \dfrac{d\ sin.\ V}{sin.\ v}$. . . . 3
- pinion, $d = \dfrac{D\ sin.\ v}{sin.\ V}$. . . . 4

Pitch (chord) of teeth in the pitch circle
- wheel, $P = D\ sin.\ v$. . . . 5
- pinion, $P = d\ sin.\ V$. . . . 6

Approximate pitch in the wheel $P = 0{\cdot}028\ D$. . . . 7

Number of teeth about
- wheel, $M = \dfrac{3{\cdot}14D}{P}$. 8
- pinion, $m = \dfrac{d\ M}{D}$. 9

Thickness of tooth, $a = 0{\cdot}46\ P$* 10

Bottom clearance, $b = 0{\cdot}4\ P$. 11

Depth to pitch line, $c = 0{\cdot}3\ P$. 12

Distance $r\ o$, $d = \dfrac{P\,(m+6)}{2\,(m-11)}$. 13

Distance r o', $e = 0{\cdot}11\,P\,\sqrt[3]{m}$ 14

* If a wheel of more than 80 teeth is to gear a pinion of less than 20 teeth, and the wheel and pinion are of the same kind of materials; take the thickness

of the tooth in the
- wheel, $a = P\left(0{\cdot}42 + \dfrac{m}{700}\right)$. . 15
- pinion, $a = 0{\cdot}5\ P\left(1 - \dfrac{m}{350}\right)$. . 16

A rack is to be considered as a wheel of 200 teeth.

Example with Plate IV.

Example. A wheel of $D = 48$ inches diameter is to gear a pinion about 8 revolutions to 1. Required a complete construction of the gearing?

Approximate pitch $P = 0{\cdot}028 \times 48 = 1{\cdot}34$ in. - - - 7

Number of teeth in the $\begin{cases} \text{wheel, } M = \dfrac{3{\cdot}14 \times 48}{1{\cdot}34} = 112. & \text{- 8} \\ \text{pinion, } m = \dfrac{112}{8} = 14 & \text{- - 9} \end{cases}$

Half the angle between two teeth in the $\begin{cases} \text{wheel, } v = \dfrac{180}{112} = 1^\circ 36'.\ sin = 0{\cdot}028. & 1 \\ \text{pinion } V = \dfrac{180}{14} = 12^\circ 51'.\ sin = 0{\cdot}2224. & 2 \end{cases}$

Diameter of pinion $d = \dfrac{48 \times 0{\cdot}028}{0{\cdot}2224} = 6{\cdot}043$ in. - 4

Draw the pitch circle for the wheel and pinion so that they tangent one another at r on a straight line between the centres of the circles.

Pitch in the gearing $P = 48 \times 0{\cdot}028 = 1{\cdot}344$ in. - 5

Take this chordial pitch in a pair of compasses, and set it off in the pitch circles.

Thickness of tooth $\begin{cases} \text{wheel } a = 1{\cdot}344\left(0{\cdot}42 + \dfrac{14}{700}\right) = 0{\cdot}592\text{in.} & 15 \\ \text{pinion } a = 0{\cdot}5 \times 1{\cdot}344\left(1 - \dfrac{14}{350}\right) = 0{\cdot}645\text{ in.} & 16 \end{cases}$

Set off the thickness of tooth in the corresponding pitch circles.

Bottom clearance $b = 0{\cdot}4 \times 1{\cdot}344 = 0{\cdot}5376$ in. - - 11

Depth to pitch line $c = 0{\cdot}3 \times 1{\cdot}344 = 0{\cdot}4032$ in. - - 12

Distances $r\,o$ and $r\,o'$ in the wheel $\begin{cases} d = \dfrac{1{\cdot}344(112+6)}{2(112-11)} = 0{\cdot}7851\text{ in.} & \text{- 13} \\ e = 0{\cdot}11 \times 1{\cdot}344\sqrt{112} = 0{\cdot}7126\text{ in.} & 14 \end{cases}$

Set off these distances on the line $o\,o'$ from r,—d beyond and e within the pitch circle for the wheel; then o is the centre and $o\,m$ radius for the flank m. o' the centre and $o'\,n$ radius for the face n. Draw circles through o and o' concentric with the pitch circle of the wheel.

Distances $r\,o$ and $r\,o'$ in the pinion $\begin{cases} d = \dfrac{1{\cdot}344(14+6)}{2(14-11)} = 4{\cdot}48\text{ in.} & \text{- 13} \\ e = 0{\cdot}11 \times 1{\cdot}344\sqrt[3]{14} = 0{\cdot}356\text{ in.} & 14 \end{cases}$

Proceed with the pinion similar as the wheel

On the plate is a scale of inches and decimals, which will be convenient for the above measurements.

Gearing.

Plate IV.

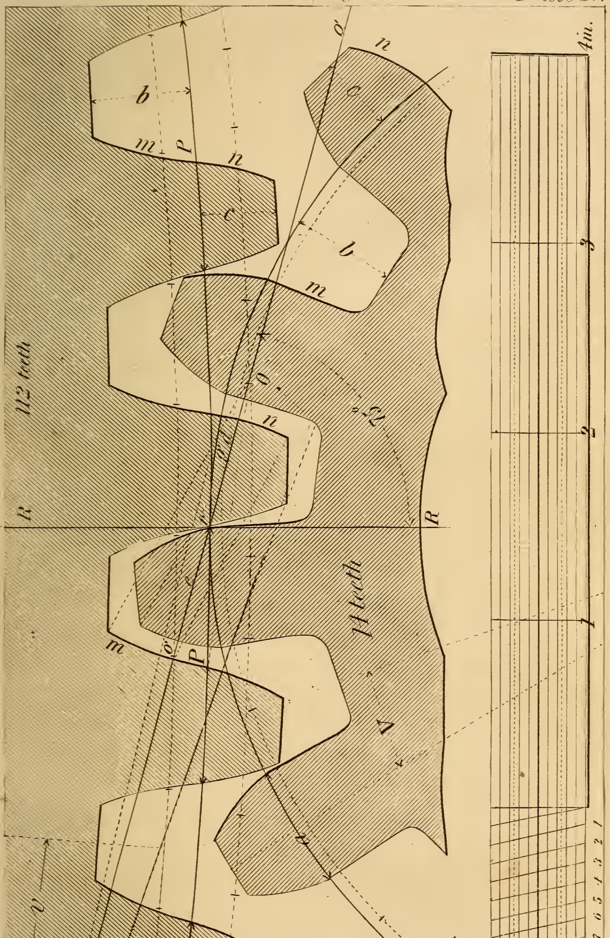


Plate V.

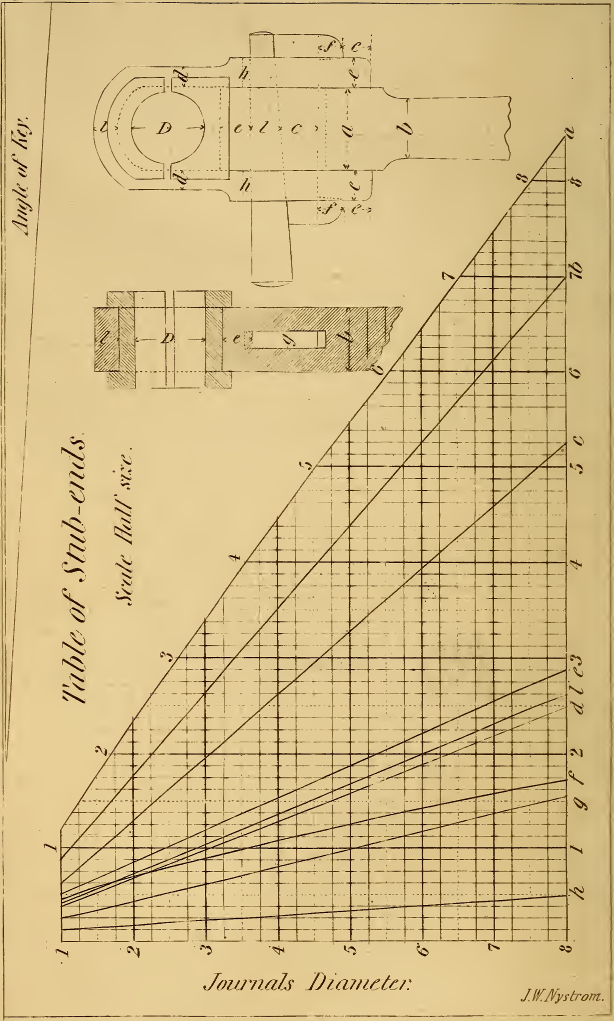
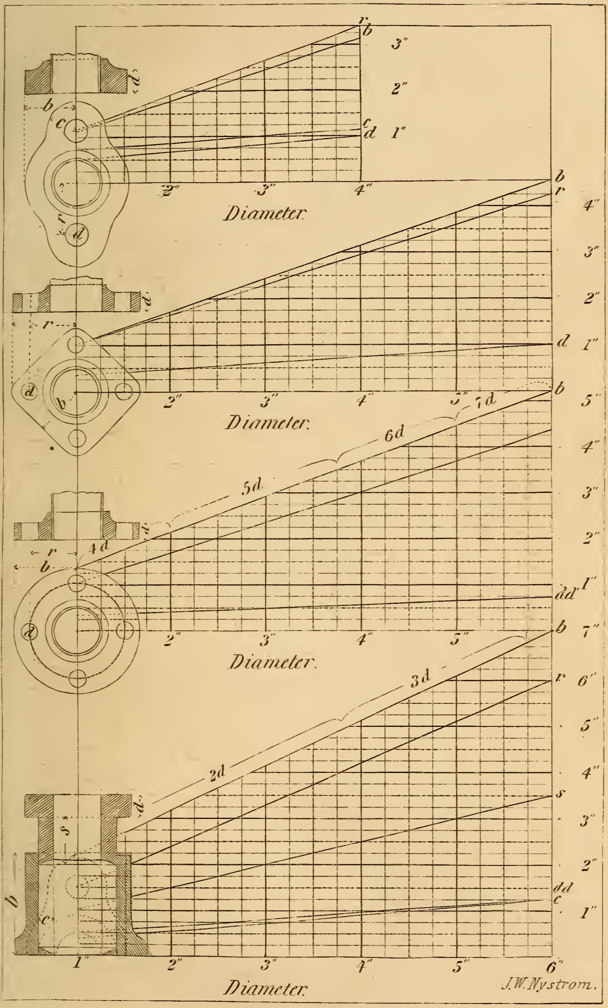


Table of Stub-ends.
Scale Half size.
Angle of Key.
Journals Diameter.
J.W. Nystrom.

Flanges & Stuffing Boxes. Plate VI



Strength of Teeth.

Letters denote.

S = strain on the teeth at the pitch-line in pounds.
a = thickness, (see figure), } inches.
h = breadth of the teeth, } inches.
v = velocity of the teeth in feet per second.
H = Horse power transmitted by the teeth.

Thickness $\begin{cases} a = 0{\cdot}025\sqrt{S}, \\ a = 0{\cdot}663\sqrt{\dfrac{H}{v}}, \end{cases}$ *Strain* $\begin{cases} S = 1600\, a^2, \\ S = 300\, h\, a. \end{cases}$

Pitch....... $\begin{cases} p = 0{\cdot}0544\sqrt{S}, \\ p = 1{\cdot}44\sqrt{\dfrac{H}{v}}, \end{cases}$ *Horses* $\begin{cases} H = 2{\cdot}275\sqrt{a\,v}, \\ H = 0{\cdot}48\sqrt{p\,v}. \end{cases}$

Abrasion included in these formulas and the breadth $h = 2{\cdot}5p$.

When great strain is required on the teeth, and the diameter or pitch of the wheel is limited, it is necessary to increase the breadth h proportionally, which will be thus,

$$h = \frac{S}{653p} = \frac{S}{300a} = \frac{Sm}{943D} = \frac{Hm}{1{\cdot}347D\,v} = \frac{H}{0{\cdot}429a\,v} = \frac{H}{0{\cdot}934p\,v},$$

Example. The pinion-wheel on a propeller shaft is to be $D = 48$ inches in diameter, and to have $m = 36$ teeth; it is driven by a pair of engines $H = 450$ horses, and the propeller to make $n = 50$ revolutions per minute. Required the breadth of the teeth $h = ?$ The velocity at pitch circle will be,

$$v = \frac{3{\cdot}14 \times 4 \times 50}{60} = 10{\cdot}5 \text{ feet per second, nearly, and}$$

$$h = \frac{Hm}{1{\cdot}347D\,v} = \frac{450 \times 36}{1{\cdot}347 \times 48 \times 10{\cdot}5} = 23{\cdot}88, \text{ or 24 inches, nearly.}$$

To Find the Diameter of Axles and Shaft.

Letters denote,

d = diameter in inches, in the bearing; and the length of the bearing $1{\cdot}5d$.
W = weight in pounds, acting in the bearing.

Water-wheels. $\begin{cases} d = \dfrac{\sqrt{W}}{18} \text{ of cast iron.} \\ d = \dfrac{\sqrt{W}}{21} \text{ of wrought iron.} \end{cases}$ Common Machinery in good order. $\begin{cases} d = \dfrac{\sqrt{W}}{24} \text{ of cast iron.} \\ d = \dfrac{\sqrt{W}}{28} \text{ of wrought iron.} \end{cases}$

Example 1. A water wheel weighs 58,680 pounds, and supported in two bearings. Required the diameter of the wheel axles? The weight acting in each bearing will be 58680 : 2 = 29340 pounds, and

$$\text{diameter } d = \frac{\sqrt{29340}}{21} = 8{\cdot}15 \text{ inches of wrought iron.}$$

Example 2. Fig. 226, page 185. Required the diameter of the axle in the wheel, when the weights $P + Q = 4864$ pounds? If the wheel is supported in two bearings $W = 4864 : 2 = 2432$ pounds.

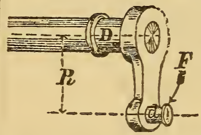
$$\text{diameter } d = \frac{\sqrt{2432}}{28} = 1{\cdot}76 \text{ inches of wrought iron.}$$

Example 3. The pressure on the steam piston, in a walking beam engine is 25000 pounds. Required the diameter of the beam journals?

$$\text{diameter } d = \frac{\sqrt{25000}}{28} = 5{\cdot}64 \text{ inches the centre one.}$$

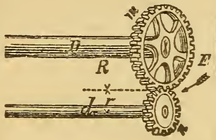
$$d = \frac{\sqrt{12500}}{28} = 4 \text{ inches at the ends.}$$

In this example it is supposed that the beam is worked by a fork connecting rod.



$$D = \frac{\sqrt[3]{FR}}{4} = 5\sqrt[3]{\frac{H}{n}}$$

$D =$ inches wrought iron.
$R =$ radius of crank in feet.
$F =$ force from the steam piston, lbs.



$$D : d = \sqrt[3]{R} : \sqrt[3]{r}$$

$$D = 4{\cdot}35\sqrt[3]{\frac{H}{\text{n}}}$$

$H =$ horse-power transmitted.
$n =$ number revolutions per minute.

When an axle or shaft not only serves as a fulcrum, but effect is transmitted by the act of twisting it, the diameter is to be caluulated as follow.

Example 4. The pressure on the piston in a steam engine is $F = 45{,}600$ pounds, applied direct on a crank of $R = 3$ feet radius. Required the diameter of the shaft and crank pin?

$$\text{Diameter of the shaft } D = \frac{\sqrt[3]{45600 \times 3}}{4} = 12{\cdot}9 \text{ inches.}$$

$$\text{Diameter of the crank pin } d = \frac{\sqrt{45600}}{28} = 7{\cdot}63 \text{ inches.}$$

Example 5. A steam engine of 368 horses is to make 32 revolutions per minute. Required the diameter of the main shaft?

$$\text{Diameter } D = 5\sqrt[3]{\frac{368}{32}} = 11\tfrac{1}{4} \text{ inches.}$$

Example 6. A cog wheel of $R = 6{\cdot}5$ feet radius is to gear with a pinion of $r = 1{\cdot}25$ feet radius, and to transmit an effect of 231 horses with 42 revolutions per minute. Required the diameter of the wheel and pinion shafts? The force F is acting uniformly at the periphery.

$$\text{Diameter of wheel shaft } D = 4{\cdot}35\sqrt[3]{\frac{231}{42}} = 7{\cdot}66 \text{ inches}$$

$$D : d = \sqrt[3]{R} : \sqrt[3]{r}$$

$$\text{Diameter of pinion shaft } d = 7{\cdot}66\sqrt[3]{\frac{1{\cdot}25}{6{\cdot}5}} = 4{\cdot}41 \text{ inches.}$$

ALLOYS.

Letters denote.

A = Antimony, B = Bismuth, C = Copper, G = Gold, I = Iron, L = Lead, N = Nickel, S = Silver, T = Tin, and Z = Zinc.

Brass, *yellow*,	2C, 1Z.
" *rolled*,	32C, 10Z, 1·5T.
Brass-casting, *common*,	20C, 1·25Z, 2·5T.
" *hard*,	25C, 2Z, 4·5T.
Brass-Propellers, (*large*),	8C, 0·5Z, 1T.
Gun-metal,	8C, 1T.
Copper-flanges, *for pipes*,	9C, 1Z, 0·26T.
Brass that bears soldering well,	2C, 0·75Z.
Muntz's metal can be rolled and worked at red heat,	6C, 4Z.
Statuary,	91·4C, 5·53Z, 1·7T, 1·37L.
German Silver,	20C, 15·8N, 12·7Z, 1·3I.
Frick's Imitative Silver,	53·39C, 17·4N. 13Z.
Medals,	100C, 8Z.
Pinchbeck,	5C, 1Z.
Chinese Silver,	65·2C, 19·5Z, 13N, 2·5S, 12 cobalt of I.
Britannia metal, — 1Z, 1A; *When fused add*, — 1A, 1B	1Z, 2A, 1B.
Babbitt's anti-Attrition metal,	25T, 2A 0·5C.

The Tin of the best quality of Banca, is to be added gradually to the melted composition.

Bell-metal, *large*,	3C, 1T.
" *small*,	4C, 1T.

Gold Metal.

$x = 21C, 13T$, $y = 62C, 9Z$, To be melted separately.

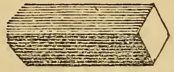
Gold = $71y + 9x$, this makes a brilliant composition.

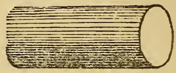
Solders.

Newton's fusible alloys,	8B, 5L, 3T,	melts at 212°.
Rose's " "	2B, 1L, 1T,	" 201o.
A more fusible composition	5B; 3L, 2T,	" 199°.
Tin solder, *coarse*,	1T, 3L,	" 500°.
" *ordinary*,	2T, 1L,	" 360°.
Soft Spelter-solder, *for common brass works*,	1C, 1Z.	
Hard " *for iron*,	2C, 1Z	
Solder for Steel,	19S, 3C, 1Z.	
Solder for fine brass works,	1S, 8C, 8Z.	
Pewterer's soft solder,	2B, 4L, 3T.	
" "	1B, 1L, 2T	
Gold Solder,	24G, 2S. 1C	
Silver solder, *hard*,	4S, 1C.	
" *soft*,	2S, 1 brass wire.	

Tempering Steel.

	Tem. Fah.
Yellow, very faint, for lancets	430°
„ pale straw, for razors scalpels	450°
„ full, for penknives and chisels for hard cast iron	470°
Brown, for scissors and chissels for wrought iron	490°
Red, for carpenter tools in general	510°
Purple, for fine watch springs and table knives	530°
Blue, bright, for swords, lock springs	550°
„ full, for daggers, fine saws, needles	560°
„ dark, for common saws	600°





Side in inches.	Weight in pounds.	Side in inches.	Weight in pounds	Diameter in inches.	Weight in pounds.	Diameter in inches.	Weight in pounds.
$\frac{1}{16}$	0·013	3⅝	44·418	$\frac{1}{16}$	0·010	3⅝	34·886
⅛	0·53	3¾	47·534	⅛	0·041	3¾	37·332
$\frac{3}{16}$	0·118	3⅞	50·756	$\frac{3}{16}$	0·119	3⅞	39·864
¼	0·211	4	54·084	¼	0·165	4	42·464
⅜	0·475	4⅛	57·517	⅜	0·373	4⅛	45·174
½	0·845	4¼	61·055	½	0·663	4¼	47·952
⅝	1·320	4⅜	64·700	⅝	1·043	4⅜	50·815
¾	1·901	4½	68·448	¾	1·493	4½	53·760
⅞	2·588	4⅝	72·305	⅞	2·032	4⅝	56·788
1	3·380	4¾	76·264	1	2·654	4¾	59·900
1⅛	4·278	4⅞	80·333	1⅛	3·360	4⅞	63·094
1¼	5·280	5	84·480	1¼	4·172	5	66·752
1⅜	6·390	5⅛	88·784	1⅜	5·019	5⅛	69·731
1½	7·604	5¼	93·168	1½	5·972	5¼	73·172
1⅝	8·926	5⅜	97·657	1⅝	7·010	5⅜	76·700
1¾	10·325	5½	102·24	1¾	8·128	5½	80·304
1⅞	11·883	5⅝	106·95	1⅞	9·333	5⅝	84·001
2	13·520	5¾	111·75	2	10·616	5¾	87·776
2⅛	15·263	5⅞	116·67	2⅛	11·988	5⅞	91·634
2¼	17·112	6	121·66	2¼	13·440	6	95·552
2⅜	19·066	6¼	132·04	2⅜	14·975	6¼	103·70
2½	21·120	6½	142·82	2½	16·688	6½	112·16
2⅝	23·292	6¾	154·01	2⅝	18·293	6¾	120·96
2¾	25·56	7	165·63	2¾	20·076	7	130·05
2⅞	27·939	7½	190·14	2⅞	21·944	7½	149·33
3	30·416	8	216·34	3	23·888	8	169·85
3⅛	33·010	8½	244·22	3⅛	25·926	8½	191·81
3¼	35·704	9	273·79	3¼	28·040	9	215·04
3⅜	38·503	10	337·92	3⅜	30·240	10	266·29
3½	41·408	12	486·66	3½	32·512	12	382·21

Cements for Cast Iron.

Two ounces Sal-ammoniac, one ounce Sulphur and sixteen ounces of borings or filings of cast Iron, to be mixed well in a mortar, and kept dry. When required for use take one part of this powder to twenty parts of clear iron borings or filings, mixed throughly in a mortar, make the mixture into a stiff paste with a little water, and then it is ready for use. A little fine grindstone sand improves the cement.

Or one ounce of Sal-ammoniac to one hundred weight of Iron borings. No heat allowed to it.

The Cubic contents of the joint in inches, divided by five, is the weight of dry borings in pounds Avoir. required to make cement to fill the joint nearly.

Cement for Stone and Brick work.

Two parts Ashes, three of Clay, and one of Sand, mixed with oil, will resist weather equal to marble.

Brown Mortar.

One part Thomaston lime, two of Sand, and a small quantity of Hair.

Hydraulic Mortar.

Three parts of Lime, four Puzzolana, one Smithy Ashes, two of Sand, and four parts of rolled stone or shingles.

Diameter inches.	Thickness 16th in.	Copper pipe pounds.	Iron pipe pounds.	Diameter inches.	Thickness 16th in.	Copper pipe pounds.	Iron pipe pounds.
5	3	12·50	10·96	9½	4	30·59	26·81
5	4	16·88	14·78	10	4	32·21	28·21
5¼	3	13·15	11·52	10½	4	33·94	29·70
5¼	4	17·75	15·55	11	4	35·20	30 84
5½	3	13·63	11·94	11½	4	36·94	32·35
5½	4	18·39	16·07	12	4	38·45	33·67
5¾	3	14·25	12·48	13	4	41·45	36 30
5¾	4	19·25	16·86	14	4	44·64	39·11
6	3	14·76	12·94	14	5	55·88	48·97
6	4	19·91	17·43	15	4	47·64	41·74
6¼	3	15·36	13·46	15	5	59·59	52·20
6¼	4	20·75	18·16	16	4	50·75	44·45
6½	3	15·90	13·93	16	5	63·47	55·60
6½	4	21·41	18·75	17	4	53·86	47·15
6¾	3	16·50	14·45	17	5	67·34	59·00
6¾	4	22·25	19·70	18	4	57·04	50·00
7	3	17·03	14·93	18	5	71·26	62·41
7	4	22·93	20·07	19	4	60·14	52·65
7¼	3	17·65	15·45	19	5	75·23	65·90
7¼	4	23·74	20·79	20	4	62·51	54·74
7½	3	18·32	16·05	20	5	78·21	68·5
7½	4	24·45	21·40	21	5	82·98	72·64
7¾	3	18·95	16·60	22	5	86·77	76·00
7¾	4	25·28	22·13	23	5	90·57	79·34
8	3	29·42	17·03	24	5	94·31	82·60
8	4	25·96	22·72	26	5	101·9	89·32
8½	3	20·58	18·03	28	5	109·4	95·68
8½	4	27·47	24·07	30	5	117·0	102·4
9	4	28·98	25·38	36	5	140·0	122·5

Weight of Cast Iron Cylinders per Foot.

Diameter in inches.	Weight in pounds,	Diameter in inches.	Weight in pounds.	Diameter in inches.	Weight in pounds.	Diameter in inches	Weight in pounds.
¾	1·39	2¾	18·74	4¾	55·92	7½	139·4
⅞	1·88	2⅞	20·48	4⅞	58·72	7¾	148·87
1in.	2·47	3in.	22·35	5in.	61·96	8in.	158·63
1⅛	3·13	3⅛	24·20	5⅛	64·66	8¼	168·15
1¼	3·87	3¼	26·18	5¼	68·31	8½	179·1
1⅜	4·68	3⅜	28·23	5⅜	71·00	8¾	189·0
1½	5·57	3½	30·36	5½	74·98	9in.	200·8
1⅝	6·54	3⅝	32·57	5⅝	78·65	9¼	211·12
1¾	7·59	3¾	34·85	5¾	81·95	9½	223·7
1⅞	8·71	3⅞	37·21	5⅞	85·81	9¾	235·3
2in.	9·91	4in.	39·66	6in.	89·23	10in.	247·9
1⅛	11·19	4⅛	41·80	6¼	96·82	10½	273·27
2¼	12·54	4¼	44·77	6½	104·7	11in.	299·9
2⅜	13·98	4⅜	47·00	6¾	112·9	11½	327·8
2½	15·49	4½	50·19	7in.	112·4	12in.	356·9
2⅝	17·08	4⅝	52·71	7¼	130·28	13in.	418·9

	Thickness of Metal.						
Bore.	¼	⅜	½	⅝	¾	⅞	1
1	3.06	5·05					
1¼	3·67	6·00					
1½	6·89	6·89	9·81				
1¾		7·80	11·04				
2		8·74	12·23	16·0			
2¼		9·65	13·48	17·52			
2½		10·57	14·66	19·05	23·8		
2¾		11·54	15·91	20·59	25·68		
3		12·28	17·15	22·15	27·56	33·30	39·31
3¼		13·24	18·40	23·72	29·64	35·46	41·77
3½		14·20	19·66	25·27	31·20	37·63	44·23
3¾		15·50	20·90	26·83	33·07	39·77	46·68
4		16·80	22·05	28·28	34·94	41·92	49.14
4¼		17·41	23·35	29·85	36·73	44·05	51·57
4½		18·00	24·49	31·40	38·58	46·19	54·00
4¾		18·89	25·70	32·91	40·43	48·34	56·45
5		19·79	26·94	34·34	42·28	50·50	58·90
5½		21·54	29·40	37·44	45·94	54·81	63·82
6		23·42	31·82	40·56	49·60	58·96	68·70
6½		25·26	34·32	43·68	53·30	63·18	73·40
7		27·15	36·66	46·80	56·96	67·60	78·39
7½		28·92	39·22	49·92	60·48	71·76	83·28
8		30·76	41·64	52·68	64·27	76·12	88·20
8½		32·82	44·11	56·16	68·00	80·50	73·28
9		34·45	46·50	58·92	71·70	84·70	97·98
9½		36·26	48·98	62·02	75·32	88·98	102·9
10		38·15	54·46	65·08	78·99	93·24	108·8
10½			53·88	68·14	82·68	97·44	112·7
11			56·34	71·19	86·40	101·83	117·6
11½			58·82	74·28	90·06	106·1	122·6
12			61·26	77·36	93·70	110·5	127·4
12½			63·70	80·40	97·40	114·7	132·5
13			66·14	83·46	101·1	118·9	137·3
13½			68·64	86·55	104·8	123·3	137·28
14			71·07	89·61	108·46	127·6	147·0
14½			73·72	92·66	112·1	131·9	151·9
15			75·96	95·72	115·8	136·2	156·8
15½			78·40	98·78	119·5	140·4	161·8
16			80·87	101·8	123·1	144·8	166·6
16½			83·30	104·8	126·8	149·0	171·6
17			85·73	107·9	130·5	135·3	176·6
17½			88·23	111·1	134·2	157·6	181·3
18				114·1	137·8	169·9	186·2
19				120·2	145·2	170·5	195·9
20				126·3	152·5	179·0	205·8
21				132·5	159·8	187·6	215·5
22				138·6	167·2	196·5	225·4
23				144·8	147·6	204·8	235·3
24				150·8	181·9	213·28	245·1
26					196·6	230·6	264·7
28					211·3	247·6	284·3
30					226·2	264·8	303·9

Weight of Flat Rolled Iron per Foot,

Look first for the thickness, and follow that line until the column where the breadth is on the top; is the number of pounds per foot.

																				2½
																			2⅜	20·06
																		2¼	18·06	19·01
																	2⅛	16·16	17·05	17·95
																2	12·36	15·21	16·05	16·89
															1⅞	12·67	13·46	14·26	15·05	15·84
														1¾	11·09	11·83	12·57	13·31	14·04	14·78
													1⅝	9·610	10·30	10·98	11·67	12·36	13·04	13·73
												1½	8.237	8·871	9·505	10·14	10·77	11·40	12·04	12·67
											1⅜	6·970	7·551	8·132	8·713	9·294	9·874	10·45	11·03	11·62
										1¼	5·808	6·337	6·864	7·393	7·921	8·448	8·977	9·505	10·03	10·56
									1⅛	4·752	5·227	5·703	6·178	6·653	7·129	7·604	8·079	8·554	9·028	9·504
								1	3·802	4·224	4·646	5·069	5·492	5·914	6·336	6·758	7·181	7·604	8·025	8·448
							⅞	2·956	3·326	3·696	4·065	4·435	4·805	5·178	5·544	5·914	6·283	6·653	7·022	7·392
						¾	2·217	2·534	2·850	3·168	3·484	3·802	4·119	4.435	4·752	5·069	5·386	5·703	6·019	6·336
					⅝	1·584	1·846	4·112	2·375	2·640	2·904	3·168	3·432	3·696	3·960	4·224	4·488	4·752	5·916	5·280
				½	1·056	1·265	1·477	1·690	1·901	2·112	2·325	2·535	2·746	2·957	3·168	3·379	3·591	3·802	4·013	4·224
			⅜	0·634	0·792	0·950	1·108	1·267	1·425	1·584	1·742	1·900	2·059	2·218	2·376	2·534	2·693	2·851	3·009	3·168
		¼	0·317	0·422	0·528	0·633	0·738	0·845	0·950	1·056	1·161	1·266	1·372	1·479	1·584	1·689	1·795	1·900	2·006	2·112
	3/16	0·162	0·237	0·316	0·396	0·474	0·553	0·633	0·712	0·792	0·870	0·949	1·029	1·109	1·188	1·267	1·346	1·425	1·504	1·584
⅛	0·081	0·108	0·158	0·211	0·264	0·316	0·369	0·422	0·475	0·528	0·580	0·633	0·686	0·739	0·792	0·845	0·950	0 898	1·00	1·056

Weight of Flat Rolled Iron per Foot,

RULE.—Look first for the thickness, and follow the line until the column where the breadth is on the top, is the number of pounds per foot.

																				6
																			5¾	116·6
																		5½	106·9	111·5
																	5¼	97·58	102·0	106·4
																5	88·71	92·93	97·16	101·4
															4¾	80·26	84·27	88·29	92·30	96·41
														4½	72·23	76·04	79·84	83·64	87·44	91·24
													4¼	64·63	68·22	71·81	75·40	78·99	82·58	86·17
												4	57·45	60·83	64·21	67·59	70·97	74·35	77·73	81·11
											3⅞	52·38	55·65	58·93	62·20	65·97	68·75	72·02	75·30	78·58
										3¾	49·09	50·69	53·86	57·03	60·20	63.36	66·53	69·70	72·87	76·04
									3⅝	45·93	47·46	49·00	52·06	55·13	58·19	61·25	64·31	67·37	70·44	73·51
								3½	42·87	44·35	45·83	47·31	50·27	53·23	56·18	59·14	62·10	65·05	68·01	70·97
							3⅜	39·92	41·34	42·77	44·19	45·62	48·47	51·24	54·17	57·03	59·88	62·73	65·58	68·43
						3¼	37·07	38·44	39·81	41·19	42·51	43·93	46·68	49·25	52·17	54·92	57·66	60·41	63·15	65·89
					3⅛	34·33	35·64	36·96	38·28	39·60	40·92	42·24	44·88	47·43	50·16	52·80	55·44	58·08	60·72	63·36
				3	31·94	32·95	34·21	35·48	36·75	38·02	39·28	40·55	43·09	45·62	48·16	50·69	53·23	55·76	58·30	60·84
			2⅞	29·42	30·49	31·57	32·78	34·00	35·21	36·43	37·64	38·86	41·29	43·72	46·15	48·58	51·01	53·43	55·87	58·31
		2¾	26·72	27·88	29·04	30·20	31·36	32·53	33·69	34·85	36·01	37·17	39·50	41·82	44·14	46·47	48·79	51·11	53·44	55·77
	2⅝	24·39	25·50	26·61	27·72	28·83	29·93	31·04	32·15	33·26	34·37	35·48	37·70	39·92	42·13	44·35	46·57	48·79	51·01	53·23
2½	22·18	23·23	24·29	25·34	26·40	27·46	28·51	29·57	30·62	31·68	32·73	33·79	35·91	38·02	40·13	42·24	44·35	46·47	48·58	50·69

Weight of Flat Rolled Iron per Foot,

The thickness is in the first column, and the breadth in the top line.

	2½	2⅝	2¾	2⅞	3	3⅛	3¼	3⅜	3½	3⅝	3¾	3⅞	4	4¼	4½	4¾	5	5¼	5½	5¾	6
2⅜	20·06	21·07	22·07	23·07	24·07	25·07	26·08	27·18	28·09	28·84	29·59	30·84	32·10	34·11	36·12	38·12	40·13	42·13	44·14	45·95	47·76
2¼	19·01	19·96	20·91	21·86	22·81	23·76	24·71	25·66	26·61	27·56	28·51	29·46	30·41	32·32	34·22	36·12	38·02	39·92	41·82	43·32	44·82
2⅛	17·95	18·85	19·75	20·65	21·54	22·44	23·34	24·23	25·13	25·78	26·43	27·57	28·72	30·52	32·31	34·11	35·90	37·70	39·49	41·09	42·69
2	16·89	17·74	18·58	19·43	20·28	21·12	21·97	22·81	23·65	24·50	25·35	26·19	27·04	28·72	30·41	32·10	33·79	35·48	37·17	38·86	39·55
1⅞	15·84	16·63	17·42	18·22	19·01	19·80	20·59	21·38	22·18	22·97	23·76	24·55	25·35	26·92	28·51	30·09	31·68	33·26	34·85	36·43	38·01
1¾	14·78	15·52	16·26	17·00	17·74	18·48	19·22	19·96	20·70	21·44	22·18	22·92	23·66	25·13	26·61	28·09	29·57	31·05	32·53	34·01	36·49
1⅝	13·73	14·42	15·10	15·79	16·47	17·16	17·85	18·53	19·22	19·90	20·59	21·28	21·97	23·33	24·71	26·08	27·46	28·83	30·20	31·53	32·86
1½	12·67	13·31	13·94	14·57	15·21	15·84	16·47	17·10	17·74	18·37	19·01	19·64	20·28	21·54	22·81	24·08	25·35	26·61	27·88	29·15	30·42
1⅜	11·62	12·20	12·78	13·36	13·94	14·52	15·10	15·68	16·26	16·84	17·42	18·00	18·59	19·74	20·91	22·07	23·23	24·39	25·55	26·72	27·90
1¼	10·56	11·09	11·62	12·14	12·67	13·20	13·73	14·25	14·78	15·31	15·84	16·37	16·90	17·95	19·01	20·07	21·12	22·18	23·23	24·29	25·35
1⅛	9·504	9·981	10·45	10·93	11·41	11·88	12·36	12·83	13·31	13·79	14·27	14·74	15·21	16·15	17·11	18·06	19·01	19·96	20·91	21·86	22·81
1	8·448	8·872	9·294	9·716	10·14	10·55	10·98	11·40	11·83	12·20	12·67	13·09	13·52	14·36	15·21	16·05	16·90	17·74	18·59	19·43	20·27
⅞	7·392	7·763	8·132	8·502	8·871	9·740	9·610	9·980	10·35	10·72	11·09	11·46	11·83	12·56	13·31	14·04	14·78	15·52	16·26	17·00	17·74
¾	6·336	6·654	6·970	7·287	7·604	7·920	8·237	8·554	8·871	9·188	9·505	9.822	10·14	10·77	11·41	12·04	12·67	13·31	13·94	14·57	15·20
⅝	5·280	5·545	5·808	6·072	6·337	6·601	6·865	7·129	7·393	7·657	7·921	8·183	8·445	8·975	9·507	10·03	10·56	11·09	11·61	12·14	12·67
½	4·224	4·436	4·647	4·858	5·069	5·280	5·492	5·703	5·914	6·125	6·336	6·547	6·759	7·181	7·604	8·026	8·449	8·871	9·294	9·716	10·23
⅜	3·168	3·372	3·485	3·644	3·802	3·960	4·119	4·277	4·436	4·594	4·752	4·910	5·069	5·386	5·703	6·019	6·386	6·653	6·970	7·287	7·594
¼	2·112	2·218	2·320	2·429	2·535	2·640	2·746	2·851	2·957	3·062	3·168	3·274	3·380	3·591	3·802	4·013	4·224	4·436	4·647	4·858	5·069
3/16	1·584	1·663	1·741	1·822	1·901	1·980	2·059	2·138	2·218	2·297	2·376	2·455	2·535	2·693	2·851	3·009	3·168	3·327	3·485	3·643	3·801
⅛	1·056	1·109	1·162	1·215	1·267	1·320	1·373	1·426	1·479	1·531	1·584	1·637	1·690	1·795	1·901	2·006	2·112	2·218	2·323	2·429	2·535

Weight Per Square Foot in Pounds.

Thickness in inches.	Cast Iron.	Wrought or Sheet Iron.	Sheet Copper.	Sheet Lead.	Sheet Zinc
$\frac{1}{16}$	2·346	2·517	2·890	3·694	2·320
$\frac{1}{8}$	4·693	5·035	5·781	7·382	4·642
$\frac{3}{16}$	7·039	7·552	8·672	11·074	6·961
$\frac{1}{4}$	9·386	10·070	11·562	14·765	9·275
$\frac{5}{16}$	11·733	12·588	14·453	18·456	11.61
$\frac{3}{8}$	14·079	15·106	17·344	22·148	13·93
$\frac{7}{16}$	16·426	17·623	20·234	25·839	16·23
$\frac{1}{2}$	18·773	20 141	23·125	29·530	18·55
$\frac{9}{16}$	21·119	22·659	26·016	33·222	20·87
$\frac{5}{8}$	23·466	25·176	28·906	36·913	23·19
$\frac{11}{16}$	25·812	27·694	31·797	40·604	25·53
$\frac{3}{4}$	28·159	30·211	34·688	44·296	27·85
$\frac{13}{16}$	30·505	32·729	37·578	47·987	30·17
$\frac{7}{8}$	32·852	35·247	40·469	51·678	32·47
$\frac{15}{16}$	35·199	37·764	43·359	55·370	34·81
1	37·545	40·282	46·250	59·061	37·13
$1\frac{1}{8}$	42·238	45·317	52·031	66·444	41·78
$1\frac{1}{4}$	46·931	50·352	57·813	73·826	46·42
$1\frac{3}{8}$	51·625	55·387	63·594	63·594	51·04
$1\frac{1}{2}$	56·317	60·422	69·375	88·592	55·48
$1\frac{5}{8}$	61·011	65·458	75·156	95·975	60·35
$1\frac{3}{4}$	65·704	70·493	80·938	103·358	65.00
$1\frac{7}{8}$	70·397	75·528	86·719	110·740	69·61
2	75·090	80·563	92:500	118·128	74·25

Weight of Copper Rods or Bolts per Foot,

Diameter. Inches.	Weight. Pounds.	Diameter. Inches	Weight. Pounds.	Diameter. Inches.	Weight. Pounds.	Diameter Inches.	Weight. Pounds.
$\frac{1}{4}$	0·1892	1	3·0270	$1\frac{7}{8}$	10·642	$3\frac{3}{8}$	34·487
$\frac{5}{16}$	0·2956	$1\frac{1}{16}$	3·4170	2	12·108	$3\frac{1}{2}$	37·081
$\frac{3}{8}$	0·4256	$1\frac{1}{8}$	3·8912	$2\frac{1}{8}$	13·668	$3\frac{5}{8}$	39·737
$\frac{7}{16}$	0·5794	$1\frac{3}{16}$	4·2688	$2\frac{1}{4}$	15·325	$3\frac{3}{4}$	42.568
$\frac{1}{2}$	0·7567	$1\frac{1}{4}$	4·7298	$2\frac{3}{8}$	17·075	$3\frac{7}{8}$	45·455
$\frac{9}{16}$	0·9578	$1\frac{5}{16}$	5·2140	$2\frac{1}{2}$	18·916	4	48·433
$\frac{5}{8}$	1·1824	$1\frac{3}{8}$	5·7228	$2\frac{5}{8}$	20·856	$4\frac{1}{4}$	53·550
$\frac{11}{16}$	1·4307	$1\frac{7}{16}$	6·2547	$2\frac{3}{4}$	22·891	$4\frac{1}{2}$	61·321
$\frac{3}{4}$	1·7027	$1\frac{1}{2}$	6·8109	$2\frac{7}{8}$	25·019	$4\frac{3}{4}$	68·312
$\frac{13}{16}$	1·9982	$1\frac{9}{16}$	7·3898	3	27·243	5	76·130
$\frac{7}{8}$	2·3176	$1\frac{5}{8}$	7·9931	$3\frac{1}{8}$	29·559	$5\frac{1}{2}$	91·550
$\frac{15}{16}$	2·6605	$1\frac{3}{4}$	9·2702	$3\frac{1}{4}$	31·972	6	109·

Thickness by the Gauge.	Thickness in Inches.	*Weight per Square Foot in Pounds.* Sheet and Boiler Iron.	Sheet Cast Steel.	Sheet Copper.	Thickness in Inches.
No. 0	0·340	13·7	14·0	15·6	11/32
" 1	0·300	12·1	12·4	13·8	5/16
" 2	0.284	11·4	11·7	13·0	9/32
" 3	0·259	10.4	10·6	11·9	1/4
" 4	0·238	9·60	9 80	11·0	7/32
" 5	0·220	8·85	9·02	10·1	"
" 6	0·203	8·17	8·33	9·32	"
" 7	0·180	7·24	7·38	8·25	3/16
" 8	0·165	6·65	6·78	7·59	"
" 9	0·148	5·96	6·08	6·80	5/32
" 10	0·134	5·40	5·51	6·16	"
" 11	0·120	4·83	4·93	5·51	1/8
" 12	0·109	4·40	4·50	5·02	"
" 13	0·095	3·83	3·91	4·37	2/32
" 14	0·083	3·34	3·41	3·81	"
" 15	0·072	2·90	2·96	3·31	1/16
" 16	0·065	2·62	2·67	3·00	"
" 17	0·058	2·34	2·39	2·67	"
" 18	0·049	1·97	2·01	2·25	"
" 19	0·042	1·69	1·72	1:93	2/64
" 20	0·035	1·41	1·42	1·61	"
" 21	0·032	1·29	1·31	1·47	"
" 22	0·028	1·13	1·15	1·29	1/32
" 23	0·025	1·00	1·02	1·14	"
" 24	0·022	0·885	0·903	1·01	"
" 25	0·020	0·805	0·820	0·918	"
" 26	0·018	0·724	0·738	0·826	1/64
" 27	0·016	0·644	0·657	0·735	"
" 28	0·014	0·563	0·574	0·642	
" 29	0·013	0·523	0·533	0·597	
" 30	0·012	0·483	0·493	0·551	
31	0·010	0·402	0·410	0·480	
32	0·009	0·362	0·370	0·420	
" 33	0·008	0·322	0·328	9·370	
" 34	0·007	0·282	0·288	0·323	
" 35	0·005	0·230	0·235	0·262	
" 36	0·004	0·170	0·173	0·194	

A new wire gauge has been started by Brown & Sharpe, of Providence, R. I., and called the "American Standard wire gauge." It is an improvement on the Birmingham gauge, but still it must be accompanied by an interpreter to explain what it is. Not one in a hundred or one in a thousand of those who have to deal with measures, would understand the thickness from the number of the wire gauge. Whenever it is written or spoken of, it must be translated into inches in order to make it clear how much it is; why not then have the numbers of the wire gauge expressed direct into inches as proposed by M. Whitworth a few years ago, published I believe in the Artizan for 1857. If the American manufacturers would take up the proposition of M. Whitworth it would be one step ahead. It is very clear that the wire gauge is patched up from different sources and ages to its present state, which is not worth to imitate in a new guage.

Proportion of Bolts and Nuts. Number of threads per In.

Diameter of Bolt.						Diameter.	Number of Threads per Inch.	
3 inch.	$4\frac{1}{2}$	$5\frac{1}{4}$	5	$7\frac{1}{16}$	$2\frac{1}{2}$	$\frac{1}{4}$	20	10
$\frac{3}{4}$	$4\frac{1}{8}$	$4\frac{3}{4}$	$4\frac{1}{2}$	$6\frac{3}{8}$	$2\frac{1}{4}$	$\frac{5}{16}$	18	9
$2\frac{1}{2}$	$3\frac{3}{4}$	$4\frac{3}{8}$	$4\frac{1}{8}$	$5\frac{13}{16}$	2	$\frac{3}{8}$	16	9
$2\frac{1}{4}$	$3\frac{3}{8}$	$3\frac{7}{8}$	$3\frac{3}{4}$	$5\frac{5}{16}$	$1\frac{3}{4}$	$\frac{7}{16}$	14	8
2	3	$3\frac{1}{2}$	$3\frac{3}{8}$	$4\frac{3}{4}$	$1\frac{5}{8}$	$\frac{1}{2}$	12	7
$1\frac{7}{8}$	$2\frac{3}{4}$	$3\frac{1}{4}$	3	$4\frac{3}{16}$	$1\frac{1}{2}$	$\frac{5}{8}$	11	7
$1\frac{3}{4}$	$2\frac{5}{8}$	3	$2\frac{3}{4}$	$3\frac{7}{8}$	$1\frac{3}{8}$	$\frac{3}{4}$	10	6
$1\frac{5}{8}$	$2\frac{1}{2}$	$2\frac{7}{8}$	$2\frac{5}{8}$	$3\frac{11}{16}$	$1\frac{1}{4}$	$\frac{7}{8}$	9	6
$1\frac{1}{2}$	$2\frac{1}{4}$	$2\frac{5}{8}$	$2\frac{1}{2}$	$3\frac{1}{2}$	$1\frac{1}{8}$	1	8	5
$1\frac{3}{8}$	2	$2\frac{3}{8}$	$2\frac{1}{4}$	$3\frac{3}{16}$	$1\frac{1}{16}$	$1\frac{1}{8}$	7	4
$1\frac{1}{4}$	$1\frac{7}{8}$	$2\frac{1}{8}$	2	$2\frac{7}{8}$	1	$1\frac{1}{4}$	7	$3\frac{1}{2}$
$1\frac{1}{8}$	$1\frac{5}{8}$	$1\frac{7}{8}$	$1\frac{7}{8}$	$2\frac{5}{8}$	$\frac{7}{8}$	$1\frac{1}{2}$	6	3
1	$1\frac{1}{2}$	$1\frac{3}{4}$	$1\frac{5}{8}$	$2\frac{5}{16}$	$\frac{3}{8}$	$1\frac{3}{4}$	5	$2\frac{1}{2}$
$\frac{7}{8}$	$1\frac{5}{16}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{8}$	$\frac{3}{4}$	2	$4\frac{1}{2}$	$2\frac{1}{4}$
$\frac{3}{4}$	$1\frac{3}{16}$	$1\frac{3}{8}$	$1\frac{5}{16}$	$1\frac{7}{8}$	$\frac{5}{8}$	$2\frac{1}{2}$	4	2
$\frac{5}{8}$	1	$1\frac{1}{8}$	$1\frac{3}{16}$	$1\frac{11}{16}$	$\frac{1}{2}$	3	$3\frac{1}{2}$	
$\frac{9}{16}$	$\frac{7}{8}$	1	1	$1\frac{7}{16}$	$\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{1}{4}$	
$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{7}{8}$	$1\frac{1}{4}$	$\frac{7}{16}$	4	3	
$\frac{7}{16}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{3}{4}$	$1\frac{1}{16}$	$\frac{3}{16}$	$4\frac{1}{2}$	$2\frac{7}{8}$	
$\frac{3}{8}$	$\frac{7}{16}$	$\frac{5}{8}$	$\frac{3}{4}$	$1\frac{1}{16}$	$\frac{5}{16}$	5	$2\frac{7}{8}$	
$\frac{5}{16}$	$\frac{9}{16}$	$\frac{5}{8}$	$\frac{9}{16}$	$\frac{11}{16}$	$\frac{5}{16}$	$5\frac{1}{2}$	$2\frac{5}{8}$	
$\frac{1}{4}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{9}{16}$	$\frac{11}{16}$	$\frac{1}{4}$	6	$2\frac{1}{2}$	

Velocity at the end. V	Space fallen through S	Time in seconds. T	Velocity at the end. V	Space fallen through S	Time in seconds. T	Velocity at the end. V	Space fallen through S	Time in seconds. T
0·1	·00015	0·0031	5·1	·40388	0·158	11	1·8789	0·342
0.2	·00062	0·0062	5·2	·41987	0·162	12	2·0652	0·373
0·3	·00139	0·0093	5·3	·43618	0·165	13	2·6242	0·405
0·4	·00248	0·0124	5·4	·45279	0·168	14	3·0435	0·436
0·5	·00388	0·0155	5·5	·46972	0·171	15	3·4938	0·467
0·6	·00559	0·0187	5·6	·48695	0·174	16	3·9751	0·498
0·7	·00761	0·0218	5·7	·50450	0·177	17	4·4876	0·530
0·8	·00994	0·0230	5·8	·52236	0·181	18	5·0310	0·560
0·9	·01257	0·0280	5·9	·55057	0·184	19	5·6056	0·591
1·	·01552	0·0311	6·	·55900	0·187	20	6·2112	0·622
1·1	·01879	0·0342	6·1	·57779	0·190	21	6·8478	0·654
1·2	·02065	0·0373	6·2	·59689	0·193	22	7·5155	0·685
1·3	·02624	0·0404	6·3	·61630	0·196	23	8·2143	0·716
1·4	·03043	0·0436	6.4	·63602	0·199	24	8·9441	0·747
1·5	·03493	0·0467	6·5	·65606	0·202	25	9·7049	0·778
1·6	·03975	0·05	6·6	·67639	0·205	26	10·497	0·810
1·7	·04487	0·052	6:7	·69705	0·209	27	11·320	0·840
1·8	·05031	0·556	6.8	·71801	0·212	28	12.174	0·872
1·9	·05605	0·0591	6·9	·73928	0·215	29	13·059	0·903
2·	·06211	0·0623	7·	·76087	0·218	30	13·975	0·933
2·1	·06847	0·0654	7.1	·78276	0·221	31	14·922	0·965
2·2	·07515	0·0685	7·2	·80497	0·224	32	15·900	0·996
2·3	·08214	0·0717	7·3	·82748	0·227	33	16·910	1·025
2·4	·08944	0·0747	7·4	·85031	0·231	34	18·789	1·058
2·5	·09705	0·0780	7·5	·87344	0·234	35	19·022	1·091
2·6	·10497	0·0810	7·6	·89689	0·237	36	20·124	1·120
2·7	·11320	0·0841	7·7	·92065	0·240	37	21·258	1·151
2·8	·12174	0·0872	7·8	·94472	0·243	38	22·422	1·184
2·9	·13059	0·0903	7·9	·96910	0·246	39	23·618	1·213
3·	·13975	0·0934	8·	·99379	0·250	40	24·844	1·243
3·1	·14922	0·0966	8·1	1·0187	0·253	41	26·102	1·276
3·2	·15900	0·0997	8·2	1·0441	0·256	42	27·391	1·308
3·3	·16910	0·1025	8·3	1·0697	0·259	43	28·57	1·338
3·4	·18788	0·1059	8.4	1·0956	0·262	44	30·062	1·370
3·5	·19022	0·1092	8 5	1·1218	0·265	45	31·444	1·400
3·6	·20124	0·1121	8.6	1·1484	0·268	46	32·857	1·431
3·7	·21257	0·1152	8·7	1·1753	0·271	47	34·301	1·463
3·8	·22422	0·1185	8·8	1·2015	0·274	48	35·776	1·495
3·9	·23618	0·1214	8·9	1·2299	0·278	49	37·282	1·525
4·	·24844	0·1246	9·	1·2577	0·281	50	38·820	1·555
4·1	·26102	0·1278	9·1	1·2858	0·283	51	40·388	1·588
4·3	·27391	0·1309	9·2	1·3143	0·287	52	41·987	1·619
4·3	·28571	0·1339	9·3	1·3430	0·290	53	43·618	1·650
4·4	·30062	0·1371	9·4	1·3720	0·293	54	45·279	1·680
4·5	·31444	0·1403	9·5	1·4041	0·296	55	46·972	1·711
4·6	·32857	0·1433	9·6	1·4310	0·300	56	48·695	1·742
4·7	·34301	0·1465	9·7	1·4610	0·302	57	50·450	1·774
4.8	·35776	0·1496	9·8	1·4913	0·306	58	52·236	1·805
4·9	·37282	0·1526	9·9	1·5219	0·309	59	55·058	1·835
5·	·38820	0·1559	10	1·5528	0·312	60	55·900	1·868

GRAVITATION.

Gravity or **Gravitation** is a mutual faculty which all bodies in nature possess, to attract one another; or *Gravity* is the force by which all bodies tend to approach each other. A large body attracting a comparatively very small one, and their distance apart being inconsiderable, the force of gravity in the small body will be very sensible compared with that in the large one; such is the case with the body, our earth, attracting small bodies on or near her surface.

Gravitation is not periodical, it acts continually *ever* and *ever*. A body placed unsupported at a distance from the earth, the *force of gravity* is instantly operating to draw it down, and then we say, "the body fell down" If it were possible to withdraw the attraction between the body and the earth, it would not fall down, but remain unsupported in the space where it was placed;—giving the body a motion upwards it would continue that, and never come back to the earth again.

Law of Gravity.

The force of Gravity is proportional to the mass of the attracting bodies, and inverse as the square of their distance apart.

This law was discovered by Sir Isaac Newton. It is this law that supports the condition of the whole universe, and enables us to calculate the distances, motions and masses, &c., of the heavenly bodies.

The unit or measure of force of gravity is assumed to be the velocity a falling body has obtained at the end of the first second it falls; this unit is commonly denoted by the letter g; its value at the level of the sea in New York is $g = 32{\cdot}166$ feet per second, in vacuum. The space fallen through in the first second is $\frac{1}{2}g = 16{\cdot}083$ feet.

This value augments with the latitude, and abates with the elevation above the level of the sea.

l = *latitude*, h = *height* in feet above the level of the sea, and r = *radius* of the earth in feet, at the given latitude l.

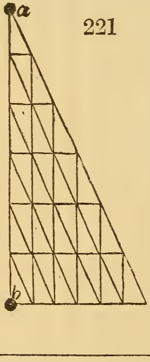
$$r = 20887510(1+0{\cdot}00164 \cos.2l),$$

$$g = 32{\cdot}16954(1-0{\cdot}00284 \cos.2l)\left(1-\frac{2h}{r}.\right)$$

Letters denote.

S = the *space* in feet, which the falling body passes through in the time T.
u = the *space* in feet, which the body falls in the Tth second.
V = *velocity* in feet per second, of the falling body at the end of the time T.
T = *time* in seconds the body is falling.

u	V	S	T
1	2	1	1″
3	4	4	2″
5	6	9	3″
[illegible]	8	16	4″
9	10	25	5″
11	12	36	6″



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The accompanying Diagram is a good illustration of the acceleration of a falling body. The body is supposed to fall from a to b, every small triangle represents the space 16·08 feet which the body falls in the first second; when the body has reached the line 3″ seconds, it will be found that it has passed 9 triangles, and $9 \times 16{\cdot}08 = 144{\cdot}72$ feet the space which a body will fall in 3″ seconds. The number of triangles between each line is the space u which the body has fallen in that second. Between 3″ and 4″ are 7 triangles and $7 \times 16{\cdot}08 = 112{\cdot}56$ feet, the space fallen through in the fourth second. Under the line 3″ will be found 6 triangles, which represents the velocity V the body has obtained at the end of the third second or $6 \times 16{\cdot}08 = 96{\cdot}48$ feet per second. For every successive second the body will gain two triangles or $2 \times 16{\cdot}08 = 32{\cdot}16$ feet per second.

Formulas for Accelerated Motion.

$$V = g\,T = \frac{2\,S}{T} = \sqrt{2g\,S} = 8{\cdot}02\sqrt{S}, \quad \dots \quad 1,$$

$$S = \frac{g\,T^2}{2} = \frac{V\,T}{2} = \frac{V^2}{2\,g} = \frac{V^2}{64{\cdot}33}, \quad \dots \quad 2,$$

$$T = \frac{V}{g} = \frac{2\,S}{V} = \sqrt{\frac{2\,S}{g}} = \frac{\sqrt{S}}{4{\cdot}01}, \quad \dots \quad 3,$$

$$u = g(T - \tfrac{1}{2}), \qquad T = \frac{u}{g} + \tfrac{1}{2}, \quad \dots \quad 4.$$

Example 1. A body is dropped at a height of 98 feet. What velocity will it have when it reaches the ground, and what time will it take to fall down?

Formula 1. $V = 8{\cdot}02\sqrt{S} = 8{\cdot}02\sqrt{98} = 79{\cdot}39$ feet per second.

Formula 3. $T = \frac{\sqrt{S}}{4{\cdot}01} = \frac{\sqrt{98}}{4{\cdot}01} = 2{\cdot}46$ seconds.

Example 2. A body was dropped at the opening of a hole in a rock, and reached the bottom after 3·5 seconds. How deep was the hole?

Formula 2. $S = \frac{g\,T^2}{2} = \frac{32{\cdot}16 \times 3{\cdot}5^2}{2} = 196{\cdot}98$ feet.

Retarded Motion.

A body thrown up vertically will obtain inversely the same motion as when it falls down, because it is the same force that acts upon it, and causes *retarded motion* when it ascends, and *accelerated motion* when it descends.

$V =$ the *velocity* at which the body starts to ascend.
$v =$ *velocity* at the end of the line t.
$T =$ *time* in seconds in which the body will ascend.
$t =$ any *time* less than T.
$S =$ *height* in feet to which the body will ascend.
$s =$ the space it ascends in the time t.

Formulas for Retarded Motion.

$$v = V - g\,t = \frac{s}{t} - \frac{g\,t}{2}, \quad \dots \quad 5,$$

$$s = V\,t - \frac{g\,t^2}{2} = t\,v + \frac{g\,t^2}{2}, \quad \dots \quad 6,$$

$$V = v + g\,t = \frac{s}{t} + \frac{g\,t}{2}, \quad \dots \quad 7,$$

$$t = \frac{V - v}{g} = \frac{V}{g} - \sqrt{\frac{V^2}{g^2} - \frac{2\,S}{g}}, \quad \dots \quad 8.$$

Formulas for T and S, is the same as for accelerated motion.

Example 3. A ball starts to ascend with a velocity of 135 feet per second. At what velocity will it strike an object 60 feet above? Find the time t, by the Formula 8.

$$t = \frac{135}{32{\cdot}16} - \sqrt{\frac{135^2}{32{\cdot}16^2} - \frac{2 \times 60}{32{\cdot}16}} = 0{\cdot}41 \text{ seconds, until it strikes, and from}$$

Formula 5, we have,

$$v = 135 - 32{\cdot}16 \times 0{\cdot}41 = 121{\cdot}83 \text{ feet, per second.}$$

Example 4. A ball thrown up vertically from a cannon, occupied 9 seconds, until it arrived at the same place it started from. How high up was the ball, and at what velocity did it start?

One half of 9 = $4\frac{1}{2}$ seconds. Formula 2.

$$S = \frac{32{\cdot}16 \times 4{\cdot}5^2}{2} = 326 \text{ feet high.}$$

$$V = 32{\cdot}16 \times 4{\cdot}5 = 144{\cdot}7 \text{ feet per second.}$$

If a cannon ball be shot from A, in the direction AB, at an angle BAC to the horizon, there are two forces acting on the ball at the same time, namely,—the force of gunpowder, which would propel the ball uniformly in the direction AB, and the force of gravity which only acts to draw the ball down at an accelerated motion; these two different (uniform and accelerated) motions will cause the ball to move in a curved line, (Parabola) AaC. Fig. 225.

V = velocity of the ball at A. W = weight of the ball in pounds.
S = the greatest hight of ball over the horizontal line AC.
t = time from A to C, via a. p = pounds of powder in the charge.
b = the distance from A to C, called *horizontal range*.

$$V = 2800\sqrt{\frac{p}{W}}, \quad p = \frac{WV^2}{7840000}, \quad b = 243781 \sin. x \cos. x \frac{p}{W}$$

Example 5. The cannon being loaded sufficiently to give the ball a velocity of 900 feet per second, the angle $x = 45°$. Required the distance b = ? and the time t = ?

$$b = \frac{900^2 \times \sin.45° \times \cos.45°}{32{\cdot}16} = 1259 \text{ feet, the distance from } A \text{ to } C.$$

It will be observed that the distance b will be longest when the angle x is 45°, because the product of *sine* and *cosine* is greatest for that angle. $\sin.45° \times \cos.45° = 0{\cdot}5$.

Example 5. What time will it take for a ball to roll 38 feet on an inclined plane angle, $x = 12° \, 20'$, and what velocity has it at 38 feet from the starting point.

$$T = \sqrt{\frac{2S}{g \sin.x}} = \sqrt{\frac{2 \times 38}{32{\cdot}16 \times \sin.12° \, 20'}} = 3{\cdot}33 \text{ seconds.}$$

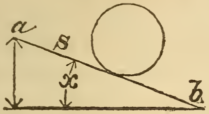
$$V = g \, T \sin.x = 32{\cdot}16 \times 3{\cdot}33 \times \sin{\cdot}12° \, 20' = 22{\cdot}8 \text{ feet per second.}$$

Power Concentrated in Moving Bodies.

It is highly important to distinguish betweeu power simply, and power when concentrated in a moving body. The former is the *force multiplied by its velocity*,—but the power concentrated in a moving body is equal to the *weight of the body multiplied by the square of its velocity, and the product divided by the accelleratrix g*,—or the power concentrated in a moving body is equal to the power expended in giving it the motion.

Example. A sledge weighing 20 pounds, strikes a nail with a velocity of 12 feet per second. With what effect did it strike?

$$P = \frac{20 \times 12^2}{32{\cdot}16} = 89{\cdot}55 \text{ effects.}$$

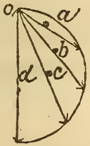


222.

$$V = g\ T \sin.x = \sqrt{2g\ S \sin.x},$$

$$S = \frac{g\ T^2}{2 \sin.x} = \frac{V^2}{2\ g \sin.x},$$

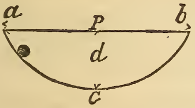
$$T = \frac{V}{g \sin.x} = \sqrt{\frac{2S}{g \sin.x}}.$$



223.

A body will fall from o the distances a, b, c, and d, in equal times.

$$T = \sqrt{\frac{2d}{g}}.$$

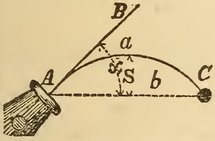


224.

A body will fall from a to b via c in the shortest time, if the curve is a Cycloid.

$S = 4d$, the length of the Cycloid.

$$T = \pi\sqrt{\frac{d}{2g}} = \pi\sqrt{\frac{p}{2\pi\ g}}.$$



225.

$$b = \frac{V^2 \sin.x \cos.x}{g},$$

$$T = \frac{V \sin.x}{g}, \qquad S = \frac{V \sin.^2 x}{2g}.$$

226.

$$S = g\frac{T^2\ F}{2M} = \frac{V^2\ M}{2g\ F},$$

$$V = g\ T\ \frac{F}{M} = \sqrt{\frac{2g\ S\ F}{M}},$$

$$T = \frac{V\ M}{g\ F} = \sqrt{\frac{2S\ M}{g\ F}},$$

$$F = \frac{V\ M}{g\ T}, = \frac{2S\ M}{g\ T^2},$$

$M = P + Q$, and $F = P - Q$.

CENTRIFUGAL FORCE.

Central Forces are of two kinds, *centrifugal* and *centripetal.*

Centrifugal Force is the tendency which a revolving body has to depart from its centre of motion.

Centripetal Force is that by which a revolving body is attracted or attached to its centre of motion.

The *Centrifugal* and *Centripetal forces* are opposites to each other, and when equal the body revolves in a circle; but when they differ the body will revolve in other curved lines, as the Ellipse, the Parabola, &c., according to the nature of the difference in the forces. If the *centrifugal force* is *o* while the other is acting, the body will move straight to the centre of motion; and if the *centripetal force* is *o* while the other is acting, the body will depart from the circle in a straight line, tangent to the circle in the point where the *centripetal force* ceased to act. The *central forces* are distinct from the force that has set the body in motion.

If the *centrifugal force* be made use of to produce an effect, such effect will be at the expense of the one producing the rotary motion.

Letters denote.

F = *Centrifugal force* in pounds.
M = the *Mass* or weight of the revolving body in pounds.
v = *Velocity* of the revolving body, in feet per second.
R = *Radii* of the circle in which the body revolves, in feet.
n = number of revolutions per minute.

Example 1. Required the *centrifugal force* of a body weighing 63 pounds, and making 163 revolutions per minute, in a circle of 4 feet, 4 inches radius?

$$F=\frac{M\,R\,n^2}{2933}=\frac{63\times4{\cdot}33\times163^2}{2933}=2475 \text{ pounds.}$$

Example 2. A Railroad train runs 43 miles per hour on a curved track of 115 feet radii. What should be the obliquity of the track?

$$\tan.x=\frac{\text{Miles}^2}{69R}=\frac{43^2}{69\times115}=0{\cdot}233,$$

or $x = 13^\circ\ 10'$, the obliquity of the track.

Example 3. A governor having its arms l = 1 foot, 6 inches, how many revolutions must it make per minute to form an angle $x = 30^\circ$?

$$n=\frac{54{\cdot}16}{\sqrt{1{\cdot}5\times\cos.30^\circ}}=47{\cdot}5 \text{ revolutions per minute.}$$

227.

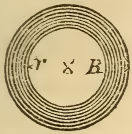
$$F=\frac{M\,v^2}{g\,R}=\frac{M\,v^2}{32{\cdot}16R}, \qquad 1,$$

$$F=\frac{4M\,R\,\pi^2\,n^2}{60^2g}=\frac{M\,R\,n^2}{2933}, \qquad 2,$$

$$M=\frac{F\,g\,R}{v^2}=\frac{2933F}{M\,n^2}, \qquad 3,$$

$$R=\frac{M\,v}{F\,g}=\frac{2933F}{M\,n^2}, \qquad 4,$$

$$n=\sqrt{\frac{2933\,F}{M\,R}}, \quad v=\sqrt{\frac{F\,R\,g}{M}}, \qquad 5,$$



228.

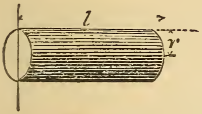
Centrifugal force of a ring.

$$F = \frac{M\, n^2 \sqrt{R^2+r^2}}{4150}.$$

229.

Centrifugal force of a grinding stone, circle-plane, cylinder, rotating round its centre.

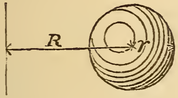
$$F = \frac{M\, R\, n^2}{4150}.$$



230.

Centrifugal force of a cylinder rotating round the diameter of its base.

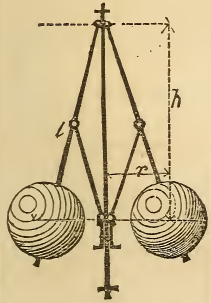
$$F = \frac{M\, n^2 \sqrt{4\, l^2+3r}}{10260}.$$



231.

Centrifugal force of a ball, (centre of gyration included.)

$$F = \frac{M\, n^2 \sqrt{R^2+\frac{2}{5}r^2}}{2933},$$



232.

Governor.

$$n = \frac{60}{2\pi}\sqrt{\frac{g}{h}} = \frac{54{\cdot}16}{\sqrt{h}} = \frac{54{\cdot}16}{\sqrt{l \cos.x}},$$

$$h = \frac{2933}{n^2}, \quad l = \frac{2933}{n^2 \cos.x} = \frac{h}{\cos.x},$$

$$\cos.x = \frac{2933}{n^2\, l} = \frac{h}{l}, \quad r = \sqrt{l^2 - h^2}.$$

PENDULUM.

Simple Pendulum is a material point under the action of gravitation, and suspended at a fixed point by a line of no weight.

Compound Pendulum is a suspended rod and body of sensible magnitude, fixed as the simple pendulum.

Centre of Oscillation is a point in which if all the matter in the compound pendulum were there collected, it would make a simple pendulum oscillate at the same times.

Angle of Oscillation is the space a pendulum describes when in motion.

The velocity of an oscillating body through the vertical position, is equal to the velocity a body would obtain by falling vertically the distance *versed sine* of half the angle of oscillation.

Letters denote.

l = *length* of the simple pendulum, or the distance between the centre of suspension, and centre of oscillation in inches.
t = *time* in seconds for n oscillations.
n = number of single oscillations in the time t.

Example 1. Required the length of a pendulum that will vibrate seconds? here $n = 1$, and $t = 1''$.

$l = 39{\cdot}109 \dfrac{t^2}{n^2} = 39{\cdot}109$ inches, the length of a pendulum for seconds.

Example 2. Require the length of a pendulum that will make 180 vibrations per minute? here $t = 60''$ and $n = 180$.

$$l = \frac{39{\cdot}109 t^2}{n^2} = \frac{39{\cdot}109 \times 60^2}{180^2} = 4{\cdot}346 \text{ inches.}$$

Example 3. How many vibrations will a pendulum of 25 inches length make in 8 seconds?

$$n = \frac{6{\cdot}254 t}{\sqrt{l}} = \frac{6{\cdot}254 \times 8}{\sqrt{25}} = 10 \text{ vibrations.}$$

Example 4. A pendulum is 137·67 inches long and makes 8 vibrations in 15 seconds. Required the unit or accelleratrix g = ?

$$g = \frac{0{\cdot}8225 l\, n^2}{t^2} = \frac{0{\cdot}8225 \times 137{\cdot}67 \times 8^2}{15^2} = 32{\cdot}209.$$

Example 5. A compound pendulum of two iron balls P and Q, having the centre of suspension between themselves: see Fig. 238. $P = 38$ pounds, $Q = 12$ pounds, $a = 25$ inches, and $b = 18$ inches. How long is the simple pendulum, and how many vibrations will the pendulum make in 10 seconds?

$$x = \frac{a\,P - b\,Q}{P+Q} = \frac{25 \times 38 - 18 \times 12}{38+12} = 14{\cdot}68 \text{ inches.}$$

$$l = \frac{a^2\,P + b^2\,Q}{x(P+Q)} = \frac{25^2 \times 38 + 18^2 \times 12}{14{\cdot}68(38+12)} = 37{\cdot}68 \text{ inches,}$$

the length of the single pendulum.

$$n = \frac{6{\cdot}254 t}{\sqrt{l}} = \frac{6{\cdot}254 \times 10}{\sqrt{37{\cdot}68}} = 10{\cdot}193 \text{ vibrations in 10 seconds.}$$

If a compound pendulum is hung up at its centre of oscillation, the former centre of suspension will be the centre of oscillation, and the pendulum will oscillate the same time.

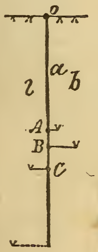
233.

Simple Pendulum.

$$l = \frac{12g\,t^2}{\pi^2\,n^2} = \frac{39{\cdot}1t^2}{n^2},$$

$$t = \frac{n\sqrt{l}}{6.25},$$

$$n = \frac{6{\cdot}254t}{\sqrt{l}},$$



234.

A = centre of gravity.

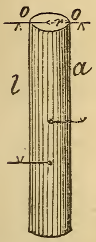
B = centre of gyration.

C = centre of oscillation.

$$a : b = b : l,$$

$$b = \sqrt{al} = 1{\cdot}1432a,$$

$$l = 1\tfrac{1}{3}a.$$



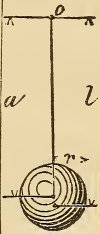
235.

Compound Pendulum.

r = radius of cylinder.

$$l = \frac{16a^2+3r^2}{12a},$$

$$l = \frac{4a}{3}+\frac{r^2}{4a}.$$



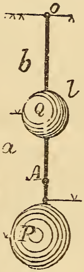
236.

$$g = \frac{l\,\pi^2\,n^2}{12t^2},$$

$$g = \frac{0{\cdot}8225l\,n^2}{t^2},$$

o = *centre of suspension.*

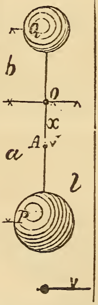
$$l = a + \frac{2r}{5a}.$$



237.

$$l = \frac{a^2\,P+b^2\,Q}{a\,P+b\,Q}.$$

P and Q expressed in pounds, or cubic contents.



238.

$$x = \frac{a\,P - b\,Q}{P+Q},$$

$$l = \frac{a^2P+b^2\,Q}{x(P+Q)}.$$

Length of a Pendulum vibrating seconds at the level of the sea, in various places.

At the Equator, lat. 0° 0′ 0″	39·0152 inches.
" Washington, lat. 38° 53′ 23″	39·0958 "
" New York, lat. 40° 42′ 40″	39·1017 "
" London, lat. 51° 31′	39·1393 "
" lat. 45°	39·1270 "
" Stockholm, lat. 59° 21′ 30″	39·1845 "

$l = 39{\cdot}127 - 0{\cdot}09982$ cos.2 lat. for seconds.

CENTRE OF GYRATION.

Centre of Gyration is a point in revolving bodies in which, if all the revolving matters were there contained, it would obtain equal *angular velocity* from, and sustain equal resistance to, the force that gives it a rotary motion.

The *centre of gyration* in different bodies will be found by the accompanying formulas, in which x = distance from the centre of motion to the centre of gyration.

Example 1. Fig. 239. Find the *centre of gyration* in a bar, rotating round one of its ends; its length is 7 feet, 3 inches?

$$x = 0{\cdot}5775 \times 7{\cdot}25 = 4{\cdot}13 \text{ feet, from the centre of motion.}$$

Example 2. Fig. 245. Find the *centre of gyration* of a cone, rotating round its vertex, its height being $h = 3{\cdot}3$ feet, and $R = 8$ inches $= 0{\cdot}666$ feet.

$$x = \sqrt{\frac{12h^2+3R^2}{20}} = \sqrt{\frac{12\times3{\cdot}3^2+3\times0{\cdot}666^2}{20}} = 2{\cdot}569 \text{ feet}$$

from the centre of motion.

Example 3. Fig. 249. A ring or fly wheel having its outer radius $R = 6$ feet 4 inches, the inner radius $r = 5$ feet 8 inches. Required its *centre of gyration* $x = ?$ from the centre of motion.

$$x = \sqrt{\frac{R^2+r^2}{2}} = \sqrt{\frac{6{\cdot}33^2+5{\cdot}66^2}{2}} = 6 \text{ feet.}$$

CONCLUSIONS.

The object of finding the *centre of gyration* of revolving bodies is to ascertain what effect is necessary to give a mass a certain angular velocity; or how much effect is concentrated in a body having a certain angular velocity.

Angular velocity is the number of revolutions a body makes in a unit of time, it is herein denoted by the letter n.

Letters denote.

P = *power* in effects.
H = *horse-power.*
F = the *Force* which is applied to rotate a body, in pounds.
s = the *radius* on which the force acts, in feet.
M = *Mass* of the revolving body, in pounds.
x = the *distance* from the *centre of motion* to *centre of gyration*, in feet.
T = *time* the force F is applied in seconds.
N = number of revolutions in the time T.
n = *angular velocity* or number of revolutions per minute, at the end of the time T.
g = 32·166 *accelleratrix* of the *force of gravity.*
G = *accelleratrix* of the force F, then,

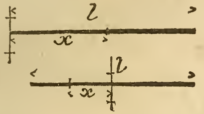
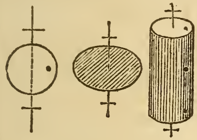
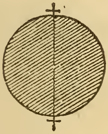
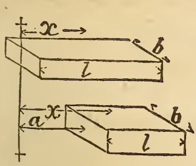
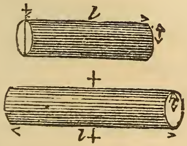
$$G : g = Fs^2 : Mx^2, \quad \text{or} \quad G = \frac{g\,Fs^2}{Mx^2}.$$

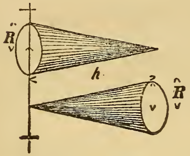
Example 4. Fig. 249. In connection with the preceding example (3) the fly-wheel weighs 7400 pounds. What force F must be applied at the radius $r = 2$ feet, to give the fly wheel an *angular velocity* of $n = 128$ revolutions per minute, at the end of the time $T = 40$ seconds?

Formula 6. $$F = \frac{n\,M\,x^2}{153{\cdot}5g\,Ts} = \frac{128\times7400\times6^2}{153{\cdot}5\times40\times2} = 2773 \text{ pounds.}$$

How many revolutions did the wheel make in the 40 seconds?

Formula 9. $$N = \frac{2{\cdot}56T^2\,Fs}{M\,x^2} = \frac{2{\cdot}56\times40^2\times2773\times2}{7400\times6^2} = 85{\cdot}27 \text{ revolutions.}$$

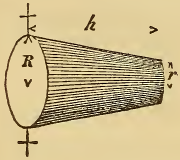
	239. *A line or bar.* $x = 0.5775l,$ $x = 0{\cdot}2887l.$
	240. *A circumference round its diameter,* *A circle-plane round its centre,* *A cylinder round its axis.* $x = 0{\cdot}7071r.$
	241. *A circle-plane round its diameter.* $x = 0{\cdot}5r.$
	242. *A Sphere round its diameter.* Convex surface, $x = 0{\cdot}8165r,$ Solid, . . . $x = 0{\cdot}6324r.$
	243. *Parallelopiped.* $x = \sqrt{\frac{4l^2+b^2}{12}},$ $x = \sqrt{\frac{4l^2+b^2}{12}+a^2+a\,l}.$
	244. *Cylinder.* $x = \sqrt{\frac{4l^2+3r^2}{12}},$ $x = \sqrt{\frac{l^2+3r^2}{12}}.$



245. *Cone.*

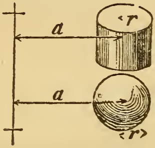
$$x = \sqrt{\frac{2h^2+3R^2}{20}},$$

$$x = \sqrt{\frac{12h^2+3R^2}{20}}.$$



246. *Conic Frustrum.*

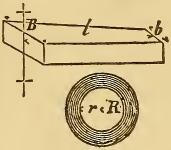
$$x = \sqrt{\frac{h}{10}\left(\frac{R^2+3R\,r+R\,r^2}{R^2+R\,r+r^2}\right) + \frac{3}{20}\left(\frac{R^5-r^5}{R-r^3}\right)}$$



247. *Cylinder and Sphere.*

$$x = \sqrt{a^2+\tfrac{1}{2}r^2},$$

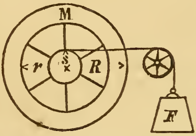
$$x = \sqrt{a^2+\tfrac{2}{5}r^2}.$$



248. *Wedge and Ring.*

$$x = 0{\cdot}204\sqrt{12l^2+B^2+b^2},$$

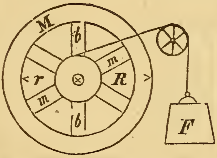
$$x = \sqrt{\frac{R^2+r^2}{2}}.$$



249. *Fly Wheel.*

$$x = \sqrt{\frac{R^2+r^2}{2}},$$

$$F\,G : M\,g = x^2 : s^2.$$



250. *Fly Wheel with Arms.*

$$x^2(M+m) = M\,\frac{R^2+r^2}{2} + m\,\frac{4r^2+b^2}{12},$$

$$x = \sqrt{\frac{6M(R^2+r^2)+m(4r^2+b^2)}{12(M+m)}}.$$

Formulas for Force and Power of Acceleration.

$T = \sqrt{\dfrac{4\pi\, s\, N}{G}}$, - 1,	$N = \dfrac{2{\cdot}56\, T^2\, F\, s}{M\, x^2}$ - - 9,
$T = x\sqrt{\dfrac{4\pi\, N\, M}{g\, F\, s}}$, 2,	$n = \dfrac{153{\cdot}5\, T\, F\, s}{M\, x^2}$, - - 10,
$T = \dfrac{2\pi\, s\, n}{60 G}$, - - - 3,	$P = \dfrac{n^2\, M\, x^2}{244 T}$, - - - 11,
$T = \dfrac{4\pi\, n\, M\, x^2}{60 g\, F\, s}$, - - 4,	$M = \dfrac{244 T\, P}{n^2\, x^2}$, - - - 12,
$F = \dfrac{N\, M\, x^2}{2{\cdot}56\, T^2\, s}$, - - 5,	$T = \dfrac{M\, x^2\, n^2}{244 P}$, - - - 13,
$F = \dfrac{n\, M\, x^2}{153{\cdot}5\, T\, s}$, - - 6,	$n = \sqrt{\dfrac{244\, T\, P}{M\, x^2}}$, - 14,
$s = \dfrac{N\, M\, x^2}{2{\cdot}56\, T^2\, F}$, - - 7,	$H = \dfrac{n^2\, M\, x^2}{134100\, T}$, - - 15,
$s = \dfrac{n\, M\, x^2}{153{\cdot}5\, T\, F}$, - - 8,	$M = \dfrac{134100\, T\, H}{n^2\, x^2}$, 16,

Fly-Wheels. Weight of.

The weight of a fly-wheel will be determined by the *formula* 16 in which the time $T = 130$ seconds, the time in which the fly-wheel would *concentrate* the same power as the steam-engine. When the works or resistance is very irregular it will be better to take the time $T = 170$. The *centre of gyration* (including ring and arms,) can in practice be assumed at $x = r$ the inner radius of the ring.

Example 5. Required the weight of a fly-wheel for ordinary work, the steam engine being 56 horse power, making 42 revolutions per minute, and the inner radius $r = 10$ feet?

$$M = \frac{134100\, T\, H}{n^2\, r^2} = \frac{134100 \times 130 \times 56}{42^2 \times 10^2} = 5535 \text{ pounds.}$$

CENTRE OF PERCUSSION.

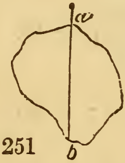
Centre of Percussion is a point in which the momentums of a moving body are concentrated. *Centre of Percussion* is the same as *centre of oscillation*, and to be calculated by the same formulas.

Take an iron bar in one hand, and strike heavily over a sharp edge, if the *centre of percussion* of the bar strikes over the edge, the whole momentum will there be discharged, but if it strikes at a distance from the *centre* of *percussion* a part of the momentum will be discharged in the hand, and a shock felt.

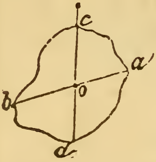
It is sometimes of great importance to properly place the *centre of percussion*. If it is dislocated, the moving body not only fails to properly transmit its effect, but the lost momentum acts to wear out the machinery.

CENTRE OF GRAVITY.

Centre of Gravity is a point around which the momentums of all matters (under the action of the force of gravity) in a body, or system of bodies, are equally divided.



251



A body or system of bodies suspended at its *centre of gravity*, will be in equilibrium in all positions.

A body or system of bodies, suspended in a point *out* of its *centre* of *gravity*, will hang with its *centre of gravity* vertical under the point of suspension.

A body or system of bodies suspended in a point out of its *centre of gravity*, and having two different positions, the two vertical lines through the point of suspension will meet in the *centre of gravity*; thus if a plane be hung up in two different positions, the vertical lines a, b, and c, d, will meet in the *centre of gravity* o.

z = distance to the centre of gravity as noted in the figures.

Example 1. The radius of a circle being 3 feet, how far is its centre of gravity from the centre of the half circle?

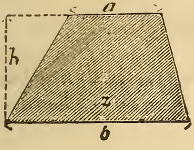
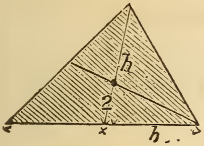
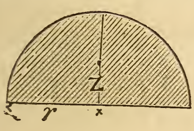
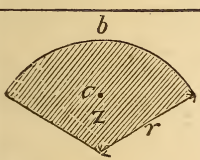
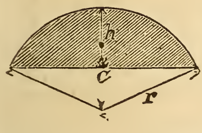
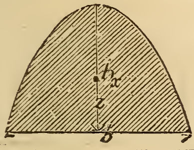
$$z = 0{\cdot}6367 \times 3 = 1{\cdot}91 \text{ feet.}$$

Example 2. How far from the bottom of a cylindric shell, open at one end, is its centre of gravity? The cylinder is 4 feet long, radius $r = 0{\cdot}8$ feet.

$$z = \frac{h}{r+2h} = \frac{4}{0{\cdot}8+2\times4} = 0{\cdot}625 \text{ feet.}$$

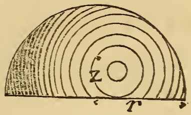
Example 3. Fig. 264. An irregular figure weighing $P = 138$ pounds, is suspended between a fulcrum and a weight, $l = 5{\cdot}6$ feet, $W = 57$ pounds. Required the distance to the centre of gravity $z = ?$

$$z = \frac{57\times5{\cdot}6}{138} = 2{\cdot}31 \text{ feet.}$$

	252. *Quadrangle.*—**a** *and b parallel.* $z = \frac{h}{2} - \frac{h}{6}\left(\frac{b-a}{b+a}\right).$
	253. *Triangle.* $z = \frac{h}{3}.$
	254. *Half a circle plane or Elliptic plane.* $z = 0{\cdot}424r.$
	255. *Circle sector.* $z = \frac{2c\,r}{3b}.$
	256. *Circle Segment.* **a** = *area.* $z = \frac{c^3}{12\mathbf{a}}.$ $x = h + z - r.$
	257. *Parabola.* $z = \frac{2h}{5}.$ For half a Parabola $x = \frac{3}{8}b.$

258.

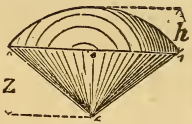
Half Sphere.



Convex surface . . . $z = \frac{1}{2}r.$
Solid . . . . $z = \frac{3}{8}r.$

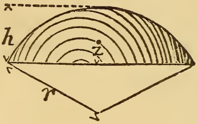
259.

Spherical Sector.



Solid, $z = \frac{1}{4}\left(r - \frac{h}{2}\right).$

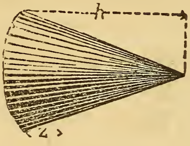
260. *Spherical Segment.*



Convex surface $z = \frac{h}{2},$

Solid $z = \frac{h}{2}.\left[\frac{2r^2+h^2}{3r^2+h^2}.\right]$

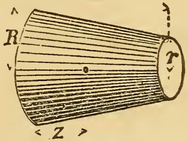
261. *Cone.*



Convex surface $z = \frac{h}{3},$

Solid $z = \frac{h}{4}.$

262. *Conic Fustrum.*

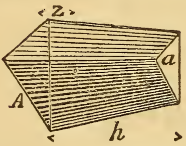


Con. sur. $z = \frac{h}{2} - \frac{h}{6}\left[\frac{R-r}{R+r}.\right]$

Solid $z = \frac{h}{4}.\left[\frac{R^2+r(2R+3r)}{R^2+r(R+r)}.\right]$

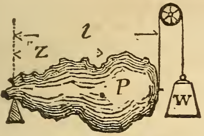
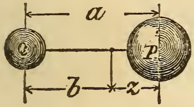
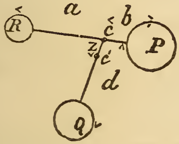
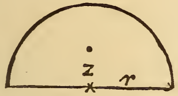
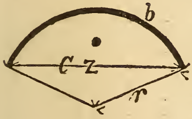
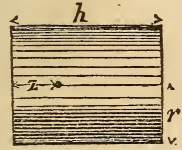
263.

Pyramidic Fustrum.



A and a = area of the two bases.

Solid $z = \frac{h}{4}\left[\frac{A+3a+2\sqrt{A\,a}}{A+a+\sqrt{A\,a}}.\right]$

	264. *Irregular Figure.* $P : W = l : z.$ $z = \frac{W\,l}{P}.$
	265. *To find the Centre of Gravity of two bodies, P and Q.* $z = \frac{Q\,a}{P+Q}, \qquad b = \frac{P\,a}{P+Q}.$
	266. *To find the Centre of Gravity of a system of bodies.* $b = \frac{R\,a}{P+R}, \qquad z = \frac{Q\,d}{P+R+Q}.$
	267. *Half a circumference of a Circle or Ellipse.* $z = 0{\cdot}6367r.$
	268. *Circle arc or Elliptic arc.* $z = \frac{c\,r}{b} = \frac{c(c^2+4h^2)}{8h\,b}.$
	269. *Cylindric Surface with a bottom in one end.* $z = \frac{h}{r+2h}.$

SPECIFIC GRAVITY.

Specific Gravity is the comparative density of substances. The unit for measuring the specific gravity is assumed to be the density of rain water, or distilled water.

One cubic foot of distilled water weighs 1000 ounces, or 62·5 pounds avoirdupois.

To Find the Weight of a Body.

RULE 1. Multiply the contents of the body in cubic feet by 62·5, and the product by its specific gravity, will be the weight of the body in pounds avoirdupois.

RULE 2. Multiply the contents of the body in cubic inches by 0·03616, and the product by its specific gravity, will be the weight of the body in pounds avoirdupois.

RULE 3. Divide the specific gravity by 0·016 and the quotient is the weight of a cubic foot.

Example 1. A bottle full of mercury is 3 inches, inside diameter, and 6 inches high. How much mercury is there in the bottle in pounds?

One cubic inch of mercury weighs 0·491 pounds, and by the formula for Fig. 119 we have the

$$\text{weight} = 0{\cdot}491 \times 0{\cdot}785 \times 3^2 \times 6 = 20{\cdot}85 \text{ pounds.}$$

Example 2. Required the weight of a cone of cast iron, diameter at the base $d = 1{\cdot}33$ feet, height $h = 4$ feet? One cubic foot of cast iron weighs 450·5 pounds, and by formula for Fig. 117 we have the

$$\text{weight} = 450{\cdot}5 \times 0{\cdot}2616 \times 1{\cdot}33^2 \times 4 = 834 \text{ pounds.}$$

Example 3. The section area of the lower hole in a steam boat is 245 square feet; how much space must be taken in the length of the hole for 131 tons of anthracite coal?

Anthracite coal are 42·3 cubic feet per ton.

$$\text{length} = \frac{42{\cdot}3 \times 131}{245} = 22{\cdot}6 \text{ feet, the space required.}$$

Weight and Bulk of Substances.

Names of Substances.	*Cubic feet in pounds.*	*Cubic foot per ton.*	*Names of Substances.*	*Cubic feet in pounds.*	*Cubic foot per ton.*
Cast iron, - -	450·5	4·97	Sand, - - - -	94·5	23·7
Wrought iron, -	486·6	4·60	Granite, - - -	139	16·1
Steel, - - -	489·8	4·57	Earth, loose, - -	78·6	28·5
Copper, - - -	555·	4·03	Water, salt, (sea) -	64·3	34·8
Lead, - - -	707·7	3·16	" fresh - -	62·5	35·9
Brass, - - -	537·7	4·16	Ice, - - - -	58·08	38·56
Tin, - - - -	456	4·91	Gold, - - -	1013	2·21
Pine, white - -	29·56	75·6	Silver, - - -	551	4·07
" yellow, - -	33·81	66·2	Coal, Anthracite -	53	42·3
Mahogany, - -	66·4	33·8	" Bituminous -	50	44·8
Marble, common, -	141·0	15·9	" Cumberland -	53	42·3
Mill-stone, - -	130	17·2	" Charcoal -	18·2	123
Oak, live - - -	70	32·0	Coke, Midlothian -	32·70	68·5
" white, - -	45.2	49·5	" Cumberland -	31·57	70·9
Clay, - - -	101·3	22·1	" Natural Virginia	46·64	48·3
Cotton Bales, - -			Conventional rate of		
Brick, - - -	100	22·4	Stone coal, 28 bushels		
Plaster Paris, - -	105	21·3	(5 pecks) = 1 ton, -		43·56

To Find the Specific Gravity.

W = weight of a body in the air.
w = weight of the body (heavier than water) immersed in water.
S = specific gravity of the body. Then,

$$W-w : W = 1 : S. \quad S = \frac{W}{W-w}, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 1,$$

Example 4. Required the specific gravity of a piece of iron-ore weighing 6·345 pounds in the air, and 4·935 pounds in water, $S = ?$

$$S = \frac{6\cdot345}{6\cdot345 - 4\cdot935} = 4\cdot5 \text{ the specific gravity.}$$

When the body is lighter than water, annex to it a heavier body that is able to sink the lighter one.

S = specific gravity of the heavier annexed body.
s = specific gravity of the lighter body.
W = weight of the two bodies in air.
w = weight of the two bodies in water.
V = weight of the heavier body in air.
v = weight of the lighter body in air.

$$s = \frac{v}{W - w - \frac{V}{S}}, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 2,$$

Example 5. To a piece of wood, which weighs $v = 14$ pounds in the air, is annexed a piece of cast-iron $V = 28$ pounds; the two bodies together weigh $w = 11\cdot7$ pounds in water. Required the specific gravity of the wood?

$W = V + v = 28 + 14 = 42$ pounds.
$S = 7\cdot2$ specific gravity of cast-iron.

Formula 2. $$S = \frac{14}{42 - 11\cdot7 - \frac{28}{7\cdot2}} = 0\cdot529, \text{ the specific}$$

gravity of the wood, (Poplar White Spanish.)

A simple way to obtain the specific gravity of woods, is to form it to a parallel rod, and place it vertically in water, then when in equilibrium, the immersed end is to the whole rod as the specific gravity is to 1.

Example 6. A cylinder of wood is 6 feet, 3 inches long, when immersed vertically in water it will sink 3 feet, 9 inches by its own weight. Required its specific gravity.

$$3\cdot75 : 6\cdot25 = S : 1, \qquad S = \frac{3\cdot75}{6\cdot25} = 0\cdot600.$$

To discover the Adulteration in Metals. or to find the proportions of two Ingredients in a Compound.

$$V = \frac{W - s(W-w)}{1 - \frac{s}{S}}, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 3,$$

Example 7. A metal compounded of silver and gold weighs $W = 6$ pounds in the air, and in water $w = 6\cdot636$ pounds. Require the proportions of silver and gold?

$S = 19\cdot36$ specific gravity of gold.
$s = 10\cdot51$ specific gravity of silver.

$$\text{weight } V = \frac{6 - 10.51(6 - 5.636)}{1 - \frac{10\cdot51}{19\cdot36}} = 4\cdot755 \text{ pounds of gold.}$$

and 1·245 pounds of silver.

Names of Substances.	*Specific gravity.*	*Weight per cubic inch.*
Metals.		
Platinum, rolled	22·669	·798
" wire,	21·042	·761
" hammered,	20·337	·736
" purified,	19·50	·706
" crude, grains	15·602	·565
Gold, hammered	19·361	·700
" pure cast	19·258	·697
" 22 carats fine	17·486	·733
" 20 " "	15·702	·568
Mercury, solid at — 40°	15·632	·566
" at +32° Fahr.	13·619	·493
" " 60° "	13·580	·491
" " 212° "	13·375	·484
Lead, pure	11·330	·410
" hammered	11·388	·412
Silver, hammered	10·511	·381
" pure	10·474	·379
Bismuth,	9·823	·355
Red Lead,	8·940	·324
Cinober,	8·098	·293
Manganese,	8·030	·290
Copper, wire and rolled	8·878	·321
" pure	8·788	·318
Bronze, gun metal	8·700	·315
Brass, common	7·820	·282
Steel, cast steel	7·919	·286
" common soft	7·833	·283
" hardened & temp.	7·818	·283
Iron, pure	7·768	·281
" wrought and rolled	7·780	·282
" hammered	7·789	·282
" cast-iron	7·207	·261
Tin, from Böhmen	7·312	·265
" English	7·291	·264
Zinc, rolled	7·191	·260
" cast	6·861	·248
Antimony,	6·712	·244
Aluminium	2·5	0·09
Arsenic,	5·763	·208
Stones and Earths.		
Topaz, oriental	4·011	·145
Emery,	4·000	·144
Diamond,	3·521	·127
Limestone, green	3·180	·115
" white	3·156	·114
Asbestos, starry	3·073	·111
Glass, flint	2·933	·106
" white	2·892	·104
" bottle	2·732	·0987
" green	2·642	·0954
Marble, Parian	2·838	·103
" African	2·708	·0978
" Egyptian	2·668	·0964
Mica,	2·800	·1000
Hone, white razor	2·838	·104
Chalk,	2·784	·100
Porphyry,	2·765	·0999
Spar, green	2·704	·0976
" blue	2·693	·0971
Alabaster, white	2·730	·0987
" yellow	2·699	·0974
Coral, red	2·700	·0974
Granite, Susquehanna	2·704	·0976
" Quincy	2·652	·0958
" Patapsco	2·640	·0954
" Scotch	2·625	·0948
Marble, white Italian	2·708	·0978
" common	2·686	·0968
Talc, black	2·900	·0105
Quartz,	2·660	·0962
Slate,	2·672	·0965
Pearl, oriental	2·650	·0957
Shale,	2·600	·0940
Flint, white	2·594	·0936
" black	2·582	·0933
Stone, common	2·520	·0910
" Bristol	2·510	·0906
" Mill	2·484	·0897
" Paving	2·416	·0873
Gypsum, opaque	2·168	·0783
Grindstone,	2·143	·0775
Salt, common	2·130	·0770
Saltpetre,	2·090	·0755
Sulphur, native	2·033	·0735
Common soil,	1·984	·0717
Rotten stone,	1·981	·0416
Clay,	1·930	·0698
Brick,	1·900	·0686
Nitre,	1·900	·0636
Plaster Paris,	1·872 2·473	·0677 ·0894
Ivory,	1·822	·0659
Sand,	1·800	·0651
Phosphorus,	1·770	·0640
Borax,	1·714	·0620
Coal, Anthracite	1·640 1·436	·0593 ·0592
" Maryland	1·355	·0490
" Scotch	1·300	·0470
" New Castle	1·270	·0460
" Bituminous	1·270	·0460
Charcoal, triturated	1·380	·0500
Earth, loose	1·500	·0542
Amber,	1·078	·0387
Pimstone,	1·647	·0596
Lime, quick	0·804	·0291
Charcoal,	0·441	·0160
Woods (Dry.)		
Alder,	·800	·0289
Apple-tree,	·793	·0287
Ash, the trunk	·845	·0306
Bay-tree,	·822	·0297
Beech,	·852	·0308
Box, French	·912	·0330
" Dutch	1·328	·0480
" Brazilian red	1·031	·0373
Cedar, wild	·596	·0219
" Palestine	·613	·0222

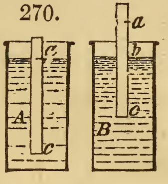
Names of Substances.	*Specific gravity.*	*Weight per cubic inch.*
Cedar, Indian - -	1·315	·0476
" American	·561	·0203
Citron, - - -	·726	·0263
Cocoa-wood, - -	1·040	·0376
Cherry-tree, - -	·715	·0259
Cork, - - -	·240	·0087
Cypress, Spanish -	·644	·0233
Ebony, American -	1·331	·0481
" Indian -	1·209	·0437
Elder-tree, - -	·695	·0252
Elm, trunk of - -	·671	·0243
Filbert-tree, - -	·600	·0217
Fir, male - - -	·550	·0199
" female - -	·498	·0180
Hazel, - - -	·600	·0217
Jasmine, Spanish -	·770	·0279
Juniper-tree, - -	·556	·0201
Lemon-tree, . - -	·703	·0254
Lignum-vitæ, - -	1·333	·0482
Linden-tree, - -	·604	·0219
Log-wood, - -	·913	·0331
Mastic-tree - -	·849	·0307
Mahogany, - -	1·063	·0385
Maple, - - -	·750	·0271
Medlar, - - -	·944	·0342
Mulberry - -	·897	·0324
Oak, heart of, 60 old	1·170	·0423
Orange-tree, - -	·705	.0255
Pear-tree, - -	·661	·0239
Pomegranate-tree, -	1·354	·0490
Poplar, - - -	·383	·0138
" white Spanish	·529	·0191
Plum-tree, - -	·785	·0284
Quince-tree, - -	·705	·0255
Sassafras, - -	·482	·0174
Spruce, - - -	·500	·0181
" old - -	·460	·0166
Pine, yellow - -	·660	·0239
" white - -	·554	·0200
Vine, - - -	1·327	·0480
Walnut, - - -	·671	·0243
Yew, Dutch - -	·788	·0285
" Spanish - -	·807	·0292
Liquids.		
Acid, Acetic - -	1·062	·0384
" Nitric - -	1·217	·0440
" Sulphuric -	1·841	·0666
" Muriatic -	1·200	·0434
" Fluoric - -	1·500	·0542
" Phosphoric -	1·558	·0563
Alchohol, commercial	·833	·0301
" pure -	·792	·0287
Ammoniac, liquid -	·897	·0324
Beer, lager - -	1·034	·0374
Champagne, - -	9·97	·0360
Cider, - - -	1·018	·0361
Ether, sulphuric -	·739	·0267
Egg, - - - -	1·090	·0394
Honey, - - -	1·450	·0524
Human blood, -	1·054	·0381
Milk, - - -	1·032	·0373

Names of Substances.	*Specific gravity.*	*Weight per cubic inch.*
Oil, Linseed - -	·940	·0340
" Olive - - -	·915	·0331
" Turpentine -	·870	·0314
" Whale - -	·932	·0337
Proof Spirit, - -	·925	·0334
Vinegar, - - -	1·080	·0390
Water, distilled	1·000	·0361
" Sea - -	1·026	·0371
" Dead sea -	1·240	·0448
Wine, - - -	·992	·0359
" Port - -	·997	·0361
Miscellaneous.		
Asphaltum, - -	·905	·0327
	1·650	·0597
Beeswax, - - -	·965	·0349
Butter, - - -	·942	·0341
Camphor, - -	·988	·0357
India rubber, - -	·933	·0338
Fat of Beef, - -	·923	·0334
" Hogs, - -	·936	·0338
" Mutton, -	·923	·0334
Gamboge, - -	1·222	·0442
Gunpowder, loose -	·900	·0325
" shaken	1·000	·0361
" solid -	1·550	·0561
	1·800	·0650
Gum Arabic, -	1·452	·0525
Indigo, - - -	1·009	·0365
Lard, - - -	·947	·0343
Mastic, - - -	1·074	·0388
Spermaceti, - -	·943	·0341
Sugar, - - -	1·605	·0580
Tallow, sheep - -	·924	·0334
" calf - -	·934	·0338
" ox, - -	·923	·0334
Atmospheric air, -	·0012	43
Gases. Vapours.		*Weight cub. ft. grains.*
Atmospheric air, -	1·000	527·0
Ammoniacal gas, -	·500	263·7
Carbonic acid, - -	1·527	805·3
Carbonic oxid, -	·972	512·7
Carburetted hydrogen,	·972	512·7
Chlorine, - - -	2·500	1316
Chlorocarbonous acid,	3·472	1828
Chloroprussic acid,	2·152	1134
Flouboric acid,	2·371	1250
Hydriodic acid, -	4·346	2290
Hydrogen, - -	·069	36·33
Oxygen, - - -	1·104	581·8
Sulphuretted hydrogen,	1·777	9370
Nitrogen, - -	·972	512·0
Vapour of Alchohol,	1·613	851·0
" turpen'e spir.,	5·013	2642
" water, -	·623	328·0
Smoke of bitumin. coal,	·102	53·80
" wood, -	·90	474·0
Steam at 212° - -	·488	257·3

HYDROMETER.

A BODY wholly immersed in a liquid will lose as much of its weight, as the weight of the liquid it displaces.

270.

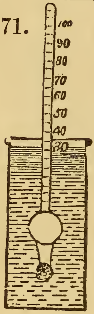


A floating body will displace its own weight of the liquid in which it floats.

A cylindrical rod of wood or some light materials, being set down in two liquids, A and B, of different specific gravities, when in equilibrium it will sink to the mark a in the liquid A, and to b in the liquid B; then the specific gravity of $A : B = b, c : a, c$, or inverse as the immersed part of the rod. This is the principle upon which a hydrometer is constructed.

Table showing the comparative Scales of Guy Lussac and Baumè, with the Specific Gravity and Proof, at the temperature of 60° *Fahr.*

271.



	Guy Lussac's.	Baumè s	Specific Grav.	Proof.	
Per centage of pure alcohol.	100	46	·796	100	Per centage over Proof.
	95	40	·815	92	
	90	36	·833	82	
	85	33	·848	72	
	80	31	·863	62	
	75	28	·876	52	
	70	26	·889	42	
	65	24	·901	32	
	60	23	·912	22	
	55	21	·923	12	
	50	19	·933	0 Proof.	
	45	18	·942	8	Under Proof.
	40	17	·951	18	
	35	16	·958	29	
	30	15	·964	35	
	25	14	·970	48	

HYDROSTATICS.

Letters denote.

A and **a** = areas of the pressed surfaces in square feet.

P and p = hydrostatic pressure in pounds.

d = depth of the centre of gravity of **A** or **a** under the surface of the liquids in feet.

S = specific gravity of the liquid.

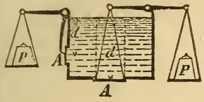
Example 1. Fig. 272. The plane $\mathbf{A} = 3{\cdot}3$ square feet, at a depth of $d = 6$ feet under the surface of fresh water. Required the pressure $P = ?$ Specific gravity of fresh water $S = 1$.

$$P = 62{\cdot}5\mathbf{A}\,d = 62{\cdot}5 \times 3{\cdot}3 \times 6 = 1237{\cdot}5 \text{ pounds.}$$

Example 2. Fig. 275. The area of the pistons $\mathbf{A} = 8{\cdot}5$ square feet, $\mathbf{a} = 0{\cdot}02$ square feet, $l = 4$ feet, $e = 9$ inches, and $F = 18$ pounds. Required the pressure $P = ?$

$$P = \frac{F\,l\,\mathbf{A}}{e\,\mathbf{a}} = \frac{18 \times 4 \times 8{\cdot}5}{0{\cdot}75 \times 0{\cdot}02} = 40800 \text{ pounds.}$$

It must be distinguished that the centre of pressure and centre of gravity of the planes, are two different points; the centre of pressure is below the centre of gravity, when the plane is inclined or vertical.

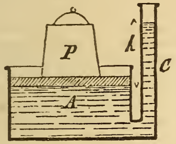


272.

$$P = 62{\cdot}5\ SAd,$$

$$A = \frac{P}{62{\cdot}5\ S\ d},$$

$$d = \frac{P}{62{\cdot}5\ S\ A}.$$

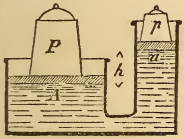


273.

The Hydrostatic paradox.

The pressure P is independent of the width of column C.

$P = 62{\cdot}5\ S\ A\ h$. (same as above.)

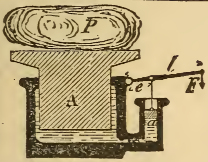


274.

$$P = A\left(62{\cdot}5 S h + \frac{p}{a}\right)$$

$$p = a\left(\frac{P}{A} - 62{\cdot}5 S h\right),$$

$$h = \frac{P\ a - p\ A}{62{\cdot}5\ SA\ a}$$

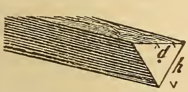


275. *Bramah's Hydraulic Press.*

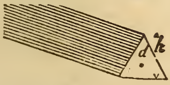
$$P = \frac{F\ l\ A}{e\ a}, \qquad A = \frac{P\ e\ a}{F\ l},$$

$$F = \frac{P\ e\ a}{A}, \qquad a = \frac{F\ A\ l}{P\ e}.$$

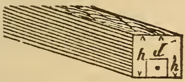
276. *Centre of Pressure of a rectangle, the upper edge at the surface of the liquid* $d = \frac{2}{3}\ h$.



277. *Centre of Pressure of a triangle, the base being at the surface of the liquid,* $d = \frac{1}{2}\ h$.



278. *Centre of Pressure of a triangle, the vertex being at the surface of the liquid.* $d = \frac{3}{4}\ h$.



279.

$$d = \frac{h}{3} + \sqrt{4\,(h - h')^2 + h^2}.$$

STABILITY OF VESSELS IN WATER.

Letters denote.—D = displacement of the vessel in pounds. $\mathfrak{X}$ = greatest immersed section in square feet. B = breadth of beam in feet in the water-line. L = length of the vessel in feet in the water-line. a = the vertical distance in feet between the centres of gravity of the vessel and displacement when in equilibrium. When the centre of gravity of the vessel c is below that of the displacement e, as in Fig. 1, then a is positive, or $+a$; and when c is above e, as in Fig. 2, a is negative, or $-a$. b = horizontal distance in feet from the centre of gravity of the displacement when in equilibrium to the same centre when out of equilibrium. d = depth of the centre of gravity of the displacement under water-line in feet when in equilibrium; and δ = depth of the same centre when out of equilibrium. f = force of wind in pounds per square inch (see page 233). h = vertical height in feet from the centre of gravity of the displacement to the centre of the weight W, Fig. 281, when the vessel is in equilibrium. r = horizontal distance in feet from the centre of the vessel to the centre of the weight W, Fig. 281. l = leverage in feet upon which any force acts to careen the vessel, to be calculated from the centre of gravity of the displacement, perpendicular to the direction of the careening force. In sailing, l is taken from the centre of gravity of the displacement to the centre of effort of the sails. m = vertical distance in feet from the centre of gravity of the displacement when out of equilibrium to the metacentre m. v = careen angle of the vessel. u = angle of the sails to the length of the vessel. z = angle of the wind to the sails. $\oplus$ = area of resistance of the vessel in sq. ft. (see page 274). A = area of all the sails in square feet. M = miles or knots per hour, by sailing. F = force in pounds acting to propel the vessel forward. W = any weight or force in pounds acting on the level l to careen the vessel.

EXAMPLE 1.—The U. S. steam frigate Niagara is $L = 329$ ft. long; $B = 55$ ft. wide; greatest immersed section, $\mathfrak{X} = 855$ sq. ft.; displacement, $D = 11{,}200{,}000$ pounds; vertical distance between the centres of gravity of displacement and vessel assumed to be $-a = 2.5$ ft. What momentum ($Wl = ?$) is required to careen her to an angle of $v = 8°$, and what force ($W = ?$) is required on a lever of $l = 35$ feet?

$$\text{Formula 1.}\quad b = \frac{55^3 \times \tan. 8°}{12 \times 855}\sqrt[3]{\frac{11{,}200{,}000}{64.3 \times 329 \times 855}} = 1.945 \text{ feet.}$$

The required careen momentum will be

$$\text{Formula 2}\quad Wl = 11{,}200{,}000\ (1.945 - 2.5 \sin. 8°) = 17{,}887{,}520 \text{ foot pounds,}$$

and the force $W = \dfrac{17{,}887{,}520}{35} = 511{,}072$ pounds $= 228$ tons.

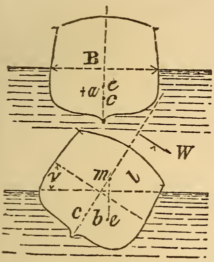
EXAMPLE 2.—It is required to find the momentum of stability of a man-of-war, by moving a number of guns of known weight from one side to the other. Each gun weighs 25,000 pounds, and four guns are moved to the opposite side, to $r = 20$ feet from the centre of the vessel; the height of the centre of gravity of the guns above the centre of gravity of displacement is $h = 16$ feet. There will be eight guns of 25,000 pounds, or $W = 200{,}000$ pounds careen weight on one side, by which the vessel is careened to an angle of $v = 7° 20'$. Dimensions of the vessel are $D = 6{,}150{,}000$ pounds, $B = 40$ feet, $L = 260$, and $\mathfrak{X} = 566$ square feet. Required the vertical distance between the centres of gravity of the vessel and displacement, $a = ?$

$$\text{Formula 1.}\quad b = \frac{40^3 \times \tan. 7° 20'}{12 \times 566}\sqrt[3]{\frac{6{,}150{,}000}{64.3 \times 260 \times 566}} = 1{,}057 \text{ feet.}$$

$$\text{Formula 6.}\quad l = \sin. 7°20'(16 - 20 \times \sin. 7° 20') + 20 \sec. 7° 20' - 1.057 = 21.36 \text{ ft.}$$

$$\text{Formula 5.}\quad \pm a = \frac{1}{\sin. 7° 20'}\left(\frac{200{,}000 \times 21.09}{6{,}150{,}000} - 1.057\right) = -\ 2.84 \text{ feet.}$$

a is negative when $\dfrac{Wl}{D} < b$.



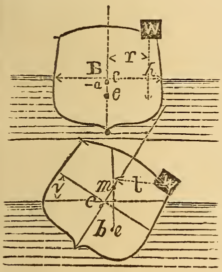
280.

$$b = \frac{B^3 \tan. v}{12 \mathfrak{X}} \sqrt[3]{\frac{D}{64.3^* \, L \, \mathfrak{X}}} \quad . \quad 1$$

$$W l = D\,(b \pm a \sin. v), \quad . \quad . \quad . \quad 2$$

Meta-centre $m = \frac{B^3}{12 \, \mathfrak{X}} \sqrt[3]{\frac{D}{64.3^* L \mathfrak{X}}}$ 3

Depth $\delta = d \cos. \tfrac{1}{2} v$, ... 4



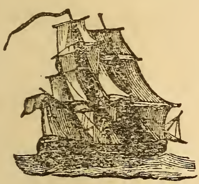
281.

$$\pm a = \frac{1}{\sin. v}\left(\frac{W l}{D} - b\right), \quad . \quad . \quad . \quad 5$$

$$l = \sin. v\,(h - r \sin. v) + r \sec. v - b, \quad 6$$

$$\cot. v = \frac{D}{W l}\,(m \pm a), \quad . \quad 7$$

Capsizes when $a \sin. v =$ or $> b$, 8

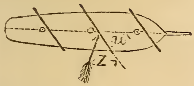


282.

Careen force $W = f A \sin. z \cos. u$, 9

Sailing force $F = f A \sin. z \sin. u$, 10

Miles per hour $M = \sqrt{\frac{F}{6 \oplus}}$, . 11



283.

Force of wind $f = \frac{4 \oplus M^2}{A \sin. z \sin. u}$, 12

* 64.3 for salt and 62.5 for fresh water.

HYDRAULICS.

Let the vessel A, Fig. 284, be kept constantly full of water up to the water line w. In two horizontal faces lower than the water line w, are made orifices a and a', through which the water will pass up vertical nearly to the water line w. Omitting the resistance of air, &c., the jet should theoretically reach the water line w; practically it reaches $0{\cdot}967h$.

It is evident that the velocity of the *jet* through the orifices, must be the velocity due to a body falling the height h, according to the law of force of gravity.

Letters denote.

Q = actual quantity of water discharged per second or in the time t, in cubic feet.
h = head, or height of water over the orifice.
t = operating time in seconds.
a = area of the orifice in square feet.
m = the coefficient for contraction. (See Fig. 299)
G = gallon of 231 cubic inches discharged in the time t.
V = velocity through the orifice in feet per second.

Example 1. Fig. 284. How many gallons of water will be discharged in five minutes, through an orifice of 0·025 square feet, applied at 8 feet under the level of the water?

$$G = 37{\cdot}75 a\, t \sqrt{h} = 37{\cdot}75 \times 0{\cdot}025 \times 5 \times 60 \sqrt{8} = 800 \text{ gallons.}$$

Fig. 285. The weight P can represent the weight of a column of water whose

height $= \dfrac{P}{62{\cdot}5A} = \dfrac{h'}{0{\cdot}967}$, acting on the area A.

Fig. 286. n = number of down strokes per minute, s = stroke of piston; the air vessel $C = 6A\,s$ at the pressure of the atmosphere.

Example 2. Fig. 286. How many double strokes must be made per minute by the lever of a hand pump, to throw up 22 cubic feet of water 18 feet high, in the time of 8 minutes and 15 seconds; the levers $l = 30$ inches, $e = 8$ inches, $s = 0{\cdot}6$ feet, $F = 20$ pounds? $8 \times 60 + 15 = 495$ seconds.

$$n = \frac{3630\,Q\,h'\,e}{t\,s\,F\,l} = \frac{3630 \times 22 \times 18 \times 8}{495 \times 0{\cdot}6 \times 20 \times 30} = 64{\cdot}5 \text{ strokes per minute.}$$

Example 3. Fig. 294. A vessel of rectangular form is of dimensions $A = 6$ square feet, the height $h = 5$ feet. What time will it take the water level to sink 2 feet, when the orifice $a = 0{\cdot}212$ square feet.

$$t = \frac{A\,(h - h')}{2{\cdot}52a(\sqrt{h} + \sqrt{h'})} = \frac{6(5-3)}{2{\cdot}52 \times 0{\cdot}212(\sqrt{5} + \sqrt{3})} = 5{\cdot}66.$$

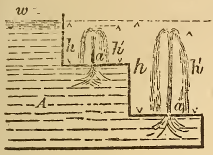
Motion of Water in Pipes,

Letters denote.

L = extreme length of the pipe in feet.
d = inside diameter in feet, and uniform throughout the length L.

Example 4. Fig. 287. What will be the velocity of the water through a pipe of 0 45 feet inside diameter, and $L = 68$ feet long, the head pressure of water being $h = 8$ feet?

$$V = 48 \sqrt{\frac{0{\cdot}45 \times 8}{68 + 50 \times 0{\cdot}45}} = 9{\cdot}6 \text{ feet per second.}$$



284.

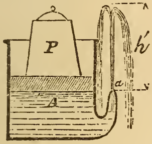
$$V = 8{\cdot}02\ \sqrt{h}$$

$$Q = m\,a\,t\,8{\cdot}02\ \sqrt{h} = 5{\cdot}05\ a\,t\ \sqrt{h}$$

$$G = 37{\cdot}77\ a\,t\sqrt{h}, \qquad m = 0{\cdot}63,$$

$$jet = 0{\cdot}967\ h,$$

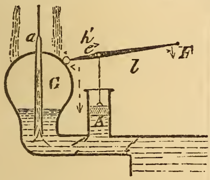
$$h = \frac{Q^2}{25{\cdot}5\ a^2\,t^2},$$



285.

$$V = 1{\cdot}015\sqrt{\frac{P}{A}}, \quad Q = a\,t\sqrt{\frac{P}{A}},$$

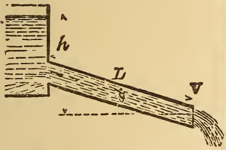
$$G = 7{\cdot}5\,a\,t\sqrt{\frac{P}{A}}, \quad h' = \frac{P}{60{\cdot}5\ A}.$$



286.

$$Q = a\,t\sqrt{\frac{F\,l}{A\,e}}, \quad n = \frac{3630\ Q\ h'\ e}{t\ s\ F\ l},$$

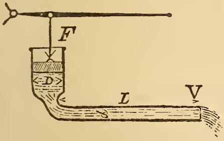
$$G = 7{\cdot}5\ a\,t\sqrt{\frac{F\,l}{A\,e}}, \quad h' = \frac{F\,l}{60{\cdot}5\ A\ e}.$$



287. *Motion of Water in Pipes.*

$$V = 48\sqrt{\frac{d\,h}{L+50d}}, \quad Q = 37{\cdot}7d^2\sqrt{\frac{d\,h}{L+50d}},$$

$$d = 0{\cdot}24\sqrt[5]{\frac{L\,Q^2}{h}}, \qquad h = \frac{Q^2\,(L+5\,d)}{142\ d^5}.$$

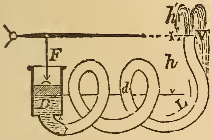


288.

Motion of Water in Pipes.

$$V = 6{\cdot}86\sqrt{\frac{d\ F}{D(L+50d)}}, \quad Q = 5{\cdot}38d^2\sqrt{\frac{d\ F}{D(L+50d)}}$$

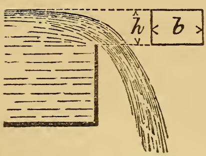
$$d = 1{\cdot}68\sqrt[5]{\frac{L\,D\,Q^2}{F}}, \qquad F = \frac{Q^2D(L+5d)}{2{\cdot}9d^5}.$$



289.

$$V = 6{,}86\sqrt{\frac{d(F-49D^2h)}{D(L+50d)}} \qquad Q = 5.38\,Vd^2t$$

$$h' = \frac{V^2}{66{\cdot}5}, \qquad h' = \frac{D\sqrt{s\,n}}{57{\cdot}65d}.$$

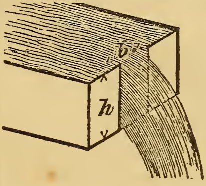


290.

Weirs.

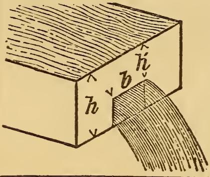
$Q = k\,b\,t.$ See Table for *Weirs*.

$$t = \frac{Q}{k\,b}, \quad b = \frac{Q}{k\,t},$$



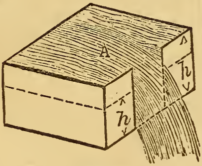
291.

$$Q = 5{\cdot}35m\,b\,h\,t\sqrt{h},$$
$$G = 40m\,b\,h\,t\sqrt{h},$$
$$t = \frac{Q}{5{\cdot}35\,m\,b\,h\sqrt{h}},$$



292.

$$Q = 5{\cdot}35m\,b\,t(h\sqrt{h} - h'\sqrt{h'}),$$
$$G = 40m\,b\,t(h\sqrt{h} - h'\sqrt{h'}),$$
$$t = \frac{Q}{5{\cdot}35m\,b(h\sqrt{h} - h'\sqrt{h'})},$$

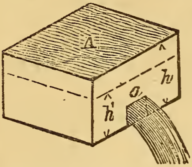


293.

$$t = \frac{0{\cdot}95m\,A(\sqrt{h} - \sqrt{h'})}{b\sqrt{h\,h'}},$$

A = area of the vessel in square feet.

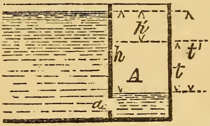
t = time in seconds, in which the water level will sink the space $h - h'$.



294.

$$t = \frac{A(h - h')}{4m\,a\,(\sqrt{h} + \sqrt{h'})},$$

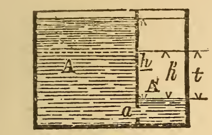
$$Q = 4m\,a\,t(\sqrt{h} + \sqrt{h'}),$$



295.

$$t = \frac{A}{3{\cdot}85a\,m}(\sqrt{h} - \sqrt{h'}),$$

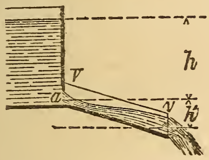
$$a = \frac{A\sqrt{h}}{3{\cdot}85t'\,m}, \quad t' = \frac{A\sqrt{h}}{3{\cdot}85am},$$



296.

$$t = \frac{A\,A'\sqrt{h}}{13{\cdot}7\,m\,a\sqrt{A+A'}},$$

$$h = \frac{A\,h'}{A + A'},$$

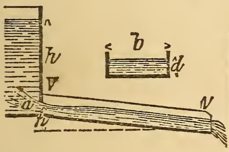


297. *Short Drain.*

$$V = \frac{8{\cdot}02\sqrt{h}}{\sqrt{1+(\frac{1}{m}-1)^2}},$$

$$v = \sqrt{V^2 + 32{\cdot}1\,h'},$$

$Q = a\,m\,V\,t$. *from* V *to* v *about* $6\sqrt{a}$,

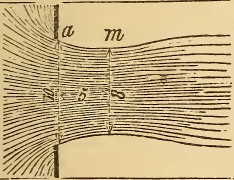


298. *Long Drain.*

$$V = \frac{8{\cdot}02\sqrt{h}}{\sqrt{1+(\frac{1}{m}-1)^2}},$$

$$v = \sqrt{V^2 + 64{\cdot}32h' - 0{\cdot}007\frac{s\,l\,V^2}{a}}$$

$s = b + 2d$ $\quad$ $l = V$ *to* v, *feet.*



299. *Proportions of the contracted Vein.*

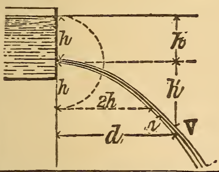
$a : m = 10^2 : 8^2$. $\quad$ $m = 0{\cdot}64\,a$.

$m = 0{\cdot}64$ *when contracted on* 4 *sides.*

$m = 0{\cdot}72$ " " " 3 *sides.*

$m = 0{\cdot}8$ " " " 2 *sides.*

$m = 0{\cdot}9$ " " " 1 *side.*



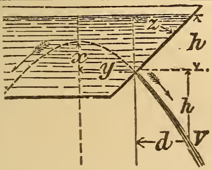
300.

The form of the Vein is a Parabola.

$d = 2\sqrt{hh'}$, $\quad$ $V = 8\sqrt{h + h'}$

$Q = 8\,m\,a\,t\sqrt{h}$,

$\tan v = \frac{2\,h'}{d}$, $\quad$ $h = \frac{d^2}{4\,h'}$.



301.

$x = \sin^2 z\,h$ $\quad$ $y = \sin. z \cos. z\,h$.

$d = 2\sqrt{(h' + x)(h - x)} - y$

$Q = 8\,m\,a\,t\sqrt{h}$ $\quad$ $V = 8\sqrt{h + h'}$.

Example 5. Fig. 289. Required the velocity and quantity of water discharged in a long pipe or hose of $L = 135$ feet long, and $d = 0{\cdot}17$ feet, attached to a hand-pump of $D = 0{\cdot}2$ feet in diameter $P = 44$ pounds, and the end of the pipe elevated $h = 20$ feet above the piston D?

$$V = 6{\cdot}86\sqrt{\frac{0{\cdot}17(44 - 49\times0{\cdot}2^2\times20)}{0{\cdot}2(135+50\times0{\cdot}17)}} = 1{\cdot}95 \text{ feet per second.}$$

$Q = 1{\cdot}95\times5{\cdot}38\times0{\cdot}2^2 = 0{\cdot}042$ per second$\times60 = 2{\cdot}52$ cubic feet per minute.
$s = 0{\cdot}8$ feet the stroke of piston, we shall have

$$n = \frac{2{\cdot}52}{0{\cdot}8\times0{\cdot}785\times0{\cdot}2^2} = 100 \text{ strokes per minute.}$$

Table for Water flowing over Weirs.

This Table is set up from careful experiments on a large scale, and is suited for *weirs* only. See Fig. 290.

RULE. Multiply the width b in feet, of the *weir* by the coefficient k, and the product is the quantity of water discharged per second, in cubic feet. h is the height as represented by Fig. 290. The width b should be $b > h$.

Example 6. How much water will flow over a wier of $b = 5$ feet, $h = 0{\cdot}5$ feet in one minute?

$Q = k\,b\,t = 1{\cdot}1295\times5\times60 = 338{\cdot}35$ cubic feet.

h. inches.	*h.* feet.	*m.*	*k.*
0·4	0·033	0·424	0·01365
0·8	0·066	0·417	0·05452
1·2	0·100	0·412	0·10592
1·6	0·133	0·407	0·16616
2·4	0·200	0·401	0·29171
3·2	0·266	0·397	0·44480
4·	0·333	0·395	0·63111
6·	0·500	0·393	1·1295
8·	0·666	0·390	1·7464
9·	0·750	0·385	2·0331
12	1·000	0·376	3·1350

HYDRODYNAMICS.

Water Power.

THE natural effect concentrated in a fall of water, is equal to the weight of the quantity of water passed through per second multiplied by the vertical space it falls.

Fig. 297. Let Q be the quantity of water which passes through the orifice a in the time $t = 1''$ second, in cubic feet of 62·5 pounds each.

h = the vertical space the water falls; then the value or natural effect of the fall is at the orifice a.

$$P = 62{\cdot}5Q\,h, \text{ effects.}$$

But, $Q = 5{\cdot}06a\sqrt{h}$, then we have

$$P = 315{\cdot}5\,a\,h\sqrt{h}.$$

This will be in horse-power,

$$H = 0{\cdot}573\,a\,h\sqrt{h}, \qquad h = \frac{1}{1{\cdot}07}\sqrt[3]{\frac{H^2}{a^2}},$$

$$H = 0{\cdot}1134Q\,h, \qquad h = \frac{H}{0{\cdot}1134Q}.$$

Example 1. In a creek passes 18 cubic feet of water per second. How high must that creek be dammed up to produce an effect of 10 horses?

$$h = \frac{10}{0{\cdot}1134\times18} = 4{\cdot}9 \text{ feet, the answer.}$$

WATER-WHEELS.

Water-wheels are of two essential kinds, namely, *Vertical* and *Horizontal.*
The *Vertical* are subdivided into
Overshot-wheels, Undershot-wheels, Breast-wheels, and *High-breast* and *Low-breast wheels.*
The *Horizontal* are *with Floats, Screw-wheels, Turbin, Reaction-wheels, &c.*
Waterwheels do not transmit in full the natural effect concentrated in a fall of water; under most favourable circumstances 80 per cent. has been utilized, but under poor arrangements only 20 per cent. may be expected.

Example 1. Fig. 302. The vertical section of the immersed floats of an undershot-wheel in a mid-stream is $\mathbf{a} = 27$ square feet, velocity of the stream $V = 8{\cdot}6$, and $v = 4$ feet per second. Required the horse-power of the wheel $H = ?$

$$H = \frac{\mathbf{a}\ v}{200}(V - v)^2 = \frac{27 \times 4}{200}(8{\cdot}6 - 4)^2 = 11{\cdot}4 \text{ horses.}$$

Example 2. Fig. 307. On a breast-wheel is acting $Q = 88$ cubic feet of water per second, the head $h = 8$ feet, velocity of the wheel at the centre of the buckets $v = 5$ feet per second; the water strikes the buckets at an angle $u = 8^{\circ}$ and velocity $V = 7$ feet per second. Required the horse-power of the wheel, $H = ?$

$$H = \frac{88}{11{\cdot}4}\left(8 + \frac{5}{25}(7 \times \cos.8^{\circ} - 5)\right) = 65 \text{ horses.}$$

Example 3. Required the effect of Poncelet's wheel, the head $h = 4$ feet, and the orifice $a = 5$ square feet, the velocity of the wheel at the centre of pressure of the floats is $v = 6{\cdot}78$ feet per second?

$$V = 6{\cdot}91\sqrt{4} = 13{\cdot}82 \text{ feet per second.}$$

$$Q = 6{\cdot}5 \times 5 \times \sqrt{4} = 65 \text{ cubic feet per second.}$$

$$H = \frac{65 \times 6{\cdot}78}{197}(13{\cdot}82 - 6{\cdot}78) = 15{\cdot}8 \text{ horses.}$$

Example 4. Fig. 309. A saw-mill wheel is to be built under a fall of $h = 18$ feet, and to make $n = 110$ revolutions per minute. Required the proper diameter of the wheel.

$$D = \frac{100}{110}\sqrt{18} = 3{\cdot}857 \text{ feet,}$$

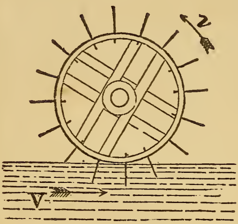
at the centre of pressure of the buckets.
Velocity $V = 8\sqrt{18} = 33{\cdot}94$ feet per second.

Velocity $$v = \frac{3{\cdot}14 \times 3{\cdot}857 \times 110}{60} = 22{\cdot}2 \text{ feet per second.}$$

The fall discharged 30 cubic feet of water per second. Required the horse-power of the wheel. $H = ?$

$$H = \frac{30 \times 22{\cdot}2}{200}(33{\cdot}94 - 22{\cdot}2) = 39 \text{ horses.}$$

How many square feet of dry Pine can it saw per hour?
See page 150. $30 \times 39 = 1170$ square feet.
The saw is meant to be applied direct on the wheel shaft.



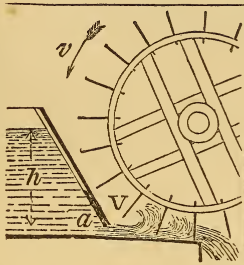
302.

Undershot wheel in a mid-stream.

$$H = \frac{\mathbf{a}\, v}{200}(V - v)^2,$$

When $V = 2v$ about, the effect will be,

$$H = \frac{\mathbf{a}\, V^3}{1600}, \quad \mathbf{a} = \text{area of float.}$$



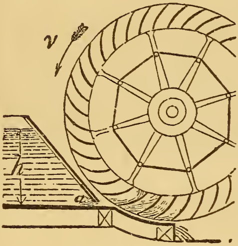
303.

Undershot-Wheel.

$$H = \frac{Q\, v}{454}(V - v),$$

$$H = \frac{m\, a\, v}{86{\cdot}8}(V - v)\sqrt{h},$$

When $V = 2v$, about, $H = \dfrac{a\, h\sqrt{h}}{0{\cdot}47}$.



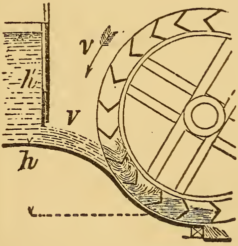
304.

Poncelet's Wheel.

$$H = \frac{Q\, v}{228}(V - v), \text{ when } h > 5 \text{ feet},$$

$$H = \frac{Q\, v}{197}(V - v) \text{ when } h < 5 \text{ feet},$$

$$Q = 8m\, a\sqrt{h}, \qquad V = 6{\cdot}91\sqrt{h}.$$

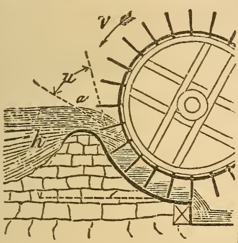


305.

Breast-Wheel with Parabolic drain.

$$H = \frac{Q}{12}\left[h + \frac{v}{28}(V - v)\right],$$

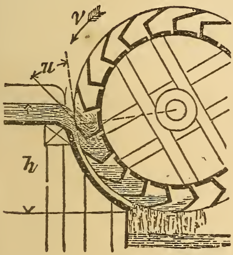
$$Q = 6{\cdot}5a\sqrt{h}.$$



306. *Low-breast Wheel.*

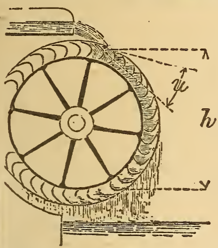
$$H = \frac{Q}{11{\cdot}2}\left[h + \frac{v}{32}(V\cos u - v)\right]$$

$Q = kb.\ V = \dfrac{Q}{a}\cdot$ See table for weirs.



307. *Breast Wheel.*

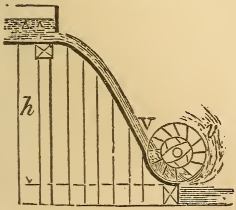
$$H = \frac{Q}{11{\cdot}4}\left[h + \frac{v}{25}(V\cos u - v)\right]$$



308. *Over-shot Wheel.*

$$H = \frac{Q}{13{\cdot}7}\left[h + \frac{v}{21{\cdot}5}(V\cos u - v)\right]$$

Proper velocity about $n = \dfrac{35\,D + 100}{D}$, *revolutions per minute.*



309. *Saw-Mill Wheel.*

$$H = \frac{Q\,v}{200}(V - v)$$

Proper diameter of the Wheel,

$D = \dfrac{100}{n}\sqrt{h}$, *in feet,*

n = *revolutions per min.*

TURBINES.

Letters denote.

Q = cubic feet of water passed through the turbine per second.
h = height of fall in feet.
D = diameter in inches of circle of effort in the turbine.
a = area in sq. in. of the conduit passage into the turbine wheel.
b = depth in inches of turbine buckets.
c = depth in inches of leading buckets.
r = breadth of turbine buckets in inches.
m = number of buckets in the turbine wheel.
m' = number of leading buckets.
n = number of revolutions of turbine per minute.
S and s = height of conduit and discharge in inches.
t = thickness of steel plate buckets in 16ths of an inch.
H = actual horse power of the turbine.
l = length in feet, d = diameter in inches } of conduit pipe.
d' = diameter in inches of the discharge pipe.
W = Hydraulic pressure on the turbine wheel bearing on the end of the shaft.

$$D=\frac{k\sqrt{h}}{n}, \quad 1$$
$$D=\frac{a}{0{\cdot}436\,r}, \quad 2$$
$$n=\frac{k\sqrt{h}}{D}, \quad 3$$
$$n=\frac{20\,k\,Q}{a\,D}, \quad 4$$
$$r=\frac{a}{0{\cdot}436D}, \quad 5$$
$$r=\frac{46\,k\,Q}{D^2n}, \quad 6$$
$$r=\frac{D}{5}\text{ to }\frac{D}{8}, \quad 7$$
$$t=\frac{m}{10}, \quad 8$$
$$a=\frac{20\,Q}{\sqrt{h}}, \quad 9$$
$$a=\frac{20\,k\,Q}{D\,n}, \quad 10$$
$$a=0{\cdot}436\,Dr, \quad 11$$
$$a=m'rs, \quad 12$$
$$a'=mrs, \quad 13$$
$$a'=0{\cdot}98\,a, \quad 14$$
$$Q=\frac{a\sqrt{h}}{20}, \quad 15$$
$$Q=\frac{a\,D\,n}{20\,k}, \quad 16$$
$$m=5\sqrt{D}, \quad 17$$
$$m'=4{\cdot}5\sqrt{D}, \quad 18$$
$$b=\frac{0{\cdot}625\,D}{\sqrt{m}}, \quad 19$$
$$c=\frac{0{\cdot}78\,D}{\sqrt{m'}}, \quad 20$$
$$s=0{\cdot}86\,S, \quad 21$$
$$d=D+r+\sqrt[3]{l}, \quad 22$$
$$d'=D+2\,r, \quad 23$$
$$W=\frac{D^2h}{3}, \quad 24$$

$$H=0{\cdot}1134\,Q\sqrt{h}\ \text{natural effect of the fall,} \quad 25$$

$$\left.\begin{array}{l} H=\dfrac{30\,Q^3}{a^2}, \\[2ex] H=\dfrac{a\,h\sqrt{h}}{267{\cdot}5}, \end{array}\right\} \text{actual horse power, 66 per cent of the natural.} \quad 26, 27$$

The coefficient k can vary from 800 to 1200 without seriously affecting the per centage of the ultilized power, but it is best between 900 and 1000. This is a great advantage of the turbine over water wheels, that under the same head of fall it can run at different velocities and still utilizing the maximum effect. Whatever coefficient k adopted it must be kept the same throughout the construction of the turbine.

Jonval's Turbine has so many advantages above other hydraulic motors that it is considered sufficient to describe the construction of that one only, but the principal formulas will answer for any kind of turbines.

On the accompanying plate is a drawing of a Jonval Turbine such as the Author of this Pocket Book has built in Russia. The buckets are not supported by concentric rings, but are fastened only on one side, which is considered more simple and convenient for replacing new buckets. For falls over 30 feet it may be better to make it with concentric rings. When a turbine is to be constructed we have on the one side given the natural effect of the fall, and on the other side the actual work to be done, which latter should not exceed 66 per cent. of the former. Between these two points the turbine is to be so proportioned as to utilize the greatest possible effect with smallest expense of Machinery.

Jonval's turbine in good condition generally utilizes 60 to 80 per cent. Suppose a fall of $h=25$ feet, discharging $Q=12$ cubic feet of water per second, the natural effect will be,

$$H=0{\cdot}1134\times12\times\sqrt{25}=6{\cdot}8 \text{ horses,}$$

of which $6{\cdot}8\times0{\cdot}66=4{\cdot}5$ horses to be counted upon as the actual effect of the turbine.

Turbine shaft to make $n=200$ revolutions per minute with the assumed coefficient $k=960$. From these dates we will obtain all the principal dimensions of the turbine, namely,

$$D=\frac{960\sqrt{25}}{200}=24 \text{ inches.} \quad - \quad - \quad 1$$

$$a=\frac{20\times960\times12}{24\times200}=48 \text{ sq. in.} \quad 10$$

$$m=5\sqrt{24}=24{\cdot}5 \text{ say } 25. \quad - \quad - \quad 17$$

$$m'=4{\cdot}5\sqrt{24}=22 \text{ buckets.} \quad - \quad 18$$

$$r=\frac{48}{0{\cdot}436\times24}=4{\cdot}6 \text{ in.} \quad - \quad - \quad - \quad 5$$

$$b=\frac{0{\cdot}625\times24}{\sqrt{25}}=3 \text{ in.} \quad - \quad - \quad - \quad - \quad 19$$

$$c=\frac{0{\cdot}78\times24}{\sqrt{22}}=4 \text{ inches.} \quad - \quad - \quad 20$$

$$t=\frac{25}{10}=2{\cdot}5, \text{ 16ths.} \quad - \quad - \quad - \quad - \quad 8$$

In calculating the breadth r from formula 5, it must come inside of formula 7, if not the diameter D must be altered.

Now proceed with the construction as shown at the bottom of the plate, which represents a section of the buckets through the circle of effort of the turbine.

The drawing of the turbine is ¾ of an inch to the foot, and the construction of the buckets 3 inches to the foot.

Draw the base line AB, set off the angle of the leading buckets$=10°$. The distance between the leading buckets will in this case be $24\times3{\cdot}14:22=3{\cdot}43$ inches, set off this from S towards A, draw the straight part of the second bucket parallel to the first one, draw from S the line $d\ d$ at right angle to the buckets, and e will be the centre for the curved part. From the centre of S draw the line o to the end of the second buckets, divide this line into eight equal parts take five of them as radus and draw from the end of the second bucket a circlearc of about 50°, which will be the propelling part of the turbine wheel bucket.

Distance between the wheel buckets will be $24\times3{\cdot}14:25=3{\cdot}02$ inches, set off this from A towards S, draw the second propelling arc. Set off from A the depth of the wheel buckets $b=3$ inches, set off 2 b to s, which will be the length of the first wheel bucket. Set off from s to u the distance between the buckets 3·02 inches. Make $s=0{\cdot}86\ S$. Draw from u a curved line in the form of a parabola that will leave the space s and tangent the propelling circlearc somewhere about x. Care must be taken that the discharging area a' of all the wheel buckets will be about 2 per cent. less than the conduit area a of all the leading buckets. The surface of the buckets should be made as smooth as possible, or even polished.

For very high falls the Hydraulic pressure W becomes very considerable

and may necessitate another arrangement, namely, to lay the shaft horizontally and place on it two turbines so that the leading buckets are either between or outside of the wheels, but then comes another disadvantage, namely, that the number of revolutions will be greatly increased and may be required to gear it down 10 to 20 times to the proper speed of the main shaft.

To avoid this as much as possible take $k=800$ and make $r=\frac{D}{8}$.

One great advantage with Jonval's turbine is that it can be placed almost anywhere between the high and low levels to suit the location, though it should not be more than 20 feet above the lower level; then in order to utilize the whole fall, care must be taken to make the discharge pipe perfectly air tight. It is not necessary to make the discharge straight down from the turbine, it can be carried horizontally or inclined, as may suit the location. The Author has built turbines similar to that represented on the accompanying plate, at General Maltzof's Establishment, Kaluga, Russia.

Velocity of Water in Rivers.

The velocity of the water at the bottom in rivers is to that at the surface, as 8 is to 10.

MOTION OF WATER IN PIPES.

For City Water works. Du Buat's formula.

Letters denote.

Q = cubic feet of water passed through the pipe per minute.
D = inside diameter of the pipe in feet.
L = length of the pipe in feet increased by 50 diameters.
H = differential head in feet.
v = velocity of the water in the pipe in feet per minute.

$$Q=\frac{2356\sqrt{D^5}}{\sqrt{\frac{L}{H}}}. \qquad D=\frac{1}{22{\cdot}329}\sqrt[5]{\frac{Q^2L}{H}}, \qquad v=\frac{3000\sqrt{D}}{\sqrt{\frac{L}{H}}}.$$

Example 1. A water pipe of $D=1{\cdot}75$ feet in diameter, $L=36{,}000+50\times1{\cdot}75=36087{\cdot}5$ feet long, head pressure $H=390$ feet. Required how much water it can discharge per minute?

$$Q=\frac{2356\sqrt[3]{1{\cdot}75^5}}{\sqrt{\frac{36087{\cdot}5}{390}}}=992{\cdot}26 \text{ cubic feet.}$$

Example 2. At a distance of 27960 feet from a water work is required $Q=564$ cubic feet of water per minute, head pressure being $H=256$ feet. Required the diameter of the pipe? $L=27960+50=28010$ feet.

$$D=\frac{1}{22{\cdot}329}\sqrt[5]{\frac{564^2\times28010}{256}}=1{\cdot}4436 \text{ feet.}$$

Example 3. A water pipe of $D=0{\cdot}75$ feet in diameter, $L=8650+50=8700$ feet, have a head pressure of $H=128$ feet. Required the velocity $v=?$ of the discharge.

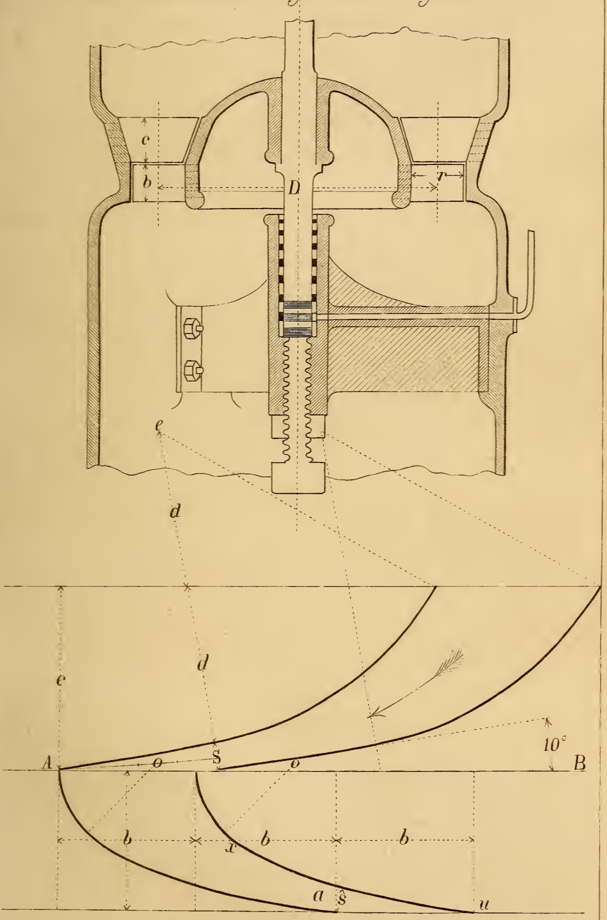
$$v=\frac{3000\sqrt[3]{0{\cdot}75}}{\sqrt{\frac{8700}{128}}}=41{\cdot}424 \text{ feet per second.}$$

Consumption of water in cubic feet per head of population, including all uses, as for manufactories, fires, &c., &c., in 24 hours.

January,	2·58,	April,	2·73	July,	4·58	October,	4·46
February,	2·40,	May,	3·37	August,	4·75	November,	4·12
March,	2.64,	June,	3·50	Sept.,	4·61	December,	3·61

JONVAL'S TURBINE,

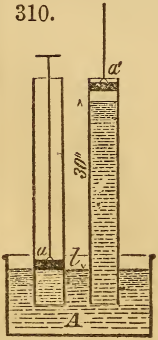
as constructed by John W. Nystrom.



ATMOSPHERE. AEROSTATIC.

The atmosphere round our earth, as well as all other gaseous matters, endeavours to occupy a larger space to infinity, (no known limits,) but as it is a material substance, it is under the action of force of gravity, and cannot expand farther than when its density is in equilibrium with the said force. Conceive the atmosphere to consist of a great number of layers, one on the top of the other; the density of the under layers will evidently be greatest, because the upper ones press on them, and they are all elastic; hence the density of the atmosphere is greater at the surface of the earth than higher up. We can now find out the weight and density of all these layers.

310.



A is a vessel full of mercury, in which is placed vertically a glass tube about 3 feet high above the surface l; in the glass tube is fitted, air-tight, piston a, just one square inch area, which can be moved by the piston-rod c; now the piston stand is at a on the level l, and in contact with the mercury in the tube; raise the piston by the piston rod and handle c, the mercury in the tube will follow until the height of 30 inches, the piston still continues to move higher in the tube, but the mercury will maintain its position at 30 inches from l. Now it may be supposed that it is some force of the piston that draws the mercury up in the tube; if so why did it separate at 30 inches? If the column becomes too heavy it could separate at l, and the 30 inches of mercury follow the piston; as this is not the case, but the weight of the atmosphere pressing on the surface l and forcing the mercury up in the tube until it (the mercury and the atmosphere) comes in equilibrium, which occurs at the 30 inches; and the piston only served to remove the atmospheric pressure in the tube; hence we have the weight of a column of atmospheric air with one square inch base equal to the weight of a column of mercury 30 inches high and one sq. in. base. One cubic inch of mercury at 60° Fahr. weighs 0·491 pounds, this multiplied by the height, 30 inches, gives 14·73 pounds, the weight of the columns of mercury or atmosphere; this is generally termed "the atmospheric pressure per square inch."

The specific gravity of mercury at 60° Fahr. is 13·58, and

$$\frac{13{\cdot}58 \times 30}{12} = 33{\cdot}95$$ feet, the height of a column of water required to balance the atmosphere.

If the temperature and force of gravity were uniform throughout the atmosphere, the density would decrease in an arithmetical progression by the height from the ground, and by observing the altitude of columns of mercury at two different heights, the extreme height of the atmosphere would be found simply by the formula 7, page 64, in which

$a = o$ the altitude of the column of mercury at the top of the atmosphere.
$b = 30$ inches, the column of mercury at the level of the sea.
$\delta =$ the difference of the columns of mercury at the level of the sea, and at a height h above the sea.

Then, n multiplied by the height h, should be the extreme height of the atmosphere, or

$$H = h\left(\frac{b}{\delta} + 1\right) \quad \cdots \cdots \quad 1,$$

Example. The mountain Chimborazo, Eucador, (South America,) is $h = 3{\cdot}87$ miles high above the level of the sea; at its top the column of mercury is observed to be only 27·63 inches, and $\delta = 30 - 27{\cdot}63 = 2{\cdot}37$.

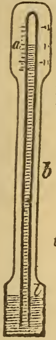
$$H = 3{\cdot}87\left(\frac{30}{2{\cdot}37}+1\right) = 52{\cdot}825 \text{ miles, the extreme height}$$

of the atmosphere.

This is about the true height, but the calculation is incomplete by lack of many circumstances accompanied with higher calcules in mathematics, which can not be allowed to occupy room in this work.

The column is 30 inches when the temperature of the atmosphere is 32° Fah. (See Barometer.)

311. BAROMETER.



THE **Barometer** is based upon the same principle as the preceding experiment. It consists of a glass tube b, about 35 inches high, open at one end, c, and filled with distilled mercury; inverted in a small vessel A, also containing mercury. About the top of the column of mercury is placed a scale to indicate the height from l. When disturbances take place in the atmosphere by heat, condensation, &c., its weight and density will differ, and the column of mercury will fall and rise accordingly; hence by connection of the scale at a, it indicates the disturbance.

To Find the Density of the Atmosphere about a Barometer,

Letters denote.

h = altitude of the column of mercury in inches.
t = temperature of the atmosphere, Fah.
S = specific gravity of the air around the Barometer at the time h and t are observed.

$$S = 1 \text{ when } t = 32^{\circ}.$$

$$S = \frac{h}{0{\cdot}0624(448{\cdot}77+t)} \quad \cdots\cdots \quad 2,$$

Example. The Barometer has fallen to 26·31 inches and the temperature is $t = 60^{\circ}$. Required the specific gravity of the air?

$$S = \frac{26{\cdot}31}{0{\cdot}0624(448{\cdot}77+60)} = 0{\cdot}83.$$

This formula does not include the expansion of mercury from 32° to 60°, which must be separately reduced by the following formula.

$$h = H(0{\cdot}9967962 - 0{\cdot}0001001t) \quad \cdots\cdots \quad 3,$$

in which H = the observed column at the temperature t, and h = the true column to be inserted in the formula 2.

To Measure Vertical Heights by the Barometer.

Letters Denote,

H = column of mercury, T = temperature of the air } at the lower station.
h = column of mercury, t = temperature of the air } at the higher station.
l = latitude of the place.
f = vertical height, in feet, between the higher and lower station.

$$x = \log.\left(\frac{H}{h} \times \frac{1}{1+0{\cdot}0001001(T-t)}\right), \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 4,$$

$$f = 60345{\cdot}51x\left(1+0{\cdot}002551 \cos.2l\right)\left(1+0{\cdot}00208(T+t-64^\circ)\right), \quad \cdot \quad \cdot \quad 5.$$

If the atmosphere is very calm the observations may be made one after the other by one Barometer and detached Thermometer; but the least disturbance of wind requires the observations at the upper and lower stations to be made at the same time. The reduction of the columns of mercury is included in the formula 5.

WIND. AREODYNAMIC.

THE motions and effects of gases by the force of gravity, are precisely the same as that of liquids. (See Hydraulics.)

The altitude or head of the atmosphere at uniform density will be the altitude of a column of water 33·95 feet, divided by the specific gravity of the air, 0·0012046, or,

$$\frac{33{\cdot}95}{0{\cdot}0012046} = 28183 \text{ feet},$$

the velocity due at the foot of this head is (Formula 1, page 183.)

$V = 8{\cdot}02\sqrt{28183} = 1346{\cdot}4$ feet per second, the velocity at which the air will pass into a vacuum.

Velocity of Wind.

When air passes into an air of less density, the velocity of its passage is measured by the difference of their density.

$\left.\begin{array}{l} H = \\ h = \end{array}\right\}$ density of the air in inches of mercury.

$t =$ temperature at the time of passage.

$V =$ velocity of the wind in feet per second.

$$V = 1346{\cdot}4\sqrt{\frac{H-h}{h}\left(1+0{\cdot}00208t\right)}, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 6.$$

The force of wind increases as the square of its velocity.

$\mathbf{a} =$ area exposed at right-angles to the wind, in square feet.

$F =$ force of the wind in pounds.

$H =$ horse-power.

$v =$ velocity of the plane $\mathbf{a}$ in direction of the wind, $+$ when it moves opposite, and $-$ when it moves with the wind.

$$F = 0{\cdot}002288\mathbf{a}\,V^2, \qquad \text{when } v = o, \quad - \quad \cdot \quad 7,$$

$$F = 0{\cdot}002288\mathbf{a}(V \mp v)^2, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 8,$$

$$H = \frac{\mathbf{a}v(V \mp v)^2}{241400}, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 9.$$

Example. A Rail-train running *ENE* 25 miles per hour, exposes a surface of 1000 square feet to a pleasant brisk gale *NE* by *E*. Required the resistance to the train in the direction it moves, and the horse-power lost?

$ENE - NE$ by $N = 3$ points $= 33^\circ\ 45'$.

$V = 14$ feet per second, a brisk gale.

$v = 25 \times 1{\cdot}467 = 36{\cdot}6$ feet per second.

$F = 0{\cdot}002288 \sin.^2 33^\circ\ 46' \times 1000(14+\cos.33^\circ\ 45' \times 36{\cdot}6)^2 = 305{\cdot}1$ pounds.

$$H = \frac{305{\cdot}1 \times 36{\cdot}6}{550} = 20 \text{ horses}.$$

Miles per hour.	*feet per second.*	*Force per sq. ft pound.*	*Common Appellation of the Force of Wind.*
1	1·47	0·005	Hardly perceptible.
2	2·93	0·020	Just perceptible.
3	4·4	0·044	
4	5·87	0·079	Gentle pleasant wind.
5	7·33	0·123	
6	8·8	0·177	
7	10·25	0·241	
8	11·75	0·315	Pleasant brisk gale.
9	13·2	0·400	
10	14·67	0·492	
12	17·6	0·708	
14	20·5	0·964	
15	22·00	1·107	
16	23·45	1·25	
18	26·4	1·55	Very brisk.
20	29·34	1·968	
25	36·67	3·075	
30	44·01	4·429	High wind.
35	51·34	6·027	
40	58·68	7·873	
45	66·01	9·963	Very high.
50	73·35	12·30	
55	80·7	14·9	
60	88·02	17·71	Storm or tempest.
65	95·4	20·85	
70	102·5	24·1	Great storm.
75	110	27·7	Hurricane.
80	117·36	31·49	
100	146·66	50·	Tornado, tearing up trees, &c.

BALLOON.

To Find what Weight, and to what Height a Balloon can raise.

Letters denote.

C = cubic contents of the balloon, in feet.
s = specific gravity of the gas used to inflate the balloon, air = 1 at 32°.
W = the weight in pounds, it can raise from the ground.
w = the weight with which it is loaded, including the weight of the materials of which it is made.
f = height in feet to which it will raise.
T = temperature at the ground.
t = temperature at the height f.
H = Barometer column in inches, at the ground.
l = latitude of the place.

$$W = 0{\cdot}07529C(1 - s), \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 10,$$

$$x = \log.\left(\frac{Hw}{W} \times \frac{1}{1 + 0{\cdot}0001001(T - t)}\right) \quad \cdot \quad \cdot \quad \cdot \quad 11,$$

$$f = 60345{\cdot}51x\left(1 + 0{\cdot}002551 \cos.2l\right)\left(1 + 0{\cdot}00208(T + t - 64)\right) \quad \cdot \quad 12.$$

Balloons are commonly filled with Hydrogen gas whose specific gravity is $s = 0{\cdot}07$, when pure, or about 14 times lighter than air, of which say 10 times to be relied upon, as some foreign heavier gases may accompany it.

WIND-MILLS.

The sail-shaft of vertical wind-mills should have an inclination from 12° to 15° with the level when built on low flat ground; on high ground, elevated from 1000 to 1500 feet within a circle of about two miles, the sail-shaft should incline from 3° to 6° with the level

Effect of Wind-Mills.

Letters denote.

A = projecting area of sails exposed to the wind, in square feet.
V = velocity of the wind in feet per second.
H = horse-power of the mill.
R = extreme, r = inner } radii of sails in feet.
$l = \sqrt{\frac{R^2+r^2}{2}}$, radius of centre of percussion in feet.
n = number of revolutions of sails per minute.
v = mean angle of sails to the plane of motion.

The angle of the sails should be from 20° to 30° at the inner radius r, at the extreme radius R from 7° to 12°, and the mean angle v = 15° to 17°.

$$H = \frac{A\, l\, n\, sin.v\, cos.v}{1{,}540{,}000}\left(V - \frac{b\, n\, sin.v}{9\cdot5}\right)^2$$

assume the mean angle v = 16°, we have the horse power.

$$H = \frac{A\, l\, n}{5{,}800{,}000}\left(V - \frac{l\, n}{34\cdot5}\right)^2$$

In order to utilise the maximium effect of wind, it is necessary to load the mill that the number of revolutions of the sails are proportional to the velocity of the wind.

Proper revolutions will be found by $n = \frac{3\cdot16\, V}{l\, sin.\, v}$.

If $v = 16°$, $n = \frac{11\cdot5\, V}{l}$, $H = \frac{A\, V^3}{1{,}135{,}000}$, and $A = \frac{1{,}135{,}000\, H}{V^3}$.

Example 1. A wind-mill is to be built of six horse power in brisk wind, V = 20 feet per second. Required the area of sails A = ?

$$A = \frac{1{,}135{,}000 \times 6}{20^3} = 851 \text{ sq. feet.}$$

Example 2. Four sails $\frac{851}{4}$ = 212·75 sq. feet each. 212·75 = 6 feet wide by 35·5 long, dimensions of the sails. Inner radius r = 5 feet and R = 5+35·3 =40·5 feet. Required the radius of centre of percussion l = ?

$$l = \sqrt{\frac{40\cdot5^2+5^2}{2}} = \sqrt{832\cdot6} = 28\cdot85 \text{ feet.}$$

Example 3. The mean angle of sails to be v = 16°. Required the proper number of revolutions of the sails per minute in brisk wind of V = 20 feet per second n, = ?

$$\text{Revolutions } n = \frac{11\cdot5 \times 20}{28\cdot85} = 8 \text{ per minute.}$$

Example 4. A wind-mill has an area of A = 750 sq. feet exposed to high wind of V = 50 feet per second, and makes n = 26 revolutions per minute,—centre of percussion l = 25. Required the horse power of the mill H = ? and proper number of revolutions per minute n = ?

$$H = \frac{750 \times 25 \times 26}{5{,}800{,}000}\left(50 - \frac{25 \times 27}{34\cdot5}\right)^2 = 7\cdot8 \text{ horses.}$$

$$n = \frac{11\cdot5 \times 50}{25} = 23 \text{ rev. per minute.}$$

$$H = \frac{750 \times 25 \times 23}{5{,}800{,}000}\left(50 - \frac{25 \times 23}{34\cdot5}\right)^2 = 8\cdot26 \text{ horses.}$$

LIGHT.

Light is the sensation transmitted by the eye and produces the sense of seeing. Heat and Electricity produce light by making bodies luminous.

Intensity of Light is inverse as the square of the distance from the luminous body.

Velocity of Light is 192500 miles per second.

Light passes from the sun 95000000 miles in 8 minutes.

Light can pass around the whole earth in one-eighth of a second.

Solids must be heated to at least 600° to produce light in the dark; and o 1000° in day-light.

MOTION OF GAS IN PIPES.

Letters denote.

Q = cubic feet of gas passed through the gas pipe per hour.

L = length in feet, D=diameter in inches of the pipe.

H= head of water in inches which presses the gas through the pipe.

s = specific gravity of the gas, air being 1.

n = number of candles required for giving the same light as Q cubic feet of gas per hour.

$$Q = 780\,D^2\sqrt{\frac{HD}{sL}}, \qquad Q = \sqrt{n}, \quad n = Q^2 \qquad D = \frac{1}{14{\cdot}35}\sqrt[5]{\frac{sLQ}{H}},$$

Example. At a distance of L=6450 feet from the gas work is required Q=940 cubic feet of gas per hour. Head of water being H=6 inches, specific gravity s=0·5. Required the diameter of the pipe D= ?

$$D = \frac{1}{14{\cdot}35}\sqrt[5]{\frac{0{\cdot}5\times6450\times940^2}{6}} = 3{\cdot}7887 \text{ inches.}$$

SOUND.

Velocity of Sound through Air.

v = velocity in feet per second.

t = temperature of the air, Fah. scale.

D = distance in feet the sound travels in the time T.

$$v = 1089{\cdot}42\sqrt{1+0{\cdot}00208(t-32)},$$

Velocity of sound in water is about 4 times that in air, and 8 times that through solids.

Intensity of sound is inversely as the square of the distance.

$$D = 1089{\cdot}42T\sqrt{1+0\,00208(t-32)},$$

$$T = \frac{D}{v}.$$

Example. A ship at sea was seen to fire a cannon, and 6·5 seconds afterwards the report was heard, the temperature in the air was 60°. Required the distance to the ship?

$$D = 1089{\cdot}42\times6{\cdot}5\sqrt{1+0{\cdot}0028(60^\circ-32)} = 7300 \text{ feet, or } 1{\cdot}38 \text{ miles.}$$

Descriptions of Sound.	*Audible at a distance of* feet.	miles.
A powerful human voice in the open air, no wind,	460	0·087
Report of a musket, - - - - - - - -	16000	3·02
Drum, - - - - - - - - - -	10560	2
Music, strong brass band, - - - - - -	15840	3
Cannonading, very strong, - - - - - -	575000	90
In a barely observable breeze a strong human voice with the wind can be heard, - - - -	15840	3

RINGING BELLS.

Letters denote.

D = diameter of the bell at the mouth, in inches.
d = diameter of the bell at the crown, in inches.
h = heighth of the bell from the mouth to the crown in inches.
S = thickness of sound bow in inches.
W = weight of the bell in pounds avoirdupois.
n = number of vibrations per second, corresponding with the key note of the bell, and to be found in the accompanying table I.
k = from 0·07 to 0·08, or a coefficient expressing the relative thickness of the sound bow to the diameter of the bell. In peals of bells, the sound bow is generally $S = 0{\cdot}08D$ for the triple, and $S = 0{\cdot}07D$ for the tenor; the intermediate bells in the peal having the intermediate proportions of sound bow.

Example 1. Required the weight of a bell $D = 62$ inches in diameter, and $S = 4\frac{1}{2}$in. thickness of the sound bow, $W = ?$

$$\textit{Formulae } 1. \quad W = 0{\cdot}25 \times 62^2 \times 4{\cdot}5 = 4324{\cdot}5 \text{ pounds.}$$

Example 2. A bell of 2,500 pounds is to be constructed with a sharp note, taking the sound bow $k = 0{\cdot}075$. Required the diameter of the bell $D = ?$

$$\textit{Formulae } 10. \quad D = \sqrt[3]{\frac{4 \times 2500}{0\,075}} = 51{\cdot}084 \text{ inches}$$

Example 3. It is required to construct a bell with the key note $D\sharp$ in the first octave above zero, n = 152·22. To be of light weight with a full good note, for which latter case take $k = 0{\cdot}07$. Required the diameter of the bell, $D = ?$

$$\textit{Formulae } 11. \quad D = \frac{58000 \times 0{\cdot}07}{152{\cdot}22} = 26{\cdot}665 \text{ inches.}$$

Example 4. Required the key note of a bell with $D = 36{\cdot}5$ in. diameter and $S = 2{\cdot}75$ in., $n = ?$

$$\textit{Formulae } 4. \quad n = 58000 \times \frac{2{\cdot}75}{36.5} = 119{\cdot}7 \text{ vibrations.}$$

In the table the nearest number 120·82, in the first octave below zero, answers to the key note B, which will be the note of the bell.

Example 5. A bell of 6860 pounds is to be constructed with the key note C in the first octave below nero $n = 64$, see table I. Required the diameter of the bell $D = ?$

$$\text{Formulae } 9. \quad D = 21{\cdot}947 \sqrt[4]{\frac{6860}{64}} = 70{\cdot}6175 \text{ inches}$$

Example 6. Required the thickness of sound bow for the bell in the preceding example? $D = 70.6175$ inches and $n = 64$. $S = ?$

$$\textit{Formulae } 12. \quad S = \frac{64 \times 70.6175^2}{58000} = 5{\cdot}5027 \text{ inches.}$$

Example 7. Required the weight of a bell $D = 48$ inches diameter at the mouth, $d = 25$ inches at the crown, and $h = 34$ inches height from the mouth to the crown, $S = 3{\cdot}5$ in., $W = ?$

Formulae 17.

$$W = 48 \times 25 \times 3{\cdot}5\left(0{\cdot}5 - 0{\cdot}002816 \times 25\right) + 0{\cdot}00375 \times 34 \times 25^2 \times 3{\cdot}5 = 2126{\cdot}226 \text{ pounds.}$$

Formulas for Ringing Bells.

$$W = 0{\cdot}25\,D^2 S \quad \cdots \quad 1$$

$$W = \frac{D^4 n}{232000} \quad \cdots \quad 2$$

$$W = 0{\cdot}25 D^3 k \quad \cdots \quad 3$$

$$n = 58000 \frac{S}{D^2} \quad \cdots \quad 4$$

$$n = 232000 \frac{W}{D^4} \quad \cdots \quad 5$$

$$n = 58000 \frac{k}{D} \quad \cdots \quad 6$$

$$D = 2\sqrt{\frac{W}{S}} \quad \cdots \quad 7$$

$$D = 240{\cdot}83\sqrt{\frac{S}{n}} \quad \cdots \quad 8$$

$$D = 21{\cdot}947\sqrt[4]{\frac{W}{n}} \quad \cdots \quad 9$$

$$D = \sqrt[3]{\frac{4W}{k}} \quad \cdots \quad 10$$

$$D = 58000 \frac{k}{n} \quad \cdots \quad 11$$

$$S = \frac{n\,D^2}{58000} \quad \cdots \quad 12$$

$$S = \frac{4W}{D^2} \quad \cdots \quad 13$$

$$S = k\,D \quad \cdots \quad 14$$

$$k = \frac{S}{D} \quad \cdots \quad 15$$

$$k = \frac{4W}{D^3} \quad \cdots \quad 16$$

$$W = D\,d\,S\,(0{\cdot}5 - 0{\cdot}0002816\,d) + 0{\cdot}00375\,h\,d^2\,S \quad \cdots \quad 17$$

Table I. Vibrations per Second = n.

Key note.	Bass. 3rd Oct.	2nd Oct.	1st Oct.	Zero.	Descant. 1st Oct.	2nd Oct.	
C	16·000	32·000	64·000		128·00	256·00	512·00
C♯	16·947	33·385	67·790		135·58	271·00	542·32
D	17·960	35·920	71·840		143·68	287·36	574·72
D♯	19·027	38·055	76·110		152·22	304·44	608·88
E	20·159	40·318	80·636		161·27	322·54	645·09
F	21·357	42·715	85·430		170·86	341·72	683·44
F♯	22·627	45·255	90·510		181·02	362·04	724·08
G	23·972	47·945	95·890		191·78	383·56	767·12
G♯	25·398	50·797	101·59		203·19	406·37	812·75
A	26·908	53·817	107·63		215·27	430·53	861·07
A♯	28·508	57·017	114·03		228·07	456·13	912·27
B	30·204	60·409	120·82		241·63	483·27	966·54
C	32·000	64·000	128·00		256·00	512·00	1024·0

Table V.

Abscissa x	Ordinate y	Thickness of Metal. $S = 1$	$S = 0{\cdot}07D$	$S = 0{\cdot}75D$	$S = 0{\cdot}08D$
1	0·4142	1	0·700	0·750	0·800
1½	0·686	0·800	0·560	0·600	0.640
2	0·867	0·653	0·459	0·490	0·522
2½	0·974	0·547	0·382	0·410	0·437
3	1·025	0·474	0·331	0·355	0·379
3½	1·030	0·423	0·295	0·317	0·338
4	1·000	0·380	0·266	0·285	0·304
4½	0·955	0·351	0·245	0·263	0·281
5	0·875	0·327	0.228	0·245	0·261
5½	0·775	0·301	0·211	0·226	0·241
6	0·665	0·291	0·203	0·218	0·233
6½	0·530	0·286	0·200	0·214	0·228
7	0·390	0.279	0·195	0·209	0·223
7½	0·235	0.272	0·190	0·204	0·217
8	0·075	0.267	0·186	0·200	0·213
8·74	0·78	0.333	0.233	0.250	0·266

To Construct a Bell.

When a bell is to be constructed, we generally have the weight or key note given by contract, the diameter and sound bow are calculated by the preceding formulas and examples, and then ready to proceed with the construction. See fig. 1.

The diameter of the bell at the mouth, is divided into 10 equal parts, called strokes, which then is the scale and measurement for the construction. Make a decimal scale, as shown on plate VII.

Shrinkage to be allowed for 3 sixteenths of an inch per foot.

The section of a bell is generally laid out on a piece of board represented by the dotted lines *a*, *b*, *c*, *d*, which then is cut out and used for turning up the mould for the bell. The board should be about 11 strokes long, and 2·5 strokes wide. Through the centre of the board draw the line *p*, *q*, parallel to *b*, *c*, bisect the line *p*, *q*, and set four (4) strokes from the bisecting point towards each end, divide the strokes into halves, and number them as shown on the accompanying drawing. Through each division draw lines at right angles to *p*, *q*, set off the corresponding ordinates *y* expressed in strokes, Table II. and join them by a curve-line, which then will be the centre of thickness of metal in the bell.

At the end of the first ordinate, as a centre, draw a circle with a diameter equal to the desired thickness of the sound bow, which should be from 0·7 to 0·8 strokes. At every succeeding ordinate draw a circle with the diameter noted in Table II; for instance, if the thickness of the sound bow is 4½ inches, then the thickness of metal or diameter of the circle at the third ordinate will be $4{\cdot}5 \times 0{\cdot}474 = 2{\cdot}133$ inches; but if the sound bow is 0·7, 0·75 or 0·8 strokes, the thickness of metal at the third ordinate will be 0·331, 0·355, or 0·379 strokes. When all the thicknesses are thus drawn, draw the two lines tangenting the circles on each side of the centre line of the metal.

From 0 to 1 make a moulding of 0·1 stroke thick over the dotted line as shown by fig. 2.

Prolong the 6½ ordinate, and set off 1·79 strokes to *e*, which then is the centre for the curve on the top, draw the arc through the centre of the small circle at the 8th ordinate; join *e*, 8, set off from *e*, 0·46 strokes to the centre for the inside curve at the top.

Thickness of metal of the top should be 0·3 the sound bow at 8, and 0·333 at *r*. Draw the ordinate at 8·74, set off 0·78 to *r*, join *r* and the abscissa 8·48, and prolong the line through *r*; then finish the drawing as shown on the plate.

When the board is cut out and ready for turning the mould, it must be carefully set, so that the outside diameter of the crown will be half the diameter of the mouth of the bell.

This form of Bells gives the greatest possible gravity of tone with the least possible quantity of metal. Bells can be made almost in any form without seriously affecting the quality of tone, but the thickness of metal should always be in proportion as the square of the diameter taken at the centre of the metal as in fig. 3.

Proportions of a Peal of Eight Bells.

Bells.	*Keynote.*	*n*	*k*	*S. in.*	*D. in.*	*W. lbs.*	*Clapper.*
Tenor,	*D*	71·84	0·070	3·95	56·5	3156	63 *lbs.*
2nd,	*E*	80·64	0·071	3·62	51·1	2366	48·6
3rd,	*F*♯	90·51	0·072	3·32	46·1	1765	37·2
4th,	*G*	95·89	0·073	3·22	44·2	1575	34·1
5th,	*A*	107.63	0·075	3·08	40·5	1262	28·1
6th,	*B*	120·82	0·077	2·85	37·0	976	22·4
7th,	*C*♯	135·58	0·079	2·67	33·8	763	18·2
Triple,	*D*	143·68	0·080	2·58	32·3	673	16·8

Clapper. The weight of the clapper should be from one fortieth to one fiftieth the weight of the Bell, the smaller bells takes the largest clappers.

Bell Metal. Thirty of Tin to one hundred of Copper, is a good proportion.

RINGING BELL.

Fig. 1.

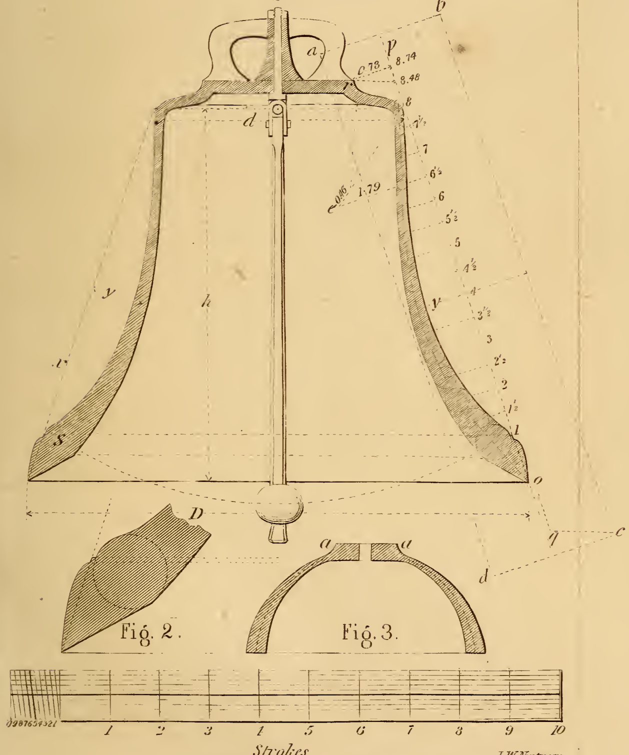


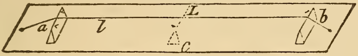
Fig. 2.

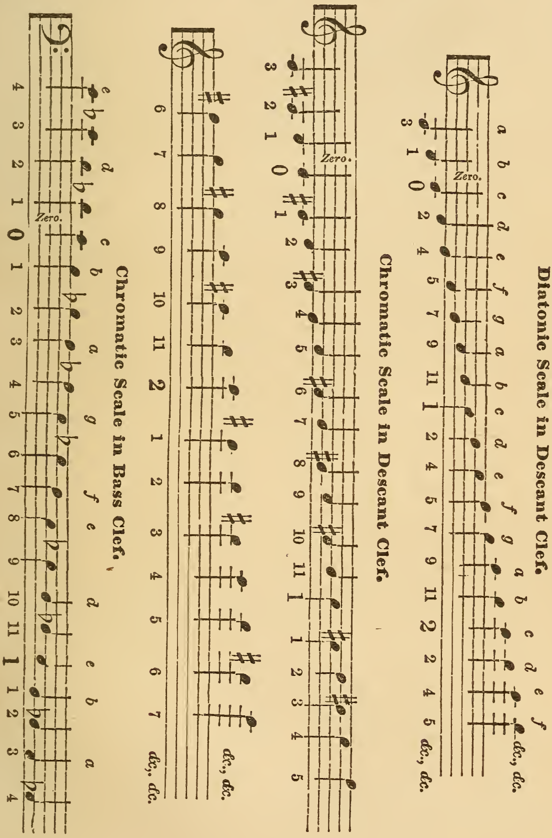
Fig. 3.

Strokes.

J. W. Nystrom.

312.


a
l
L
b
c


Diatonic Scale in Descant Clef.
Zero.
&c., &c.
Chromatic Scale in Descant Clef.
Zero.
&c., &c.
&c., &c.
Chromatic Scale in Bass Clef.
Zero.

HEAT. CALORIC.

THE *Physical constitution* of heat is yet under investigation by operative minds, its well known character and effect upon matter is the base for the investigation.

Heat resembles *light*, *electricity* and *magnetism*, and is thus far assumed to be a material substance.

Heat is contained in all matters, with no known exception. Two bodies containing different quantities of heat per unit, being placed in contact,—the heat will pass from one to the other until it comes in equilibrium, that is when the two bodies contain equal quantities of heat per unit. It is this passing of heat that first comes under our notice. The body from which the heat passes will feel the other to be cold, and *vice versa*,—the one that receives the heat will feel the other to be warm, until there is no further passage, namely, when the bodies will feel neither warm nor cold to each other; hence the measure of the emplix sensible heat, is the difference between the heat per unit in the two bodies. Cold is only a want of heat.

Caloric is only another word of expression for heat.

Caloric is of two kinds, *sensible* and *latent*.

Sensible Caloric is that which is sensible to the touch, felt as temperature, and can pass freely from one body to another.

Latent Caloric is that which is insensible to the touch. It is contained in bodies without being felt as temperature, but can by chemical action become sensible; for instance, a piece of burned limestone put into water will get warm and heat it, although both were cold before; the latent caloric in the water was set free to sensible.

Influence of Heat on Matter's Coherence.

All bodies in nature expand when heated, and contract when cooled. Solid bodies vary but little by the difference in temperature. Liquids vary more, but gases are extremely susceptible to the impression of heat and cold.

Table of Linear Expansion of Solids.

Difference in temperatures.	*Length = 1 at 32°. Names of bodies.*	*Length at T.°*	*k difference length per degree.*
32° to 212°	Glass,	1·00086133	0·00000478
32 to 392	Glass,	1·00184520	0·00000546
32 to 572	Glass,	1·00303252	0·00000583
32 to 212	Wrought iron,	1·00118210	0·00000656
32 to 572	Wrought iron,	1·00440528	0·00000894
32 to 212	Soft good iron,	1·00122045	0·00000680
32 to 212	Iron wire,	1·00123504	0·00000687
32 to 212	Cast iron,	1·00111120	0·00000618
32 to 212	Soft steel,	1·00107915	0·00000600
32 to 212	Steel hardened and tem. 150°,	1·00123956	0·00000689
32 to 212	Copper,	1·00171820	0·00000955
32 to 572	Copper,	1·00564972	0·00001092
32 to 212	Lead,	1·00284836	0·00001580
32 to 212	Gold, pure,	1·00146606	0·00000815
32 to 212	Gold, hammered,	1·00149530	0·00000830
32 to 212	Silver, pure,	1·00190868	0·00001060
32 to 212	Silver, hammered,	1·00201000	0·00001116
32 to 212	Brass, common cast,	1·00187821	0·00001043
32 to 212	Brass wire or sheet,	1·00193333	0·00001075
32 to 212	Platinum, pure,	1·00088420	0·00000491
32 to 572	Platinum, pure,	1·00275482	0·00000520
32 to 572	Platinum, hammered,	1·00095420	0·00000530
32 to 212	Zinc, pure or cast,	1·00294167	0·00001633
32 to 212	Zinc, hammered,	1·00310833	0·00001722
32 to 212	Tin, hammered,	1·00270000	0·00001500
32 to 212	Tin, cast,	1·00217298	0·00001207
32 to 212	Fire brick,	1·00042280	0·00000235
33 to 212	Marble,	1·00110410	0·00000613
32 to 212	Granite,	1·00078940	0·00000438

Table of Volume Expansion of Liquids.

Difference in temperatures.	*Names of Liquids.*	*Volume at T.°*	*k difference in vol. per degree.*
32° to 212°	Mercury, -	1·018018	0·0001000
212 to 392	" - - -	1·018433	0·0001025
392 to 572	" - - -	1·018868	0·0001048
32 to 212	Water, - - -	1·046600	0·0002595
32 to 212	Salt, dissolved, - -	1·050000	0·0002778
32 to 212	Sulphuric acid, - -	1·060000	0·0003333
32 to 212	Oil of Turpentine and Ether,	1·070000	0·0003890
32 to 212	Oil, common, - -	1·080000	0·0004444
32 to 212	Alcohol and Nitric acid,	1·100000	0·0005555

All gases expand and contract equally and uniformly; 0·0020825 its volume per degree of Fah. thermometer. The accompanying Table is the result of Mr. Dalton's experiments with air. The volume at 32° is equal to 1 or the unit.

Table for Volume Expansion of Air.

Degrees.	*Volume.*	*Degrees.*	*Volume.*	*Volume*
32°	1·000	80	1·1110	$1\frac{7}{64}$
33	1·002	85	1·121	$1\frac{1}{8}$
34	1·004	90	1·132	$1\frac{1}{7\cdot56}$
35	1·007	100	1·152	$1\frac{1}{6\cdot57}$
40	1·021	200	1·354	$1\frac{1}{3}$
45	1·032	212	1·376	$1\frac{1}{2\cdot66}$
50	1·043	302	1·558	$1\frac{9}{16}$
55	1·055	392	1·739	$1\frac{3}{4}$
60	1·066	482	1·919	$1\frac{15}{16}$
65	1·077	572	2·098	$2\frac{3}{32}$
70	1·089	680	2·312	$2\frac{5}{16}$
75	1·099			

Letters Denote,

L = length or any linear measure of the body of the temperature T.
l = length or linear measure at the temperature t.
V = volume of liquids at the temperature T:
v = volume at the temperature t.
k = coefficient for the linear measure or volume as noted in the Tables.
The volume of solids is as $L^3 : l^3$.
The linear measure of liquids is a $\sqrt[3]{v} : \sqrt[3]{V}$.

Formulas of Linear Expansion of Solids.

$$L = l\Big(1+k(T-t),\Big), \qquad l = \frac{L}{1+k(T-t)},$$

$$T = \frac{L-l}{k\,l}+t, \qquad t = T - \frac{L-l}{k\,l},$$

Example 1. A copper rod of $L = 22{\cdot}55$ feet long is 140° warm. To what temperature must it be cooled to fit in a space of $l = 22{\cdot}52$ feet?

$$t = 140 - \frac{22{\cdot}55 - 22{\cdot}52}{22{\cdot}52 \times 0{\cdot}0000158} = 55{\cdot}7^\circ \text{ the answer.}$$

Formulas of Volume Expansion of Liquids.

$$V = v\left(1+k(T-t)\right), \qquad v = \frac{V}{1+k(T-t)},$$

$$T = \frac{V-v}{k\,v}+t, \qquad t = T - \frac{V-v}{k\,v}.$$

Example 2. A vessel containing 5·68 cubic feet of water at $t = 42°$, is closed up round the water, but a cylindrical pipe of 0·008 square feet, inside section, is raised up vertically from it; now let the temperature of the water be raised to $T = 130°$. How high will the water rise in the pipe?

$$V = 5{\cdot}68\left[1+0{\cdot}000002595(130-42)\right] = 5{\cdot}681297 \text{ cubic feet,}$$

and $$\frac{5{\cdot}681297 - 5{\cdot}68}{0{\cdot}008} = 0{\cdot}162 \text{ feet} = 1{\cdot}945 \text{ inches,}$$

the height to which the water will raise in the pipe.

This is the principle upon which Thermometers are constructed, but the scale can only be approximated by this formula. The substances adopted for thermometers are spirits of wine and mercury; oil and ether has also been proposed, but the two former are best, and mercury is most generally used.

THERMOMETERS.

THERE are three different graduated Thermometers in use, namely *Fahrenheit's Celcius's*, and *Reamur's*.

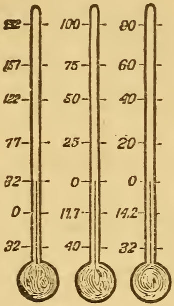
The first one, or *Fahrenheit's* is used in North America, England, and Holland.

The second or *Celcius's* in France, Sweden and Germany.

The third one, or *Reamur's*, was formerly used in France and some parts of Germany, but now only in Spain.

The Figures exhibit their difference.

Fahr. Celci. Ream.



Proportional Formulas for the Thermometrical Scales.

Celci. $= \frac{5}{4}$ Ream. $= \frac{5}{9}$ (Fah. $-$ 32.)

Ream. $= \frac{4}{5}$ Celci. $= \frac{4}{9}$ (Fah. $-$ 32.)

Fahr. $= \frac{9}{5}$ Celci.$+32 = \frac{9}{4}$ Ream.$+32$.

Example. How much is 68° Celcius on Fahrenheit's scale.

Fahrenheit's $= \frac{9}{5} \times 68 + 32 = 154{\cdot}4°$, the answer.

Fluid boils when its vapour has the same density as the atmosphere where it boils, hence, fluid will boil sooner high up in the atmosphere than at the ground.

In vacuum water boils at 88°.

The mean temperature of the earth is about 50°; at the torrid zone 75°. temperate zone 50°; and in the polar regions 36°.

Water can be kept in liquid to 20°.

Table of Temperatures when Bodies change Form.

SMELTING POINTS.	
Cast iron, fully sm.,	2754°
Gold, fine,	1983°
Silver, fine,	1850°
Copper,	2160°
Brass, common,	1900°
Zinc,	740°
Lead,	594°
Bismuth,	476°
Tin,	421°
1 Tin, 1 Bismuth,	283°
3 Tin, 2 Lead, 5 Bismuth,	212°
1 Tin, 1 Lead, 4 Bismuth,	201°
Antimony,	790°
Sulphur,	228°
Phosphorus,	109°
Beeswax, white,	155°
" yellow,	142°
Tallow,	92°
Ice,	32°
Oil of Turpentine,	14°
Ice of strong Brandy,	7°
1 Snow and 1 Salt,	0°
Mercury,	— 39°

BOILING POINTS.	
Mercury,	630°
Oil of Linseed,	600°
Sweet Oil,	412°
Sulphuric acid,	410°
Sulphur,	390°
Phosphorus,	374°
Oil of turpentine,	315°
Sea-water, salt,	217°
Water, distilled,	212°
Alcohol,	174°

MISCELLANEOUS.	
Metals, red, daylight,	1077°
Iron red, daylight,	884°
Common fire,	790°
Iron bright red, in dark,	752°
Human blood is	98°
Cold greatest ever produced,	— 90°
Venous fermentation,	— 60 to 70°
Acetous fermentation begins,	— 78°
Acetification ends,	— 88°
Phosphorous burns,	— 43°
A comfortable room about	60° to 70°

Table of Power for Transmission of Heat.

CONDUCTING POWER.	
Gold,	1000
Silver,	973
Iron,	347
Tin,	304
Copper,	898
Zinc,	363
Lead,	180
Platinum,	981
Marble,	24
Fire-brick,	11
Fire-clay,	11·4
Porcelain,	12·2
Water as the Standard.	
Water,	10
Pine,	39
Lime,	39
Oak,	33
Elm,	32
Ash,	31
Apple,	28
Ebony,	22

RELATIVE CONDUCTING POWERS OF SOLIDS.	
Hare's fur,	1·315
Eider-down,	1·305
Beaver's fur,	1·296
Raw silk,	1·284
Wool	1·118
Lamp-black,	1·117
Cotton,	1·046
Lint,	1·032
Charcoal,	0·936
Ashes of wood,	0·927
Sewing-silk,	0·917
Air,	0·577

RELATIVE CONDUCTING POWER OF FLUIDS.	
Mercury,	1000
Water,	357
Proof spirit,	312
Alcohol pure,	332

RADIATING POWER.	
Water,	100
Lampblack,	100
Paper, writing,	98
Rosin,	96
Sealing wax,	95
Glass, common,	90
India ink,	88
Ice,	85
Red Lead,	80
Graphit,	75
Lead, tempered,	45
Mercury,	20
Lead, polished,	19
Iron, polished,	15
Tin and Silver,	12
Copper and Gold,	12

REFLECTING POWERS.	
Brass,	100
Silver,	90
Tinfolium	85
Tin,	80
Steel,	70
Lead,	60
Glass,	10
Glass, oiled or waxed,	5
Lampblack,	0

Mixtures of.		*Cold produce.*	*Degrees Fahr.*
Nitrate of Ammonia,	1	46°	From $+50°$ to $+4°$.
Water,	1		
Sulphate of Soda,	8	50°	From $+50°$ to $+0°$.
Muriatic Acid,	5		
Dilute Sulphuric Acid,	5	23°	From $-68°$ to $-91°$.
Snow,	4		

SPECIFIC CALORIC.

Specific Caloric is the relative quantity of heat contained in bodies of equal weight or volume, and of the same temperature.

Let two different substances of known weight or volume and temperature, be mixed together; the temperature of the mixture will dissolve the relative quantity of caloric in the ingredients.

Mixture of the same Substances.

Letters denote.

W = weight or volume of a substance of temperature T.
w = weight or volume of a similar substance but temperature t.
t' = temperature of the mixture $W+w$. We shall have,

$$t'(W+w) = WT+wt, \qquad t' = \frac{WT+wt}{W+w},$$

$$W = \frac{w(t'-t)}{T-t'}, \qquad T = \frac{w(t'-t)}{W}+t'.$$

Example 1. Let $W = 4{\cdot}62$ cubic feet of water at $T = 150°$ be mixed with $w = 5{\cdot}43$ cubic feet at $t = 46$. Required the temperature of the mixture $t' = ?$

$$t' = \frac{4{\cdot}62\times150°+5{\cdot}43\times46°}{4{\cdot}62+5{\cdot}43} = 97{\cdot}6° \text{ the answer.}$$

Example 2. How much water of $T = 107°$ must be mixed to $w = 27{\cdot}3$ gallons of $t = 58°$, the mixture of the water to be $75°$?

$$W = \frac{27{\cdot}3(75-58)}{107-75} = 14{\cdot}5 \text{ gallons.}$$

Mixture of different Substances.

W and w expressed by *weights* only. S and s = *Specific caloric* as given in the accompanying Table. We shall have,

$$WS(T-t') = w\,s(t'-t), \qquad t' = \frac{WST+wst}{WS+ws}.$$

$$W = \frac{w\,s(t'-t)}{S(T-t)}, \qquad T = \frac{t'(WS+ws)-wst}{WS}.$$

Example 3. To what temperature must $W = 20$ pounds of iron be heated to raise $w = 131$ pounds of water of $t = 54°$ to a temperature $t' = 64°$. $T = ?$
From the Table we have $s = 1$, and $S = 0{\cdot}1218$.

$$T = \frac{64(20\times0{\cdot}1218+131)-131\times1\times54}{20\times0{\cdot}1218} = 602°.$$

the required temperature, supposing no vapour escapes from the water.

If any chemical action takes place in the mixture, these formulas will not answer, because part of the *sensible caloric* may become *latent*, or *latent caloric* may be set free.

Table of Specific Caloric, Water as Unit.

Names of Substances.	*Specific Caloric.* 32° to 212°.		32°, 572°.
Water,	1.0000		
Iron,	0.1105		
Glass-crystal,	0.1929		
Mercury,	0.029	to 0·033	0·035
Lead,	0.02819	to 0·0293	
Tin,	0.04755	to 0·0514	
Sulphur,	0.2085	to 0·188	
Lime, burned,	0.2169		
9 Water, 10 Lime,	0.43912		
Sulphuric acid, sp. g. = 1·87058,	0.3346		
Nitric acid, sp. g. = 1·29895,	0.66139		
Alcohol, sp. g. = 0·81,	0.7		
Platinum,	0.0344	to 0·0335	0·0355
Antimony,	0.0507		0·0547
Zinc,	0.0927		0·1015
Copper,	0.094	to 0·0949	0·1013
Iron,	0.1098	to 0·1105	0·1218
Glass,	0.1770		0·19
Gold,	0.0288		
Bismuth,	0.0298		
Woods in average,	0.48	to 0·6	
Sweet Oil,	0.30961		
Nickel,	0·1035		
Cobalt,	0·1498		
Tellurium,	0·0912		

SPECIFIC CALORIC OF GASES AT EQUAL DENSITY.	*Volume.* *air* = 1.	*Weight.* *air* = 1.	*Weight.* *water* = 1.
Air, atmospheric,	1·000	1·000	0·2669
Hydrogen,	0·9033	12·34	3·2936
Oxygen,	0·9764	0·8848	0·2361
Nitrogen,	1·0000	1·0318	0·2754
Carbonic-oxid gas,	1·034	1·0805	0·2884
Carbonic acid,	1·2583	0·828	0·221
Nitro-oxid gas,	1·3505	0·8878	0·2369
Gas of oils,	1·553	1·5763	0·4207
Steam,	1·96	3·136	0·847

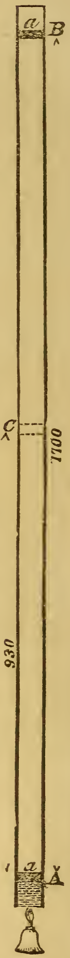
Capacity for *Caloric* is the relative ability of bodies to retain the *specific caloric.* Capacity for caloric is inverse as the density of the substances. The *specific* caloric multiplied by the atom weight of a substance, gives the constant number 0·375 (average) which proves that the atoms have equal capacity for caloric in all substances. This is a fact with no known reason, but by it valuable results may be opened.

Table of Relative Capacity for Caloric.

Names.	*Equal Weights.*	*Equal Volume.*	*Names.*	*Equal Weights.*	*Equal Volume.*
Water	1·000	1·000	Zinc	0·102	
Copper	0·114	1·027	Tin	0·060	
Iron	0·126	0·993	Lead	0·043	0·487
Brass	0·116	0·971	Glass	0·187	0·443
Gold	0·050	0·966			
Silver	0·082	0·833			

When the volume diminishes the capacity for caloric will also be diminished and thus part of the caloric will profuse the body. A volume of air compressed to $\frac{1}{5}$ its bulk will fire tinder, which requires a temperature of about 550°.

STEAM.



Steam is the vapour into which water is converted by the application of heat.

Let *AB* be a cylindrical glass tube in which is fitted a piston *a* of one square inch area; consider this piston to have no friction or weight, and can be moved steam-tight from *A* to *B*. Let the tube be 1728 inches from *A* to *B*, the space under the piston *a* just one inch from the bottom being filled with water of 32° Fah., which will be one cubic inch; weigh the whole apparatus. Now, place a lamp under the tube in a position as represented by the Figure, and notice the time, (say 10*h*, 5*m*.) The temperature of the water will gradually increase, and the piston *a* maintain a contact with it until the water begins to boil, which time is to be carefully noticed; now (10*h*, 15*m*.) It will be found that temperature of the water has raised from 32° to 212°, which took 10*h*, 15*m* — 10*h* 5*m* = 10 minutes.

Let the lamp still remain and the boiling be continued. The piston *a* will now leave the water, and gradually ascend towards *B*, apparently leaving a space between itself and the water, the latter will gradually diminish as the piston ascends, which indicates that steam is gradually formed, and occupies the space between the water and the piston, and as the piston has no weight or friction it is evident that the density of the steam must be the same as the surrounding atmosphere.

But another important faculty of steam and water will now be manifest, namely, that the temperature of both will remain the same, 212°, as at the boiling point, (10*h* 15*m*,) consequently the *heat* from the lamp which goes into the water and steam is not *sensible* but becomes *latent*. The water is now getting very low; observe carefully the moment when it apparently disappears on the bottom of the tube. - - "now" (11*h* 10*m*.) The piston *a* will be found at *B*, 1700 inches from A, and the time from the boiling-point is (11*h* 10*m*) — (10*h* 15*m*) = 55 minutes = $5\frac{1}{2}$ times that occupied to raise the water from 32° to 212° = 180°; hence the quantity of heat from the lamp now contained in the steam is $180 \times 5\frac{1}{2} + 180 = 1170°$ of which 180° is sensible and 990 latent. If the water had been enclosed in a vessel to prevent evaporation, and the same quantity of heat 1170° imparted to it, it would have a temperature of 1202, which is about that of metals when red hot in daylight.

Again to the tube *A B*, at the time before noticed, viz., 11*h*, 10*m*. Take the lamp away, weigh the apparatus, and it will be found the same weight as before; hence, the same quantity of water is still in the tube, but in the form of steam. The heat will now radiate from the tube, and it will be observed that the piston *a* gradually descends towards *A*, and the inner surface of the tube will be covered with a dew which will soon fall to the bottom as water, but still maintain the heat of 212°, until the piston *a* has fully reached its former position at *A*, when the same quantity of water (one cubic inch) will occupy the same space as before the lamp was put under it, but with a temperature of 212°.

The *heat* required to make steam of one cubic inch of water is able to raise $5\frac{1}{2}+1$ cubic inches from 32° to 212°; or steam at 212°, formed of one cubic inch of water, can raise $5\frac{1}{2}$ cubic inches of ice cool water from 32° to 212°, when mixed together, making $6\frac{1}{2}$ cubic inches.

Effect of Steam.

By the preceding experiment we find that one cubic inch of water will be 1700 cubic inches converted into steam; or one cubic inch of water makes one cubic foot of steam 212°, the same density as the surrounding atmosphere which is $14\frac{3}{4}$ pounds per square inch; the effect of steam in the experiment was consequently a weight of $F = 14\frac{3}{4}$ pounds raised 1700 inches = 142 feet in 55 minutes, or

$$P = \frac{Fs}{t} = \frac{14{\cdot}75 \times 142}{55 \times 60} = 0{\cdot}635 \text{ Effects.}$$

See Formula 5, page 148.

Advantage of Using High Steam.

Let us now make the same experiment with the tube *AB*, and load the piston with $14\frac{3}{4}$ pounds, which will be a weight $F = 29{\cdot}5$ pounds including the atmosphere. Set the lamp under as before, and the experiment is in operation. The temperature of the water will now not cease to increase when it has attained 212°; nor will the piston *a* begin to raise after 10 minutes as in the former experiment; but, when the water has attained 250°, it will cease to increase, the piston commence to ascend, and steam to generate. The piston will now only raise 930 inches from *A*, which will occupy the same time as before. The mechanical effect of the steam is therefore 29·5 pounds raised 930 = 77·5 feet in 55 minutes, or,

$$P = \frac{29{\cdot}5 \times 77{\cdot}5}{55 \times 60} = 0{\cdot}693 \text{ Effects,}$$

which exceeds the former experiment about 9 per cent.

$$\frac{0{\cdot}64363}{0{\cdot}70469} = 0{\cdot}913 \text{ or } 100 - 91{\cdot}3 = 8{\cdot}6 \text{ per cent.}$$

an advantage of using higher steam.

This per centage will increase as the steam is used higher.

Advantage of Using Steam Expansively.

We now continue the latter experiment. The piston *a* stands at 930 inches from *A*; take the lamp away, and remove the $14\frac{3}{4}$ pounds on the piston. The steam of 250° in the tube will now raise the piston *a* to *B*, at 1700 inches from *A* and the temperature of the steam will decrease from 250° to 212°; consequently the same steam has produced an additional effect by raising $14\frac{3}{4}$ pounds (the pressure of the atmosphere,) 1700 — 930 = 770 inches high = 64·165 feet, which for comparison, will here assume to be accomplished in the same time 55 minutes, we shall then have

$$P = \frac{14{\cdot}75 \times 64{\cdot}165}{55 \times 60} = 0{\cdot}29127 \text{ Effects}$$

and 0·70469+0·29127 = 0·99596 Effects produced by the same quantity of steam,

$$\text{and } \frac{0{\cdot}64363}{0{\cdot}99596} = 0{\cdot}646. \qquad 100 - 64{\cdot}6 = 35{\cdot}3 \text{ per cent.,}$$

gained by using the steam high and expansively.

Advantage taken of the Pressure of the Atmosphere by Vacuum,

Again to the latter experiment, the piston *a* stands at *B*, and the tube is full of steam at 212°; let there now be introduced among the steam $5\frac{1}{2}$ cubic inches of ice cool water, (32°), the steam will immediately condense to water, and the piston *a* begin to descend; finally, between the piston and the bottom of the tube will be found $6\frac{1}{2}$ cubic inches of water at 212°, hence the atmospheric pressure has reproduced an effect equal to that the steam before expended on it, or 0·64363 effects.

The principal features of the application of steam to produce mechanical effects are now illustrated, and we will proceed to give the principal *Rules, Formulas,* and *Tables,* respecting its property.

If steam is reduced in volume, its density and temperature will increase; and when additional heat is applied to steam its density or volume will increase the same as if it was produced direct from water.

Temperature of Steam	*Atmosph. included.* Inches of Mercury.	Pounds per square inch.	Specific gravity, air = 1.	*k* Volume compared with water.	Number of atmospheres.	*Atmosphere excluded.* Inches of Mercury,	Pounds per square inch.
32°	0·200	0·098	0·0041	187407		−29.79	−14·60
40	0·263	0·129	0·0053	144529	0·01	−29·73	−14·57
50	0·375	0·184	0·0074	103350	0·01	−29·62	−14·52
60	0·524	0·257	0·0102	75421	0·02	−29·47	−14·44
70	0·721	0·353	0·0136	55862	0·02	−29·27	−14·35
80	1·000	0·490	0·0186	41031	0·03	−29·00	−14·21
90	1·36	0·666	0·0250	30425	0·05	−28·63	−14.03
100	1·86	0·911	0·0333	22873	0·06	−28·13	−13.79
103	2·04	1·000	0·0364	20958	0·07	−27·95	−13.70
110	2·53	1·240	0·0458	16667	0·08	−27·46	−13·46
120	3·33	1·632	0·0576	13215	0·11	−26·66	−13·07
130	4·34	2·129	0·0538	10328	0·14	−25·65	−12·57
140	5·74	2·813	0·0960	7938	0·19	−24·25	−11·89
145	6·53	3·100	0·108	7040	0·22	−23·46	−11 60
150	7·42	3·636	0·122	6243	0·25	−22·57	−11 06
155	8·40	4·166	0·137	5559	0·28	−21·59	−10·54
160	9·46	4·635	0·153	4976	0·31	−20·53	−10·07
165	10·68	5·23	0·171	4443	0·35	−19·31	− 9 47
170	12·13	5·94	0·193	3943	0·4	−17·86	− 8·76
175	13·62	6·67	0·215	3538	0·45	−16·37	− 8·03
180	15·15	7·42	0·238	3208	0·50	−14·84	− 7·28
185	17·00	8·33	0·265	2879	0·56	−12·99	− 6·37
190	19·00	9·310	0·294	2595	0·63	−10·99	− 5·39
195	21·22	10·40	0·325	2342	0·71	− 8·77	− 4·30
200	23·64	11·58	0·36	2118	0·79	− 6·35	− 3·12
205	26·13	12·80	0·394	1932	0·87	− 3·86	− 1·90
210	28·84	14·13	0·431	1763	0·96	− 1·15	− 0·57
211	29·41	14·41	0·440	1730	0·98	+ 0·58	− 0·29
212	30·00	14·70	0·448	1700	1·00	∓ 0·00	∓ 0·00
212·8	30·60	15·	0·457	1669	1·02	+ 0·60	+ 0·30
214·5	31·62	15·5	0·471	1618	1·05	+ 1·62	+ 0·80
216·3	32·64	16·	0·484	1573	1·09	+ 2·64	+ 1 30
218·	33·66	16·5	0·497	1530	1·12	+ 3·66	+ 1·80
219·6	34·68	17·	0·512	1488	1·15	+ 4·68	+ 2·30
221·2	35·70	17·5	0·529	1440	1·19	+ 5·70	+ 2·8
222·7	36·72	18·	0·540	1411	1·22	+ 6·72	+ 3·3
221·2	37·74	18·5	0·554	1377	1·25	+ 7·74	+ 3·8
225·6	38·76	19·	0·567	1343	1·29	+ 8·76	+ 4·3
227·1	39·78	19·5	0·581	1312	1·33	+ 9·78	+ 4·8
228.5	40·80	20·	0·595	1281	1·36	+10·80	+ 5·3
229·9	41·82	20·5	0·608	1253	1·40	+11·82	+ 5·8
231·2	42·84	21·	0·612	1225	1·43	+12·84	+ 6·3
232·5	43·86	21·5	0·636	1199	1·46	+13·86	+ 6·8
233·8	44·88	22·	0·65	1174	1·50	+14·88	+ 7·3
235·1	45·90	22·5	0·663	1150	1·53	+15·90	+ 7·8
236·3	46·92	23·	0·677	1127	1·56	+16·92	+ 8·3
237·5	47·94	23·5	0·690	1105	1·60	+17·94	+ 8·8
238·7	48·96	24·	0·704	1084	1·63	+18·96	+ 9·3
239·9	49·98	24·5	0·717	1064	1·67	+19·98	+ 9·8
241·	51·00	25·	0·730	1044	1·70	+21·00	+10·3
243·3	53·04	26·	0·756	1007	1·77	+23·04	+11·3

Temperature of Steam.	Atmosph. included. Inches of Mercury.	Pounds per square Inch.	Specific gravity, air = 1·	k Volume compared with water.	Number of atmospheres.	Atmosphere excluded. Inches of Mercury.	Pounds per square Inch.
245·5°	55·08	27	0·784	973	1·83	+25·08	+12·3
247·6	57·12	28	0·810	941	1·90	+27·12	+13·3
249·6	59·16	29	0·836	911	1·97	+29·16	+14·3
251·6	61·20	30	0·863	883	2·04	+31·20	+15·3
253·6	63·24	31	0·889	857	2·11	+33·24	+16·3
255·5	65·28	32	0·915	833	2·18	+35·28	+17·3
257·3	67·32	33	0·941	810	2·24	+37·32	+18·3
259·1	69·36	34	0·968	788	2·31	+39·36	+19·3
260·9	71·40	35	0·993	767	2·38	+41·40	+20·3
262·6	73·44	36	1·020	748	2·45	+43·44	+21·3
264·3	75·48	37	1·045	729	2·52	+45·48	+22·3
265·9	77·52	38	1·071	712	2·59	+47·52	+23·3
267·5	79·56	39	1·097	695	2·65	+49·56	+24·3
269·1	81·60	40	1·122	679	2·72	+51·60	+25·3
270·6	83·64	41	1·148	664	2·79	+53·64	+26·3
272·1	85·68	42	1·175	649	2·86	+55·68	+27·3
273·6	87·72	43	1·200	635	2·92	+57·72	+28·3
275·	89·76	44	1·225	622	3·00	+59·76	+29·3
276·4	91·80	45	1·249	610	3·06	+61·80	+30·3
277·8	93·84	46	1·275	598	3·13	+63·84	+31·3
279·2	95·88	47	1·567	586	3·20	+65·88	+32·3
280·5	97·92	48	1·325	575	3·26	+67·92	+33·3
281·9	99·96	49	1·351	564	3·32	+69·96	+34·3
283·2	102·0	50	1·376	554	3·40	+72·00	+35·3
284·4	104·0	51	1·400	544	3·47	+74·00	+36·3
285·7	106·1	52	1·426	534	3·53	+76·1	+37·3
286·9	108·1	53	1·450	525	3·60	+78·1	+38·3
288·1	110·2	54	1·477	516	3·67	+80·2	+39·3
289·3	112·2	55	1·500	508	3·74	+82·2	+40·3
290·5	114·2	56	1·523	500	3·81	+84·2	+41·3
291·7	116·3	57	1·548	492	3·88	+86·3	+42·3
292·9	118·3	58	1·575	484	3·94	+88·3	+43·3
294·2	120·4	59	1·598	477	4·01	+90·4	+44·3
295·6	122·4	60	1·621	470	4·08	+92·4	+45·3
296·9	124·4	61	1·646	463	4·15	+94·4	+46·3
298·1	126·5	62	1·671	456	4·22	+96·5	+47·3
299·2	128·5	63	1·698	449	4·28	+98·5	+48·3
300·3	130·5	64	1·719	443	4·35	+100·5	+49·3
301·3	132·6	65	1·743	437	4·42	+102·6	+50·3
302·4	134·6	66	1·755	434	4·49	+104·6	+51·3
303·4	136·7	67	1·794	425	4·55	+106·7	+52·3
304·4	138·7	68	1·818	419	4·62	+108·7	+53·3
305·4	140·8	69	1·839	414	4·69	+110·8	+54·3
306·4	142·8	70	1·868	408	4·76	+112·8	+55·3
307·4	144·8	71	1·891	403	4·82	+114·8	+56·3
308·4	146·9	72	1·915	398	4·89	+116·9	+57·3
309·3	148·9	73	1·938	393	4·96	+118·9	+58·3
310·3	151·0	74	1·963	388	5·03	+121·0	+59·3
311·2	153·0	75	1·991	383	5·09	+123·	+60·3
312·2	155·1	76	2·011	379	5·17	+125·1	+61·3
313·1	157·1	77	2·036	374	5·23	+127·1	+62·3

Temperature of Steam.	Atmosph. included: Inches of Mercury.	Pounds per square Inch.	Specific gravity, air = 1.	k Volume compared with water.	Number of atmospheres.	Atmosphere excluded.: Inches of Mercury.	Pounds per square inch.
314·0°	159·1	78	2·060	370	5·30	+129·1	+ 63·3
314·9	161·2	79	2·081	366	5·37	+131·2	+ 64·3
315·8	163·2	80	2·105	362	5·44	+133·2	+ 65·3
316·7	165·3	81	2·128	358	5·51	+135·3	+ 66·3
317·6	167·3	82	2·152	354	5·57	+137·3	+ 67·3
318·4	169·3	83	2·178	350	5·64	+139·3	+ 68·3
319·3	171·4	84	2·203	346	5·71	+141·4	+ 69·3
320·1	173·4	85	2·228	342	5·78	+143·4	+ 70·3
321·0	175·5	86	2·248	339	5·85	+145·5	+ 71·3
321·8	177·5	87	2·275	335	5·91	+147·5	+ 72·3
322·6	179·6	88	2·295	332	5·98	+149·6	+ 73·3
323·5	181·6	89	2·322	328	6·05	+151·6	+ 74·3
324·3	183·6	90	2·343	325	6·12	+153·6	+ 75·3
325·1	185·8	91	2·365	322	6·19	+155·6	+ 76·3
325·9	187·8	92	2·389	319	6·26	+157·8	+ 77·3
326·7	189·8	93	2·411	316	6·32	+159·8	+ 78·3
327·5	191·9	94	2·435	313	6·39	+161·9	+ 79·3
328·2	193·9	95	2·459	310	6·46	+163·9	+ 80·3
329·0	196·0	96	2·483	307	6·53	+166·0	+ 81·3
329·8	198·0	97	2·505	304	6·60	+168·0	+ 82·3
330·5	200·0	98	2·530	301	6·66	+170·0	+ 83·3
331·3	202·0	99	2·558	298	6·73	+172·0	+ 84·3
332·0	204·0	100	2·583	295	6·80	+174·0	+ 85·3
335·8	214·2	105	2·703	282	7·13	+194·2	+ 90·3
339·2	224·4	110	2·815	271	7·47	+194·4	+ 95·3
342·7	234·6	115	2·947	259	7·82	+204·6	+100·3
345·8	244·8	120	3·036	251	8·15	+214·8	+105·3
349·1	255·0	125	3·178	240	8·5	+225·0	+110·3
352·1	265·2	130	3·270	233	8·83	+235·2	+115·3
355·0	275·4	135	3·405	224	9·16	+245·4	+120·3
357·9	285·6	140	3·497	218	9·51	+255·6	+125·3
360·6	295·8	145	3·626	210	9·83	+265·8	+130·3
363·4	306·0	150	3·712	205	10·2	+276·0	+135·3
368·7	326·4	160	3·941	193	10·9	+296·4	+145·3
373·6	346·8	170	4·028	183	11·5	+316·8	+155·3
378·4	867·2	180	4·375	174	12·2	+337·2	+165·3
382·9	387·6	190	4·585	166	12·9	+357·6	+175·3
387·3	408·0	200	4·82	158	13·6	+378·0	+185·3
403·8	509·	250	5·90	129	17·0	+479·	+235·3
420·3	612·	300	7·00	109	20·4	+582·	+285·3
435·0	714·	350	8·00	95	23·8	+684·	+345·3
446·5	816·	400	8·95	85	27·2	+786·	+385·3
471·3	1019·	500	10·9	70	34·0	+989·	+485·3
487·0	1223·	600	12·8	59	40·8	+1193·	+585·3
519·	1631·	800	15·7	48	54·4	+1601·	+785·3
548.	2038·	1000	19·7	38	68 0	+2008·	+985·3

Letters denote.

F = force of the steam, or pressure per square inch in pounds.
I = inches of Mercury that balances the steam.
T = temperature of the steam in degrees of Fahrenheit's Thermometer.

Formulas for Steam above 212°.

$$F = \left(\frac{T}{202} + 0{\cdot}52\right)^6, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 1,$$

$$I = \left(\frac{T}{180} + 0{\cdot}584\right)^6, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 2,$$

$$T = (\sqrt[6]{F} - 0{\cdot}52)202, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad 3.$$

Example 1. The temperature of a quantity of steam is found to be 275°. Required the density in pounds per square inch?

$$F = \left(\frac{275}{202} + 0{\cdot}52\right)^6 = 44{\cdot}3 \text{ pounds per square inch.}$$

By logarithms, $\frac{275}{202} + 0{\cdot}52 = 1{\cdot}881.$

$$\begin{aligned} \log.1{\cdot}881 &= 0{\cdot}274389 \\ &\quad\;\; \underline{\qquad\quad 6} \\ \log.44{\cdot}3 &= 1{\cdot}646334 \quad \text{or } 44{\cdot}3 \text{ pound per square inch.} \end{aligned}$$

The properties of steam are calculated and contained in the accompanying Table, as noted on the top. The two last columns contain the inches of mercury and pressure per square inch commonly expressed in practice; it is *O* at the temperature 212°, and below that temperature it is negative, which denotes so much vacuum. If the temperature in a condenser is 120, the vacuum is 13·07 pounds.

To Find the Weight of Steam,

RULE. Multiply the specific gravity of the steam by the weight of one cubic foot of air = 0·07529, and the product is the weight per cubic foot of the steam in pounds.

To Find the Quantity of Water of which a given quantity of Steam has been, or can be produced.

RULE. Divide the cubic contents of the steam by the volume k in the Table, and the quotient is the cubic content of the water.

Force or Feed Pumps.

Letters denote.

$\left.\begin{aligned} d &= \text{diameter} \\ s &= \text{stroke,} \end{aligned}\right\}$ of the force pump, single acting.

$\left.\begin{aligned} D &= \text{diameter} \\ S &= \text{stroke} \end{aligned}\right\}$ of the steam cylinder piston, in inches, double acting.

k = volume given in the Table at the given pressure of steam.

The stroke of the steam piston is only that under which steam is admitted to the cylinder.

$$d = 2D\sqrt{\frac{S}{k\,s}}, \qquad s = 4\frac{D^2 S}{k d^2}, \qquad \cdot \quad \cdot \quad 4, 5,$$

Slip water included.

Example. Required the diameter of a force-pump having the same stroke as the cylinder piston $s = 38$ inches, diameter of cylinder $D = 30$ inches, the steam is cut off at $\frac{1}{2}$ the stroke, and the steam pressure $+ 50$ pounds per square inch? Here $k = 437$, and $S = 19$ inches, because steam is cut off at $\frac{1}{2}$ the stroke.

$$d = 2 \times 30 \sqrt{\frac{19}{437 \times 38}} = 2{\cdot}03 \text{ inches.}$$

To find the Quantity of Condensing Water.

Letters denote.

q = condensing water of temperature t, in cubic feet.
Q = steam of temperature T, in cubic feet.
k = volume in the Table.
t' = temperature in the condenser when the steam and water are mixed.

$$q = \frac{1{\cdot}4Q(990 + T - t')}{k(t' - t)}, \quad . \quad . \quad . \quad . \quad 6,$$

Dimensions of the Air Pump.

$\left.\begin{array}{l} d = \text{diamter} \\ s = \text{stroke} \end{array}\right\}$ of the air pump, single acting.

$\left.\begin{array}{l} D = \text{diameter} \\ S = \text{stroke} \end{array}\right\}$ of the steam cylinder, double acting.

$$d = 2{\cdot}3D \sqrt{\frac{S(990 + T - t')}{k\,s(t' - t)}}, \quad . \quad . \quad . \quad 7,$$

Assume $t' = 100°$, and $t = 50°$, we shall have,

$$\left.\begin{array}{ll} d = 0{\cdot}326D \sqrt{\dfrac{S(940 + T)}{k\,s}}, & 8, \\ s = 0{\cdot}106D^2 \dfrac{S(940 + T)}{k\,d^2}, & 9, \end{array}\right\} \text{single acting air-pumps.}$$

$$\left.\begin{array}{ll} d = 0{\cdot}23D \sqrt{\dfrac{S(940 + T)}{k\,s}} \quad . \quad . \quad . & 10, \\ s = 0{\cdot}053D^2 \dfrac{S(940 + T)}{k\,d^2}, \quad . \quad . \quad . & 11. \end{array}\right\} \text{double acting air pumps.}$$

Example. A single acting air-pump is to be constructed for an engine $D = 38$ inches, $S = 45$ inches stroke of the cylinder; the stroke of the air-pump can be 32 inches, and the exhaust steam is 261°. Required the diameter of the air-pump? $k = 767$.

$$d = 0{\cdot}326 \times 38 \sqrt{\frac{45(940 + 261)}{767 \times 32}} = 18{\cdot}25 \text{ inches.}$$

☞ Slip water included. T and k must be taken for the exhaust steam, as the steam may have had worked expansively; the area of the foot valve must be calculated from the following formulas.

Foot Valve in the Air Pump.

To render an air-pump to work well, and with the greatest advantage, it is necessary to pay particular attention to the following formulas. The force by which the water is driven from the condenser through the footvalve into the air-pump is limited by the pressure in the condenser; this pressure is the vacuum subtracted from 14·7 pounds; it is noted in the third column where the temperature in the condenser is opposite, in the first column. Every pound of this pressure per square inch balances a column of water 27 inches high, which is the head that presses the water from the condenser.

Letters denote.

$\mathfrak{A}$ = area of the air-pump piston.
$\mathfrak{a}$ = area of the foot-valve, or bucket-valve.
$\mathfrak{D}$ = diameter of the air-pump-piston.
$\mathfrak{d}$ = diameter of the foot-valve, when round.
$\mathfrak{S}$ = stroke of air-pump piston, in feet.
$\mathfrak{P}$ = pressure in the condenser at the temperature T.
n = number of strokes of the air-pump piston per minute.

$$\mathfrak{a} = \frac{D^2 S\, n\, (990 + T)}{23000\, m\, k \sqrt{\mathfrak{p}}}.$$

$m = 0{\cdot}6$ to $0{\cdot}8$

$$\mathfrak{a} = \frac{\mathfrak{A}\,\mathfrak{S}\,n}{100\sqrt{\mathfrak{P}}}, \qquad 12,$$

$$\mathfrak{S} = \frac{100\mathfrak{a}\sqrt{\mathfrak{P}}}{n\,\mathfrak{A}}, \qquad 13,$$

$$n = \frac{100\mathfrak{a}\sqrt{\mathfrak{P}}}{\mathfrak{A}\,\mathfrak{S}}, \qquad 14,$$

$$\mathfrak{d} = \frac{\mathfrak{D}\sqrt{\mathfrak{S}\,n}}{10\sqrt[4]{\mathfrak{P}}}, \qquad 15,$$

$$\mathfrak{S} = \frac{100\mathfrak{d}^2\sqrt{\mathfrak{P}}}{n\,\mathfrak{D}^2}, \qquad 16,$$

$$n = \frac{100\mathfrak{d}^2\sqrt{\mathfrak{P}}}{\mathfrak{D}^2\,\mathfrak{S}}, \qquad 17.$$

Example. The area $\mathfrak{A}$ of an air-pump-piston is 2·35 square feet, stroke of piston $\mathfrak{S} = 3{\cdot}6$ feet, to make $n = 40$ strokes per minute, and the pressure to be $\mathfrak{P} = 3{\cdot}2$ pounds. Required the area of the foot-valve.

$$\mathfrak{a} = \frac{2{\cdot}35 \times 3{\cdot}6 \times 40}{100\sqrt{3{\cdot}2}} = 1{\cdot}85 \text{ square feet.}$$

To Find the Velocity and Quantity of the Injection Water through the Adjustage into the Condenser.

Letters denote.

v = velocity in feet per second.
h = head of the press water; + when above, and — below the adjustage.
F = vacuum, noted — or negative in the last column, but is positive in the formulas.
q = quantity of water discharged in cubic feet, per second.
a = area of all the holes in the adjustage in square feet.
d = diameter, L = length } of the injection pipe, in feet.
n = double strokes of cylinder-piston, or revolutions per minute.
A, D, and S, dimensions of the steam cylinder, in feet.
T = temperature, and k = volume coefficient of the exhaust steam.

$$a = \frac{q}{5\sqrt{2F \pm h}}, \qquad 18,$$

$$v = 8\sqrt{2F \pm h} \qquad 19,$$

$$q = \frac{n\,S\,D^2(940 + T)}{55k}, \qquad 20,$$

$$q = 5a\sqrt{2F \pm h}, \qquad 21,$$

$$d = 0{\cdot}35\sqrt[5]{\frac{L\,q^2}{2F \pm h}}, \qquad 22,$$

$$a = \frac{n\,S\,D^2(940 + T)}{275k\sqrt{2F + h}}, \qquad 23,$$

Example. Required the diameter of an injection pipe $L = 10$ feet long, which shall supply $q = 1{\cdot}3$ cubic feet of water per second into a vacuum of 12 pounds per square inch, the head of press water $h = 2$ feet?

$$d = 0{\cdot}35 \sqrt[5]{\frac{10 \times 1{\cdot}3}{2 \times 12 + 2}} = 0{\cdot}3055 \text{ feet} = 3\tfrac{11}{16} \text{ inches.}$$

Area of Steam Passages.

a = area of the steam pipe, sq. in.
A = area of the cylinder piston, sq. in.
d = diameter of the pipe, in inches.
D = diameter, S = stroke of cylinder, in inches.

$$\mathbf{a} = \frac{A\,S\,n}{35000}, \qquad d = \frac{D\sqrt{S\,n}}{186}, \qquad \cdot \quad \cdot \qquad 24, 25.$$

Example. Required the diameter of a steam-pipe for a cylinder D = 40 inches. Stroke of piston $S = 48$ inches, and $n = 38$ revolutions per minute?

$$d = \frac{40\sqrt{48 \times 38}}{186} = 9{\cdot}2 \text{ inches, nearly.}$$

Steam Ports to the Cylinder.

$$\mathbf{a} = \frac{A\,S\,n}{30600}, \qquad \cdot \quad \cdot \quad \cdot \quad \cdot \qquad 26,$$

Safety Valve.

Three-fourths of the fire grate in square feet is a good proportion for the safety valve in square inches.

Notation of Letters corresponds with Figure 3, Plate VIII.

a = area of safety valve in square inches.

$$\left.\begin{array}{l} P = \text{pressure per square inch in the boiler} \\ W = \text{weight on the safety valve lever} \\ Q = \text{weight of the safety valve and lever} \end{array}\right\} \text{in pounds.}$$

$$\left.\begin{array}{l} l = \text{lever for } W \\ e = \quad \text{"} \quad \mathbf{a}P \\ x = \quad \text{"} \quad Q \end{array}\right\} \text{in inches.}$$

Balance the lever over a sharp edge, and the centre of gravity Q is found; measure the distance x from the fulcrum C.

$$\mathbf{a}\,P\,e = W\,l + Q\,x \qquad 27, \qquad\qquad W = \frac{\mathbf{a}\,P\,e - Q\,x}{l}, \qquad 29,$$

$$P = \frac{W\,l + Q\,x}{\mathbf{a}\,e}, \qquad 28, \qquad\qquad l = \frac{\mathbf{a}\,P\,e - Q\,x}{W}, \qquad 30,$$

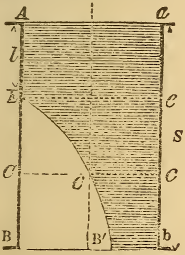
Example. Area of the safety valve **a** = 9 square inches, $e = 4\tfrac{1}{2}$ inches, $W = 50$ pounds, weight of the lever and safety valve $Q = 15$ pounds, and $x = 17$ inches. Required at what distances l, l' and l'' will the weight W indicate pressures of $P = 30$, $P' = 40$, and $P'' = 50$ pounds?

$$l = \frac{9 \times 30 \times 4{\cdot}5 - 15 \times 17}{50} = 29{\cdot}2 \text{ inches,}$$

from the fulcrum C the weight W will indicate $P = 30$ pounds.
$l' = 37{\cdot}9$ inches, when $P' = 40$ pounds.
$l'' = 45{\cdot}8$ " " $P'' = 50$ "
and thus the lever can be graduated.

EXPANSION OF STEAM.

In order to save steam, or more correctly to employ its effect to a higher degree, the admittance of steam to the cylinder is shut off when the piston has moved a part of the stroke; from the cut-off point the steam acts expansively with a decreased pressure on the piston, as represented by the accompanying figure.



Let the steam be cut off at $\frac{1}{3}$ of the stroke, and Aa represent the total pressure, say 20 pounds per square inch which will continue to the point E where the admittance of steam is shut off at one-third the stroke S. The steam $Aa\,eE$, is now acting expansively on the piston, and the pressure decreases as the volume increases, when the piston has attained Cc or two-thirds of S, the pressure $C'c=10$ pounds, only half the pressure $Aa=20$ because the volume $Aa\,eE$ is only half of $Aa\,cC$, and so on until the piston has attained $B\,b$ the pressure $B'b=\frac{1}{3}\times 20=6{\cdot}66$ pounds.

The mean pressure, or the effectual pressure, throughout the stroke, will be about 13·33 pounds per square inch, or 66 per cent., but the quantity of steam used is only 33 per cent., hence 33 per cent. is gained by using the steam expansively.

l = part of the stroke S in feet, at which the steam is cut off.
P = pressure per square inch under full admittance of steam.
F = mean pressure per square inch throughout the stroke S.
f = mean pressure per square inch during the expansion, which in double expansion cylinder engines will be the average pressure per square inch on the large piston A.
p = end pressure per square inch after expansion.
S = stroke of the cylinder Piston in feet.

$$F=\frac{Pl}{S-l}(2{\cdot}3\log.S+1) \qquad f=\frac{FS-Pl}{S-l}, \qquad p=\frac{Pl}{S}.$$

The following Tables are calculated from these formulas.

Example 1. Required from the Table I. the mean pressure F for $P=32$ lbs. at five-eights expansion.

$$\text{Add}\left\{\begin{array}{ll}\text{For 30 lbs.} & F=22{\cdot}252\\ \text{“ 2 “} & F=\ \ 1{\cdot}483\end{array}\right\}\text{from the Table I.}$$

Mean pressure of 32 lbs. $F=23{\cdot}735$ the answer.

Example 2. Required from Table II. the mean pressure f, per square inch during the expansion, or on the large piston A in double cylinder engines, when the initial pressure $P=75$ lbs. and under two-thirds expansion? $f=40{\cdot}75$ Table II.

Example 3. Required the mean pressure $f=?$ for an initial pressure $P=43$ lbs. under $\frac{3}{4}$ expansion?

$$\begin{array}{ll}\text{For } P=40\text{ lbs.} & f=18{\cdot}48\\ P=30\text{ or 3 lbs.} & f=\ \ 1{\cdot}38\end{array}\Big\}\text{ Table II.}$$

$$P=43\text{ lbs.} \qquad f=19{\cdot}86\text{ the answer.}$$

The effect gained or fuel saved by expansion and high steam is calculated from the following formulæ, in which it is supposed as a unit the work of an engine with $P=30$ pounds per square inch, or an indicated pressure of 15 lbs. without expansion.

c = per cent on 100, of effect gained or fuel saved.

$$\text{For expansion } c=100\left(1-\frac{lP}{SF}\right). \qquad \text{For high steam } c=100\left(1-\frac{26490}{kP}\right).$$

The following Table III. is calculated from these formulæ, in which the first line from 30 contains the economy per cent. from expansion alone, and the column o contains the economy per cent. from high steam above $P=30$ lbs. The balance of the table contains the jointed economy of expansion and high steam. Required the jointed economy of $P=90$ lbs. under $\frac{1}{3}$ expansion? 50·5 per cent. the answer.

Press. P.	Grade of Expansion. $\frac{1}{4}$	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{7}{8}$
1	0·9637	0·9333	0·9187	0·8765	0·7417	0·6991	0·5965	0·3848
2	1·9275	1·8666	1·8375	1·6930	1·4835	1·3982	1·1930	0·7697
3	2·8912	2·7999	2·7562	2·5395	2·2252	2·0873	1·7895	1·1546
4	3·8550	3·7333	3·6750	3·3860	2·9670	2·7964	2·3860	1·5395
5	4·8185	4·6666	4·5935	4·2325	3·7085	3·4955	2·9825	1·9240
6	5·7822	5·5999	5·5122	5·0790	4·4502	4·1946	3·5790	2·2008
7	6·7469	6·5334	6·4319	5·9255	5·2419	4·8937	4·1755	2·6946
8	7·7106	7·5666	7·3396	6·7720	5·9346	5·5928	4·7720	3·0784
9	8·6733	8·3999	8·2683	7·6185	6·6753	6·2919	5·3685	3·4632
10	9·6370	9·3333	9·1870	8·4656	7·4170	6·9912	5·9650	3·8480
11	10·601	10·266	10·106	9·3115	8·1597	7·7001	6·5615	4·2338
12	11·565	11·199	10·925	10·158	8·9014	8·3892	7·1580	4·6186
13	12·528	12·133	11·943	11·004	9·6421	9·0783	7·7545	5·0034
14	13·492	13·066	12·862	11·851	10·384	9·7874	8·5310	5·3882
15	14·456	13·930	13·781	12·698	11·126	10·486	8·9475	5·7730
16	15·420	14·933	14·700	13·544	11·268	11·185	9·5440	6·1582
17	16·383	15·866	15·618	14·390	12·609	11·884	10·140	6·5426
18	17·347	16·799	16·537	15·237	13·531	12·483	10·737	6·9274
19	18·311	17·733	17·448	16·803	14·093	13·183	11·333	7·3122
20	19·275	18·666	18·375	16·930	14·835	13·902	11·930	7·6970
21	20·238	19·599	19·293	17·776	15·576	14·501	12·526	8·0818
22	21·201	20·532	20·211	18·623	16·318	15·300	13·123	8·4667
23	22·166	21·466	21·131	19·469	17·060	15·989	13·720	8·8516
24	23·030	22·399	22·050	20·316	17·802	16·698	14·316	9·4365
25	24·093	23·333	22·968	21·162	18·573	17·477	14·912	9·6210
26	25·057	24·266	23·887	22·009	19·285	18·046	15·509	9·8978
28	26·985	26·233	25·714	23·702	20·769	19·495	16·702	10·775
30	28·912	27·999	27·562	25·395	22·252	20·873	17·895	11·546
35	33·731	32·666	32·156	29·627	25·961	24·368	20·877	13·470
40	38·550	37·333	36·750	33·860	29·670	27·964	23·860	15·395
45	43·368	42·000	41·341	38·092	33·378	31·459	26·842	17·319
50	48·187	46·666	45·937	42·325	37·067	34·955	29·825	19·243
55	53·005	51·333	50·530	46·557	40·775	38·450	32·807	21·167
60	57·822	55·999	55·122	50·790	44·520	41·946	35·790	23·090
65	62·640	60·666	59·715	55·022	48·228	45·441	38·772	24·924
70	67·460	65·333	64·300	59·255	52·419	48·937	41·755	26·694
75	72·278	69·999	68·893	63·487	56·127	52·432	44·737	28·626
80	77·096	75·666	73·500	67·720	59·340	55·928	47·720	30·790
85	81·914	80·333	78·093	71·952	63·048	59·423	50·702	32·714
90	86·730	83·999	82·680	76·180	66·750	62·919	53·680	34·638
95	91·548	88·666	87·273	80·412	70·458	66·414	56·662	36·554
100	96·370	93·333	91·870	84·650	74·170	69·910	59·650	38·480
105	101·18	97·999	96·463	88·882	77·878	73·405	62·632	40·404
110	105·99	101·66	101·05	93·120	81·586	76·900	66·614	42·328
115	110·80	106·33	105·64	97·352	85·294	80·395	69·596	44·252
125	120·46	115·66	114·83	105·81	102·83	87·387	74·562	48·101
140	134·92	130·66	128·62	118·51	103·84	97·874	85·310	53·882
150	144·56	139·33	137·81	126·47	111·26	104·86	89·470	57·730
200	192·75	186·66	183·75	169·30	148·35	139·02	119·30	76·970
250	240·93	233·33	229·68	211·62	185·43	174·77	149·12	96·210
300	289·12	279·99	275·62	253·95	222·52	208·73	178·95	115·46

Mean Pressure f during the Expansion.

Pres. P.	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{7}{8}$
30	28·549	24	23·50	20·79	17·60	16·31	13·86	8·9097
35	33·308	28	27·41	24·25	20·54	19·02	16·17	10·394
40	38·066	32	31·83	27·72	23·47	21·73	18·48	11·879
45	42·824	36	35·25	31·18	26·40	24·46	20·79	13·364
50	47·582	40	39·16	34·65	29·33	27·16	23·10	14·849
55	52·340	44	43·08	38·11	32·24	30·17	25·41	16·334
60	57·098	48	47·00	41·58	35·20	32·62	27·72	17·819
65	61·853	52	50·91	45·04	38·14	35·33	30·03	19·303
70	66·616	56	54·83	48·51	41·07	38·04	32·34	20·788
75	71·371	60	58·75	51·90	44·00	40·75	34·65	22·263
80	76·128	64	62·66	55·44	46·94	43·47	36·96	23·758
85	80·885	68	66·18	58·90	49·87	46·19	39·27	25·243
90	86·448	72	70·50	62·37	52·80	48·93	41·58	26·729
95	90·391	76	74·41	65·73	55·73	51·62	43·89	28·213
100	95·160	80	78·33	69·30	58·66	54·33	46·20	29·699
105	99·910	84	82·24	72·76	61·57	57·33	48·51	31·183
110	104·68	88	86·16	76·23	64·48	60·35	50·82	32·669
115	109·40	92	90·08	79·69	67·44	62·79	53·13	34·153
125	118·95	100	97·91	97·02	73·34	67·95	57·75	37·122
140	133·23	112	109·6	97·02	82·14	76·08	64·68	41·576
150	142·74	120	117·5	103.9	88·00	81·50	69·30	44·548
200	190·32	160	156·6	138.6	117·3	108·6	92·40	59·398
250	237·07	200	195·7	173.2	146·6	135.8	115·5	74·247
300	288·16	240	235·0	207.9	176·0	163.1	138·6	89·097

Table III. Economy of Expansion and high Steam.
Fuel saved or effect gained per cent.

Pres. P.	0	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{7}{8}$
30	0	12	29·5	32	41	49·3	52	58	67·5
35	1·6	13·6	31	33·6	42·6	51	53·6	59·6	69·1
40	2·5	14·5	32	34·5	43·5	51·8	54·5	60·5	70
45	3·4	15·4	33	35·4	44·4	52·7	55·4	61·4	71
50	4.3	16·3	33·8	36·3	45·3	53·6	56·3	62·3	71·8
55	5·2	17·2	34·7	37·2	46·2	54·5	57·2	63·2	72·7
60	6	18	35·7	38	47·	55·3	58	64	73·5
65	6·7	18·7	36.2	38·7	47·7	56	58·7	64·7	74·2
70	7·3	19.3	36.8	39·3	48·3	56·6	59·3	65·3	74·8
75	7·8	19·8	37·3	39·8	48·8	57·1	59·8	65·8	75·3
80	8·5	20·5	38	40·5	49·5	57·8	60·5	66·5	76
85	9	21	38·5	41	50	58·3	61	67	76·5
90	9·5	21·5	39	41·5	50·5	58·8	61·5	67·5	77
95	10	22	39·5	42	51	59·3	62	68	77·5
100	10·4	22·4	40	42.4	51.4	59·7	62·4	68·4	78
105	10·7	22·7	40·2	42·7	51·7	60·	62·7	68·7	78·2
115	11	23	40·5	43	52	60·3	63	69	78·5
125	11·7	23·7	41·2	43·7	52·7	61	63·7	69·7	79·2
150	14	26	43·5	46	55	63·3	66	72	81·5
200	16	28	45·5	48	57	65·3	68	74	83·5
250	17·7	29·7	46·2	49·7	58·7	67	69·7	75·7	85·2
300	19	31	48·5	51	60	68·3	71	77	86.5

Table IV.

Consumption of Coal in pounds per horse power per hour,

Grade of Expansion.

Pres. P.	0	1/4	1/3	3/8	1/2	5/8	2/3	3/4	7/8
lbs,	lbs,	lbs,	lbs,	lbs,	lbs,	lbs,	lbs,	lbs,	lbs,
30	5·6	4·93	3·95	3·81	3·30	2·84	2·69	2·35	1·82
35	5·51	4·84	3·86	3 72	3·21	2·74	2·60	2·26	1·73
40	5·46	4·79	3·81	3·67	3·16	2·70	2·55	2·21	1·68
45	5·41	4·73	3·75	3·62	3·11	2·65	2·50	2·16	1·62
50	5·36	4·68	3·71	3·57	3·06	2·60	2·45	2·11	1·58
55	5·31	4·63	3·66	3·51	3·01	2·55	2·40	2·06	1·53
60	5·26	4·59	3·60	3·47	2·97	2·50	2·35	2·02	1·49
65	5·20	4·55	3·57	3·43	2·93	2·46	2·31	1·98	1·45
70	5·19	4·52	3·54	3·40	2·90	2·43	2·28	1·94	1·41
75	5·16	4.49	3·51	3·37	2·87	2·40	2·25	1·91	1·39
80	5·12	4·45	3·47	3·33	2·83	2·36	2·21	1·88	1·35
85	5·09	4·42	3·44	3·30	2·80	2·33	2·18	1·85	1·32
90	5·07	4·39	3·41	3·28	2·77	2·31	2·16	1·82	1·29
95	5·04	4·37	3·39	3·25	2·74	2·28	2·13	1·79	1·26
100	5·01	4·34	3·36	3·23	2·72	2·26	2·10	1·77	1·23
105	5·00	4·32	3·35	3·21	2·70	2·24	2·09	1·75	1·22
115	4·98	4·31	3·33	3·19	2·69	2·22	2·07	1·73	1·20
125	4·94	4·27	3·29	3·15	2·65	2·19	2·03	1·70	1·17
150	4·81	4·14	3·16	3·02	2·52	2·05	1·90	1·57	1·04
200	4·70	4·03	3·05	2·91	2 41	1·94	1·79	1·46	0·92
250	4·60	3·93	3·01	2·81	2·31	1·85	1·70	1·36	0·83
300	4·54	3·87	2·89	2·75	2·24	1·78	1·62	1·29	0·75

Table V.

Consumption of Coal in tons per 100 horses in 24 hours.

Pres. P.	0	1/4	1/3	3/8	1/2	5/8	2/3	3/4	7/8
lbs.	tons	tons,	tons,	tons,	tons,	tons,	tons,	tons,	tons,
30	6·00	5·29	4·23	4·09	3·54	3·04	2·88	2·52	1·95
35	5·90	5·19	4·13	3·99	3·44	2·94	2·79	2·42	1·86
40	5·85	5·13	4·08	3·93	3·39	2·90	2·73	2 37	1·80
45	5·80	5·07	4·02	3·88	3·34	2·84	2·68	2·31	1·73
50	5·75	5 01	3·97	3·83	3·28	2·79	2·63	2·26	1·69
55	5·70	4·96	3·92	3·77	3·22	2·73	2·57	2·21	1·64
60	5·64	4·92	3·87	3·72	3·18	2·68	2·52	2·17	1·60
65	5·58	4·88	3·82	3·68	3·14	2·63	2·48	2·12	1·55
70	5·56	4·84	3·79	3·64	3·11	2·60	2·44	2·08	1·51
75	5·53	4·81	3·76	3·61	3·07	2·57	2·41	2·05	1·49
80	5·49	4·77	3·72	3·57	3·03	2·53	2·37	2·01	1·44
85	5·46	4·74	3·69	3·54	3·00	2·50	2·33	1·98	1·41
90	5·43	4·70	3·66	3·51	3·97	2·47	2·31	1·95	1·38
95	5·40	4·68	3·63	3·48	2·94	2·44	2·28	1·92	1·35
100	5·37	4·65	3·60	3·46	2·91	2·42	2·26	1·90	1·32
105	5·36	4·63	3·59	3·44	2·89	2·40	2·24	1·88	1·31
115	5·34	4·61	3·57	3·42	2·88	2·38	2·22	1·85	1·29
125	5·30	4·58	3·53	3·38	2·84	2·34	2·18	1·82	1·25
150	5·16	4·44	3.39	3·34	2·81	2·30	2·04	1·68	1·11
200	5·04	4·32	3·27	3·12	2·59	2·19	1·92	1·56	0·99
250	4·93	4·21	3·22	3·01	2·47	2·09	1·82	1·46	0·89
300	4·87	4·15	3·10	2 95	2·40	2·01	1·74	1·38	0·83

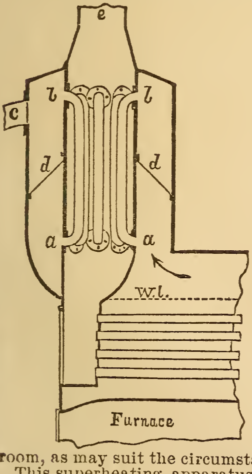
SUPERHEATED STEAM.

The Author's experience in superheated steam has been sufficient to convince him of its great importance. It appears that in order to utilize the maximum effect of steam or at least to attain the maximum quality of expansion, it is not necessary to overheat it after a pure steam is formed, that is, when all the small particles and bubbles of water are evaporated. Water which accompanies the steam in such a form has the same temperature as that due to the surrounding steam pressure, prevents it to vaporise; but when it passes through the superheating apparatus the temperature is greatly increased while the pressure remains the same because it being in connection with the steamroom in the boiler allows the water to vaporise and a pure steam may be formed.

If steam with particles of water is admitted into the cylinder part of the stroke and then allowed to expand, it is generally found that the end pressure does not come up to that due by theory, from which it has been pronounced that the expansive quality of steam does not follow that of a perfect gas, and that steam has condensed during the stroke; but if we knew the cubic containt of all the particles of water and subtracted that from the cubic containt of the steam it might be found that its expansive quality is not so far from that of a perfect gas. It appears also that the expansive quality is diminished by overheating pure steam.

The small particles of water contain a great deal more caloric per volume than the surrounding steam, consequently when admitted into the condenser a good vacuum cannot be formed so well as with pure steam. It is therefore of great importance to pay particular attention to the superheating of steam, otherwise economy by expansion will not be realized to the extent herein given by formulas and tables. It is also of great importance for expansion that the piston and steam valves are perfectly tight.

SUPERHEATING APPARATUS.



The accompanying figure represents a superheating apparatus such as the Author has built it in Russia, and is found to answer exceedingly well. The figure is a section of the forend of an ordinary tubular boiler with steamdrum and uptake. The chimney is made a great deal wider in the steamdrum and contracted to the usual size at *e*, of 0·16 times the area of the firegrate; if a strong fan blast is applied it may be better to contract it to 0·11 ▭. In the inside of the chimney are placed a number of copper tubes *a*, *a*, *b*, *b*, with flanges screwed to the side; the area of these tubes should be about four times that of the steampipe *c*. In the steamdrum is riveted steamtight a conical plate *d*, *d*, so that the steam cannot pass to the top without passing the superheating pipes. This superheating apparatus is in successful operation in three first class passenger steamers on the River Volga in Russia, each of 500 actual horses, and one in a steamer of 100 actual horses on the Black Sea.

The steamdrum can be placed around the chimney separately from the boiler and the steam led either above or below the plate *d*, *d*, by pipes from the steamroom, as may suit the circumstances.

This superheating apparatus may also be well suited for locomotives.

Diam. D	Stroke of Cylinder Piston S in feet.												
	1′	1′ 3″	1′ 6″	1′ 9″	2′	2′ 3″	2′ 6″	2′ 9″	3′	3′ 6″	4′	4′ 6″	5′
in.	H	H	H	H	H	H	H	H	H	H	H	H	H
6	3·6	3·88	4·12	4·33	4·53	4·72	4·88	5·04	5·19	5·47	5·71	5·94	6·16
7	4·9	5·27	5·61	5·90	6·17	6·43	6·65	6·86	7·07	7·44	7·78	8·10	8·38
8	6·4	6·90	7·32	7·71	8·06	8·39	8·68	8·96	9·23	9·72	10·1	10·6	11·0
9	8·1	8·72	9·27	9·75	10·2	10·6	11·0	11·3	11·7	12·3	12·9	13·4	13·9
10	10	10·8	11·4	12·0	12·6	13·1	13·6	14·0	14·4	15·2	15·9	16·5	17·1
11	12·1	13·0	13·9	14·6	15·2	15·8	16·4	16·9	17·4	18·3	19·2	20·0	20·7
12	14·4	15·5	16·5	17·4	18·1	18·9	19·5	20·2	20·8	21·9	22·9	23·8	24·6
13	16·9	18·2	19·3	20·3	21·3	22·1	22·9	23·7	24·4	25·6	26·8	27·9	28·9
14	19·6	21·1	22·4	23·6	24·7	25·7	26 6	27·4	28·3	29·7	31·1	32·4	33·5
15	2·25	24·2	25·8	27·1	28·3	29·5	30·5	31·5	32·4	34·1	35·7	37·1	38·5
16	25·6	27·4	29·3	30·8	32·2	33·5	34·7	35·8	37·0	38·9	40·6	42·2	43·8
17	28·9	31·1	33·1	34·8	36·4	37·9	39·2	40·5	41·7	43·9	45·9	47·7	49·4
18	32·4	34·9	37·1	39·0	40·8	42·5	44·0	45·4	46·8	49·2	51·4	53·5	55·4
19	36·1	38·9	41·3	43·5	45·5	47·3	49·0	50·6	52·1	54·8	57·3	59·6	61·7
20	40·0	43·1	45·8	48·2	50·4	52·4	54·3	56·0	57·7	60·7	63·5	66·0	68·4
21	44·1	47·5	50·5	53·1	55·6	57·8	59·8	61·7	63·6	67·0	70·0	72·8	75.4
22	48·4	52·1	55·4	58·3	61·0	63·4	65·6	64·8	69·8	73·5	76·8	80·0	82·8
23	52·9	57·0	60·5	63·7	66·7	69·3	71·8	74·1	76·3	80·3	84·0	87·4	90·5
24	57·6	62·0	65·9	69·4	72·6	75·5	78·1	80·7	83·1	87·4	91·5	95·2	98·6
25	62·5	67·3	71·5	75·3	78·7	81·9	84·8	87·5	90·2	94·8	99·2	103	107
26	67·6	72·8	77·3	81·5	85·2	88·6	91·7	94·7	97·5	102	107	111	116
27	72·9	78·5	83·5	87·8	91·9	95·6	99·0	102	105	111	116	120	125
28	78·4	84·4	89·8	94·5	98·8	102	106	110	113	119	124	129	134
29	84.1	90·5	96·2	101	106	110	114	118	121	128	133	139	144
30	90·0	96·9	103	108	113	118	122	126	130	137	143	149	154
31	96·1	103	110	116	121	126	130	134	139	146	153	159	164
32	102	110	117	123	129	134	138	143	148	155	163	170	175
33	109	117	124	131	137	142	147	152	157	165	173	180	186
34	115	124	132	139	145	151	157	162	167	175	183	190	198
35	122	132	140	148	154	160	166	172	177	186	194	202	210
36	129	140	148	156	163	170	176	182	187	197	205	214	222
37	137	147	156	165	172	180	186	192	198	208	217	226	234
38	144	155	165	174	182	190	196	202	209	218	229	238	247
39	152	164	174	183	192	200	206	213	220	231	241	251	260
40	160	172	183	193	202	210	217	224	231	243	254	264	274
42	176	190	202	212	222	231	240	347	254	268	280	291	302
44	193	208	221	233	244	254	263	271	280	294	307	320	331
46	211	228	242	255	266	277	287	297	306	321	336	350	362
48	230	248	264	277	290	302	313	323	332	350	366	380	394
50	250	269	286	301	315	328	339	350	360	380	397	413	427
52	270	291	309	326	340	354	367	378	390	410	429	446	463
54	291	314	333	351	367	382	396	408	420	443	463	481	500
60	360	388	412	433	453	472	488	504	519	547	571	594	616
66	435	468	498	525	548	571	591	610	628	661	690	718	744
72	518	558	593	626	653	679	704	726	748	787	822	856	886
78	608	655	696	734	766	784	825	852	877	924	964	1003	1039
84	705	759	807	851	888	924	957	989	1015	1071	1116	1166	1206
90	810	872	927	975	1020	1062	1100	1134	1168	1229	1284	1336	1385
96	921	991	1053	1110	1160	1206	1249	1291	1327	1400	1460	1505	1575

D	6′	7′	8′	9′	10′	11′	12′	13′	14′	15′	16′	18′	20′
in.	H	H	H	H	H	H	H	H	H	H	H	H	H
30	163	172	180	187	194	200	206	211	217	222	226	236	244
32	186	196	204	213	220	227	252	241	246	253	258	268	278
34	210	221	231	240	249	257	264	272	276	285	291	303	313
36	235	248	259	269	273	288	296	306	312	319	326	339	351
38	262	276	289	299	311	321	330	341	348	356	363	378	391
40	290	306	320	333	344	355	366	377	385	395	403	419	434
42	320	336	352	365	380	392	404	416	425	435	444	462	478
44	352	371	387	402	417	430	453	461	466	477	494	507	525
46	384	405	423	440	460	470	484	497	510	522	533	554	587
48	418	441	461	479	496	512	527	541	555	569	580	603	625
50	554	478	500	520	538	555	572	588	602	617	630	655	677
52	491	518	541	562	582	601	619	635	651	667	681	708	733
54	529	558	583	606	628	648	667	685	700	719	734	764	790
56	570	600	637	652	675	697	718	737	755	775	790	811	850
58	611	644	673	700	724	648	770	791	810	830	847	880	912
60	654	689	720	749	775	800	824	846	867	888	907	943	977
62	698	736	769	800	828	855	879	903	925	948	968	1007	1043
64	744	784	819	852	882	911	938	963	987	1010	1024	1073	1101
66	791	834	871	906	938	968	997	1024	1049	1074	1099	1141	1182
68	840	885	925	960	996	1023	1059	1087	1114	1140	1165	1211	1254
70	890	938	980	1019	1055	1089	1122	1152	1171	1208	1234	1283	1329
72	942	994	1037	1078	1116	1153	1187	1218	1249	1278	1306	1358	1406
74	995	1048	1095	1139	1179	1218	1254	1287	1319	1350	1380	1434	1485
76	1050	1105	1155	1201	1244	1284	1322	1358	1392	1424	1455	1512	1567
78	1105	1165	1219	1265	1310	1353	1393	1430	1466	1500	1533	1594	1649
80	1162	1225	1280	1331	1378	1423	1465	1504	1542	1578	1612	1676	1737
84	1282	1350	1411	1467	1520	1569	1615	1658	1700	1741	1778	1848	1914
88	1407	1423	1549	1610	1668	1722	1773	1820	1866	1909	1951	2029	2100
92	1538	1619	1693	1761	1823	1882	1938	1990	2039	2086	2133	2258	2297
96	1674	1763	1843	1917	1985	2049	2010	2166	2221	2272	2322	2414	2474
100	1817	1913	2000	2080	2154	2224	2290	2351	2400	2466	2520	2620	2714
104	1964	1969	2163	2250	2349	2405	2477	2542	2608	2666	2725	2833	2935
108	2119	2231	2333	2426	2512	2594	2671	2742	2806	2873	2939	3056	3165
112	2279	2399	2509	2609	2702	2790	2871	2949	3023	3092	3161	3286	3404
116	2445	2574	2691	2799	2898	2992	3081	3163	3243	3315	3391	3525	3651
120	2616	2754	2880	2995	3312	3202	3297	3385	3471	3550	3628	3772	3908
124	2793	2941	3075	3198	3101	3419	3521	3614	3706	3790	3874	4028	4172
128	2977	3133	3277	3408	3529	3643	3752	3852	3949	4038	4128	4292	4446
132	3166	3333	3485	3624	3753	3875	3990	4096	4190	4295	4390	4565	4728
136	3360	3538	3699	3847	3984	4113	4235	4348	4457	4557	4654	4846	5019
140	3561	3749	3920	4077	4222	4359	4488	4608	4716	4832	4939	5135	5319
144	3767	3966	4147	4313	4466	4611	4748	4875	4997	5111	5225	5432	5681
148	3980	4190	4381	4556	4718	4871	5016	5179	5279	5399	5519	5739	5944
152	4198	4402	4621	4805	4976	5138	5291	5431	5568	5696	5821	6053	6270
156	4421	4655	4867	5062	5242	5412	5573	5721	5865	6000	6132	6376	6604
162	4768	5020	5249	5458	5653	5836	5010	6170	6324	6469	6613	6876	7122
168	5128	5399	5645	5870	6079	6277	6463	6635	6802	6958	7112	7094	7659
174	5494	5791	6055	6297	6521	6733	6933	7117	7296	7464	7629	7932	8216
180	5887	6198	6480	6539	6979	7205	7419	7617	7809	7989	8164	8488	8793

HORSE POWER IN MACHINERY.

Horse power in Machinery is assumed to be about the effect a horse is able to produce, and has been estimated and establised by Mr. Watt to be 33,000 lbs. raised one foot per minute for one horse, which will be the same as 550 lbs. raised one foot per second. This applied to steam engines, a difficulty has been encountered, namely, to fóresay the velocity of the steam piston, for which Mr. Watt assumed a certain speed for each length of stroke, varying nearly as the cuberoot of the length of the stroke. He also adopted a standard steam pressure of 7 lbs. per square inch, established a simple rule for the nominal horse power of engines which is "*The square of the diameter of the cylinder in inches multiplied by the cuberoot of the stroke in feet, and divided by the constant number* 47 *is the nominal horse power.* This rule agreed very near to the actual performance of engines in those days, but as the improvements advanced we found that the steam piston can move with a greater velocity and the steam pressure gradually increased, that our days engines greatly exceeds the above rule.

The English Admiralty has adopted Mr. Watt's rule, with a slight modification in the assumption of the speed of the steam piston. *Area of cylinder piston in square inches multiplied by* 7 *lbs. steam pressure, multiplied into the speed of piston in feet per minute divided by* 33000. The length of stroke and relative speed of piston, and the number of revolutions per minute is assumed for

3 feet stroke	30 revolutions	and	180 feet per minute.		
9 "	$13\frac{3}{4}$ "		247 "	"	

in which the speed of the piston will be very near 120 $\sqrt[3]{S}$ feet per minute. For long strokes the Admiralty's is less, and Mr. Watt's a little more.

Another expression well known as *Indicated horse power*, which should be understood to be the gross horse power imparted by the steam on the cylinder piston, but in some cases the friction and working pumps about 10 to 25 per cent. are deducted and still called *Indicated horse power.* This makes a great confusion particularly in datas given of steamship performance. Numerous other estimates have been made of horse power, but most of them founded on that by Mr. Watt. Herein we shall assume two kinds, first, *Nominal horse power*, to express the size, weight and commercial value of engines, and to be the power spoken of when not expressly stated, the second, *Actual horsepower*, which is to express the actual power an engine is able to deliver after friction and working pumps are deducted.

Comparing propeller and paddle wheel engines we now find that the velocity of the piston is frequently greater for short stroke, still I will maintain Mr Watt's rule, because it gives an accurate estimate of the real worth of an engine; shall only alter the coefficient so as to suit our days practice.

NOMINAL HORSE POWER.

Assume a standard steam pressure of 30 lbs. per square inch, expanded two-thirds the velocity of the steam piston to be 200 $\sqrt[3]{S}$ feet and revolutions $n=\frac{100}{\sqrt[3]{S^2}}$ per minute, we will arrive to a formulæ of nominal horse power. $H=\frac{D^2\sqrt[3]{S}}{10}$, for condensing engines, which will agree very near with the actual performance of our present condensing engines. The preceding tables are calculated from this formulæ.

For high pressure engines I will assume the steam pressure to 80 lbs. per square inch, expanded $\frac{1}{2}$ which will give the nominal horse power,

$$H=\frac{D^2\sqrt[3]{S}}{4}, \text{ high pressure engines.}$$

The horse power in the accompanying table divided by 0·4 gives the nominal power o high pressure engines. The diameters D are contained in the first column in inches, and the stroke S in feet and inches on the top line.

ACTUAL HORSE POWER.

One actual horse power is 33000 lbs. raised one foot in one minute. This applied to steam engines will be the mean steam pressure on cylinder piston in pounds, multiplied by the velocity of piston in feet per minute, divided by 33,000, is the horse power imparted by the steam. From this we shall deduct 25 per cent. in condensing engines, and 13·1 per cent. in high pressure engines, for working friction and pumps, the balance to be termed the actual horse power.

Example 1. Fig. and formulæ 318. Area of steam cylinder $A=1809$ square inches, stroke of piston $S=4$ feet, indicated pressure of steam 30 lbs. to which add the atmospheric pressure 15 lbs. or $P=45$ lbs. expanded $\frac{3}{8}$, the mean pressure will be $F=31{\cdot}459$ lbs. (see Expansion Table I.), vacuum $v=12$ lbs. the engine making $n=45$ revolutions or double stroke per minute. Required the actual horse power, $H=?$ $W=31{\cdot}459+12-14{\cdot}7=$ $28{\cdot}759$ lbs.

$$H=\frac{1809\times4\times28{\cdot}759\times45}{22000}=425{\cdot}6 \text{ horses.}$$

In this example the actual horse power is 11·6 per cent. more than the nominal power from the table.

Example 2. Fig. 318. A high pressure engine of cylinder piston $A=314$ square inches, stroke $S=3$ feet, steam pressure 80 lbs, per square inch, to which add 15 lbs. $P=95$ lbs. expanded $\frac{1}{2}$, the engine making $n=56$ revolutions per minute. Required the actual horse power? From the expansion table we have the mean pressure $F=80{\cdot}412$ lbs., from which subtract the atmospheric pressure 14·7 lbs. $W=65{\cdot}712$ lbs.

$$H=\frac{314\times3\times65{\cdot}12\times56}{19000}=180{\cdot}8 \text{ horses.}$$

Annular Expansion Double Cylinder, Fig. 319.

These kind of engines are now sometimes made in Europe with a view to economise fuel, and to extend the expansion of steam. The outer cylinder A, A, is annular, similar to that made by Mouslay, but in this case it is employed only for expansion, the inner cylinder a is used for high pressure only. It is so arranged by steam valves that the high steam is acting the whole stroke on the small piston a, after which it is conducted to the annular cylinder where it acts expansively on the large piston A, A. The two pistons being connected by rods to one common crosshead as shown by Fig. 319. This arrangement has been successfully carried out by Mr. Jägerfelt in Nyköping, Sweden. The inner cylinder can be considered an ordinary high pressure engine where the utilized steam is set free into the air at each stroke; but in this case the exhaust steam accomplishes a second engagement in the annular cylinder, which according to the grade of expansion may greatly exceed the original effect imparted in the small cylinder during the first engagement.

Example 3. Fig. 319. Area of the high pressure cylinder piston $a=254{\cdot}4$ square inches, the annular expansive piston $A=763{\cdot}2$ square inches, stroke of pistons $S=3$ feet, the high steam pressure $P=60$ lbs. vacuum $v=12$ lbs., making $n=65$ revolutions per minute. Required the actual horse power of the engine $H=?$ The grade of expansion will be

$1-\frac{763{\cdot}2}{254{\cdot}4}=\frac{2}{3}$, for which the mean pressure on the annular piston will be

$f=32{\cdot}62$ lbs. See Expansion Table II. The effective pressure on the two pistons will be $V=763{\cdot}2\,(32{\cdot}62+12-14{\cdot}7)+254{\cdot}4\,(60-32{\cdot}62)=29800$ lbs.

$$H=\frac{29800\times3\times65}{22000}=264 \text{ horses.}$$

Example 4. Now we will reject the annular expansion cylinder, and take the effect of the steam without expansion, when the effectual pressure will be $60-14{\cdot}7=45{\cdot}3$ lbs. and the actual power,

$$H=\frac{254{\cdot}4\times3\times45{\cdot}3\times65}{19000}=118 \text{ horses.}$$

If we consider the last result as unit we shall have 264−118=146 horses or nearly 124 per cent. gained by the expansion, omiting the loss of steam in the steam passages.

In the first case about 11 per cent. was gained by vacuum, but that advantage is rather in favour of the utility of expansion, because the high steam cannot so well be introduced into the condenser.

The economy will be in the same proportion when the same grade of expansion is used in one cylinder.

I do not mean to maintain that this high per centage of economy is always fully realized in practice, as I am well aware of cases where expansion is of little use, caused by misconception and carelessness in its employment. There are many circumstances about an engine which are in favour of expansion, for instance, the steam passages between the main valve and cylinder, and the clearance between the piston and cylinder heads, contains a great deal of steam which is a total loss, but when expansion is used, that steam expands into the cylinder, and is consequently utilized. The expanded exhaust require a smaller air pump than would be necessary for high steam introduced in the condenser.

Half Trunk Expansion Engines. Fig. 320.

This kind of engines has been introduced by Mr. Carlsund, and are extensively used in Sweden, they are well suited for Gunboats where the machinery is required to be below the water line. The high steam is employed throughout the stroke in the annular space around the trunk, after which it is conducted to act expansively on the large piston A Fig. 320.

Example 5. Fig. 320. Area of the annular piston a=562 square inches, and A=2248 square inches, stroke of piston S=4 feet, steam pressure P=90 lbs., making n=68 revolutions per minute. Required the actual horse power?

$$\text{Grade of expansion} = 1-\frac{562}{2248} = \tfrac{3}{4},$$

From the Expansion Table II. we have f=41·58 lbs. mean pressure on A.

The effectual pressure will be V=2248 (41·58—14·7) + 562 (90—41·58) = 87639 lbs., high pressure

$$H = \frac{87639 \times 4 \times 68}{38000} = 627{\cdot}3 \text{ horses.}$$

Double Cylinder Expansion Engines, Fig. 321.

This kind of engines are now made in England and are said to be very economical. The small cylinder is used for high pressure, from which the steam is conveyed to expand in the large cylinder. In the figure it is arranged so that the pistons follow one another in one direction, when the steam must be conveyed from the top of the small cylinder to the bottom of the large one, and vice-versa; but it is sometimes arranged so that the pistons move in opposite direction, when the steam is conveyed direct at the same ends from the small cylinder to the large one, which has the advantage of making the steam passages shorter, but is more complicated in concentrating the motion.

Example 6.

$$\text{High pressure cylinder,} \begin{cases} a = 962. \text{ square inches.} \\ s = 5 \text{ feet.} \end{cases}$$

$$\text{Expansion cylinder,} \begin{cases} A = 3848 \text{ square inches.} \\ S = 10 \text{ feet.} \end{cases}$$

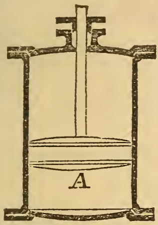
Steam pressure in the small cylinder P=40 lbs., vacuum v=12 lbs., making n=21 revolutions per minute. Required the actual horse power, H=?

$$\text{Grade of expansion} = 1-\frac{962 \times 5}{3848 \times 10} = \tfrac{7}{8}.$$

From the Expansion Table II. we have f=11.879 lbs., mean pressure on A.

w = 3848×10 (11.879+12−14:7) + 962×5 (40—11.879) = 366767 lbs. of momentum.

$$H = \frac{366767 \times 21}{22000} = 350 \text{ horses.}$$

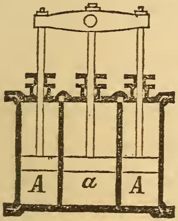


318. *One double acting Cylinder.*

Actual horse power. $\left\{ \begin{array}{ll} H = \dfrac{A\,S\,W\,n}{22000}, & \text{cond. engs.} \\ H = \dfrac{A\,S\,W\,n}{19000}, & \text{high pr. engines.} \end{array} \right.$

$W = F + v - 14{\cdot}7$ for cond. engines.

$W = F - 14{\cdot}7$ for high pressure engines.

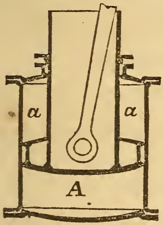


319. *Annular expansion double Cylinder.*

$$F = \frac{P\,a}{A}[2{\cdot}3\,(log.A - log.a) + 1].$$

$$f = \frac{F\,A - P\,a}{A - a}, \qquad V = A(f + v - 14{\cdot}7) + a(P - f).$$

Actual horse power. $\left\{ \begin{array}{ll} H = \dfrac{V\,S\,n}{22000}, & \text{cond. engines.} \\ H = \dfrac{V\,S\,n}{19000}, & \text{high pr. engs.} \end{array} \right.$

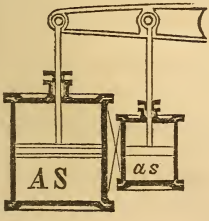


320. *Halftrunk expansion Cylinder.*

$$F = \frac{P\,a}{A}\;[2{\cdot}3(log.A - log.a) + 1].$$

$$f = \frac{F\,a - P\,a}{A - a}. \qquad V = A(f + v - 14{\cdot}7) + a(P - f).$$

Actual horse power. $\left\{ \begin{array}{ll} H = \dfrac{V\,S\,n}{44000}, & \text{cond. engines.} \\ H = \dfrac{V\,S\,n}{38000}, & \text{high pr. engs.} \end{array} \right.$



321. *Double Cylinder expansion.*

$$F = \frac{P\,a\,s}{A\,S}[2{\cdot}3(log.AS - log.as) + 1]$$

$$f = \frac{F\,A - P\,a}{A - a}, \qquad w = AS(f + v - 14{\cdot}7) + a\,s(P - f).$$

Actual horse power. $\left\{ \begin{array}{ll} H = \dfrac{w\,n}{22000}, & \text{cond. engines.} \\ H = \dfrac{w\,n}{19000}, & \text{high pr. engines.} \end{array} \right.$

SLIDE VALVES.

THE slide valve motion is one of the most important features in causing a steam engine to work well, and to employ the effect of steam economically. The author of this book being well acquainted with disarrangements on this point, has here endeavoured to give a good working-drawing of the proper proportions and arrangements of slide-valve motion. (See Plate VIII.)

Main Valve.

It will be best to assume a certain size cylinder, and at the same time give the proportions for any size.

$D = 34$ inches, diameter of the cylinder.
$S = 18$ inches stroke of piston.*
$n = 56$ double strokes per minute.

We have the area of the steamports m, from Formula 26, page 238.

$$\mathbf{a} = \frac{34^2 \times 0{\cdot}785 \times 18 \times 56}{30600} = 30 \text{ square inches, nearly.}$$

$$m = \frac{D+S}{26} = \frac{34+18}{26} = 2 \text{ inches,}$$

the width of the steamport; if the quotient gives a fraction take the nearest quarter or eighth.

$$\frac{\mathbf{a}}{m} = \frac{30}{2} = 15 \text{ inches, breadth of steamport.}$$

$r = \frac{1}{2} m$ about $= 1$ inch, the exhaust port $o = 2m - \frac{1}{2}r = 3\frac{1}{2}$ inches, and $f = o + 2r = 5\frac{1}{2}$ inches, $h = f - \frac{1}{4}r = 5\frac{1}{4}$ inches, $k = 1\frac{1}{2}m = 3$ inches, and $i = h + 2k = 11\frac{1}{4}$ inches, $e = m = 2$ inches.

* The stroke and diameter is here rather out of proportion, but we will maintain them in the calculations as they suit the drawing, which is purposely made to show the *slide valves* on a large scale. The rules will however suit any proportions of diameter and stroke.

To Find the Stroke of the Eccentric.

$s =$ stroke of the eccentric in inches.
$s = i - f - \frac{1}{4}r = 5\frac{1}{2}$ inches.

The *lap* $L = \frac{1}{2}(i - f - 2m) = \frac{7}{8}$ inches.

The *lead* of the valve, or opening of the steamport when the crank pin stands on the centre should be about

$$l = \frac{m\sqrt{n}}{80} = \frac{2\sqrt{56}}{80} = \frac{1}{4} \text{ inches, nearly.}$$

Having finished the main valve and ascertained the stroke of the eccentric, it is now required to find the position of the centre b, (Plate VI.,) of the eccentric, to the crank-pin. Suppose the crank pin of the engine stands at a on the centre nearest to the cylinder, and the eccentric rods are attached direct to the valve rods; draw the line dd, at right-angle to the centre-line aa'' of the engine, then

$$\text{the angle, sin.} W = \frac{2(L+l)}{s} = \frac{2(\frac{7}{8}+\frac{1}{4})}{5\frac{1}{2}} = 0{\cdot}409, \text{ or } W = 24^\circ\ 10'.$$

See Plates VIII. and IX.

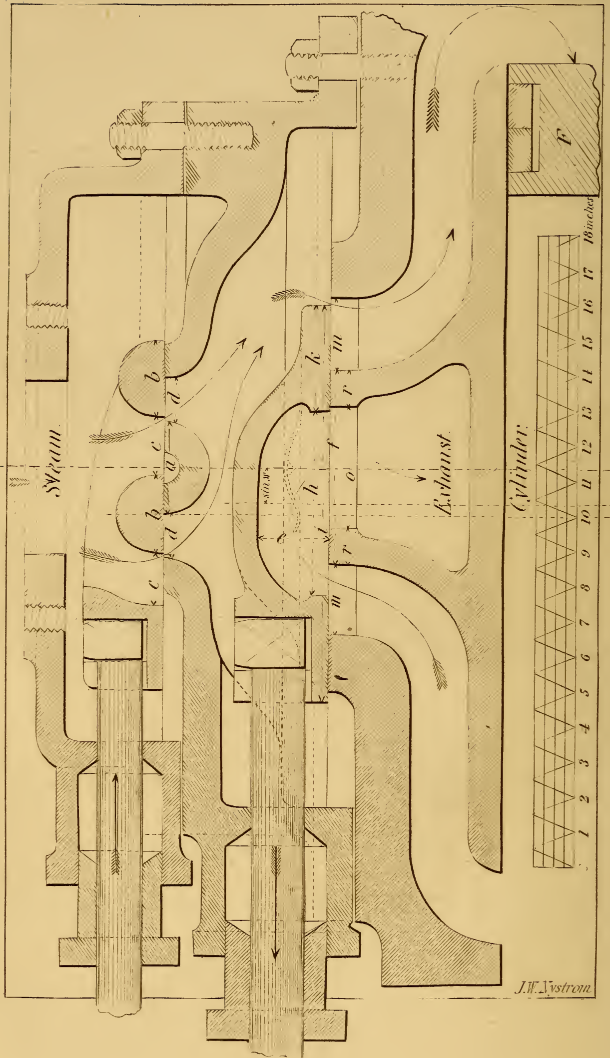
To Find the position of the Crank-Pin at the moment the Main Valve opens.

$$y = \frac{S\,l}{s \cos. W} = \frac{18 \times 0{\cdot}25}{5{\cdot}5 \times 0{\cdot}9123} = 0{\cdot}9 \text{ inches, nearly,}$$

from the centre line.

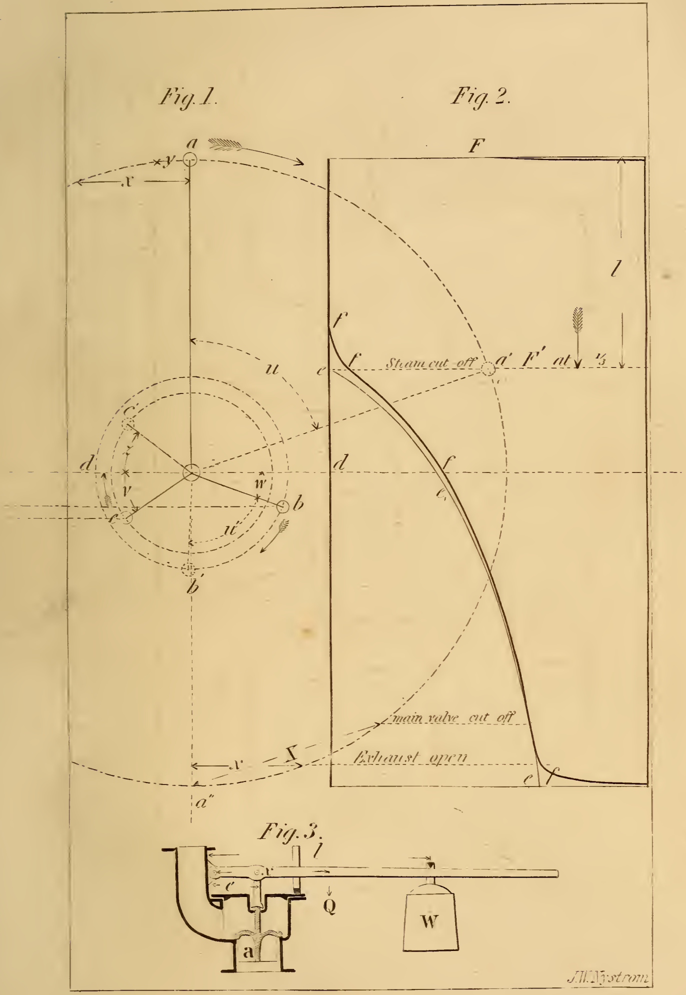
Slide Valves.

Plate VIII.



Eccentrics.

Plate IX.



To Find the position of the Crank at the moment the Exhaust opens.

$$x=\frac{S}{2}\left(\sin.W-\frac{1}{s}(f-h)\right)=\frac{18}{2}\left(0\cdot409-\frac{1}{5\cdot5}(5\cdot5-5\cdot25)\right)=3\cdot27 \text{ inches}$$

from the centre line.

To Find the position of the Crank Pin when the Main Valve cuts off the Steam.

$$X=\frac{2SL}{s}=\frac{2\times18\times\frac{7}{8}}{5\cdot5}=5\cdot727 \text{ inches.}$$

To Find at what part of the Stroke the Main Valve Cuts off the Steam,

$$\text{Will cut off at} = 1-\frac{4L^2}{s^2}=1-\left(\frac{2\times\frac{7}{8}}{5\cdot5}\right)^2=0\cdot899 \text{ of the stroke.}$$

The greater the lap is, the sooner will the main-valve cut off, but if the lap is increased the stroke of the eccentric must also be equally increased. It does not work well to cut off much by the main-valve, especially when the engine works fast; for very slow motion it may answer to cut off at $\frac{5}{8}$ the stroke.

It will be noticed that the centre of the eccentric is always ahead of the crank pin with an angle $90°+w$. Hence when the engine is to be reversed, the centre b must have the same position on the opposite side of the centre-line, or the eccentric must be moved forwards an angle of $90°-2w$.

Cut-off Valve.

The width of the cut off ports should be about $d=\frac{5}{8}m=1\frac{1}{4}$ inch, and their

$$\text{breadth } \frac{a}{2d}=\frac{30}{2\times1\frac{1}{4}}=12 \text{ inches, when two ports are used.}$$

Proportions of the Valve.

$a-b=c-d$, $a+d=b+c$, and $a=2d$, and the stroke of the cut-off valve eccentric $s=2b$, we shall have $a=2\frac{1}{2}$, $b=2\frac{1}{4}$, $c=1\frac{1}{2}$, $c=1\frac{3}{8}$, and $s=4\frac{1}{2}$ inches.

Let us assume the steam to be cut off at $\frac{1}{3}=l$ of the stroke S, the position of the crank-pin a' will then be $\sin.u=2l=0\cdot666$, or $u=70°\ 30'$; at the same time the position of the centre c' of the cut off eccentric will be

$$\sin.z=\frac{d+c}{s}=\frac{1\frac{1}{4}+1\frac{3}{8}}{4\frac{1}{2}}=0\cdot612, \text{ or } z=37°\ 50',$$

and $V=u-z=70°\ 30'-37°\ 50'=32°\ 40'$, the position of the centre c when the crank-pin a is on the centre. This Table will show the positions of the centre a and c, at different cut offs. Letters correspond with Figure 1, Plate VI.

Cut off at l.	v	$\sin.v$	*stroke of eccen. s.*	z	u	F.	p.
$\frac{1}{4}$	22° 10′	0·377	$2b$	37° 50′	60°	0·5880	0·250
$\frac{1}{3}$	32° 40′	0·539	$2b$	37° 50′	70° 30′	0·6914	0·333
$\frac{3}{8}$	31° 55′	0·527	$c+a$	43° 35′	75° 30′	0·7332	0·375
$\frac{1}{2}$	42° 35′	0·675	$b+c$	47° 25′	90°	0·8350	0·500
$\frac{5}{8}$	46° 30′	0·7193	$a+b-c$	58°	104° 30′	0·910	0·625
$\frac{2}{3}$	50° 30′	0·7933	$a+b-c$	58° 30′	109° 30′	0·985	0·666

It will now be observed that the effectual pressure F in this Table is less than in the Table on page 239, owing to the valve not cutting off the steam instantly, but gradually, so that the density of the steam in the cylinder is already diminished at the *cut off* point. The valve will cut off quicker the less the angle z is.

See Figure 2, Plate VIII. The actual pressure will not form a sharp corner at e, or follow the line e,e,e, as would be due when cut off at $\frac{1}{3}$ the stroke, but the line $ff'ff$ will be the true diagram. Including the steam in the ports and steamchest, the density at the end of the stroke will correspond nearly with the Table.

BLOWING-OFF. SALT WATER. SATURATION.

SEA water contains about 0·03 its weight of salt. When salt water boils, fresh water evaporates, and the salt remains in the boiler, consequently the proportion of salt increases as the water evaporates, until it has reached 0·36 weight to the water; the salt will then commence to saturate in the boiler, and the water in solution will hold 0·36 weight of salt to 1 of water.

To prevent this deposit in the boiler, it is necessary to keep the salt below this proportion, which is overcome by withdrawing (blow off) part of the super-salted water, while less salted (feed water,) water is replaced. It is found in practice that when the proportions are kept 0·12 of salt to 1 weight of water, the deposit will be very slight. To obtain this it will be necessary to blow off

$$\frac{0{\cdot}03}{0{\cdot}12} = 0{\cdot}25 \text{ parts of the feed water, or}$$

if a brine-pump is used, it should be at least 0·25 of the feed pump.

$W =$ cubic feet of super-salted water to be blown off per minute.

D, S, n, and k, as before, we shall have,

$$W = \frac{D^2\,S\,n}{3000k},$$

Example. $D = 30$ inches, stroke of piston 36 inches, cut off at half stroke $S = 18$, making 14 revolutions per minute, with a pressure of 30 pounds per square inch, $k = 610$. How much water must be blown off per minute?

$$W = \frac{30^2 \times 18 \times 14}{3000 \times 610} = 0{\cdot}124 \text{ cubic feet.}$$

Heat Wasted by Blowing Off.

Letters denote.

$\left.\begin{array}{l} w = \text{water evaporated} \\ W = \text{water blown off} \end{array}\right\}$ in cubic feet per unit of time.

t = temperature of the feed water.
T = " " blowing off.
H = heat wasted, per cent.

$$H = \frac{W(T - t)}{w(990 + T - t)}.$$

Example. Let the quantity of water blown off be $\frac{1}{2}$ of the feed water, we have $W = 1$, and $w = 2$, the boiling point of the water will then be $T = 215{\cdot}5°$, let the feed water taken from the hot-well be $t = 100°$. Required the quantity of heat lost?

$$H = \frac{1(215{\cdot}5° - 100)}{2(990 + 215{\cdot}5 - 100)} = 0{\cdot}052 \text{ or } 5{\cdot}2 \text{ per cent.}$$

This is a very trifling quantity of heat lost.

Proportion of Salt.	*Boiling point T.*	*Water blown off. per cent. W.*	*Heat lost. per cent.*	*Specific gravity.*
0·03	213·2°	100	100	1·03
0·06	214·4	50	10·35	1·06
0·09	215·5	33·3	5·2	1·09
0·12	216·7	25	3·5	1·12
0·15	217·9	20	2·66	1·15
0·18	219·	16·6	2·14	1·18
0·21	220·2	14·3	1·80	1·21
0·24	221·4	12·5	1·56	1·24
0·27	222·5	11·1	1·38	1·27
0·30	223·7	10·0	1·23	1·30
0·33	224·9	9·07	1·12	1·33
0·36	226	Water saturates.		1·36

Heat wasted by Incrustation.

The conducting power of iron for heat, is about 30 times that of saturated scales, hence a considerable portion of heat is lost when the scales become thick in a boiler.

t = thickness of the scale in 16th of an inch.
H = heat wasted, in per cent.

$$H = \frac{t^2}{32+t^2}.$$

Example. The scale in a boiler is 5 sixteenths of an inch thick. How much heat is lost by it?

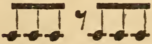
$$H = \frac{5^2}{32+5^2} = 0{\cdot}438 \text{ or 44 per cent, nearly,}$$

which goes out through the chimney.

This is merely to show that the heat lost by blowing off is but trifling, compared with the heat lost by saturation of scales, which additionally injures the boiler by softening and fracturing the iron, and final explosions.

When boilers are taken good care of by cleaning and blowing off at short intervals, the scales need not exceed 1 sixteenth of an inch.

To Command the Engineer how to Manœuvre the Engine in a Steamboat;

Go ahead - -	 - - -	one stroke.
Back - -	 - -	two strokes.
Stop - - -	 - - -	one stroke.
Slowly - -	 - -	two short.
Full speed -	 - -	three short.
Go ahead slowly	 - -	one long, two short.
Back slowly -	 -	two long, two short.
Go ahead, full speed	 -	one long, three short.
Back fast - -	 -	two long, three short.
Hurry - -		three short repeated.

SCREW PROPELLERS,

PLATE X., is a drawing of a Screw-propeller with proportions thus far known to be the most effective, particularly when the steam-engine is applied direct to the propeller shaft; its pitch is twice the diameter at the periphery, but contracts towards the centre; at the hub the pitch is lessened by the amount of slip assumed. When the propeller is geared from the engine, the pitch is generally less in proportion to the diameter.

$p = \frac{1}{4}$ of the pitch at the periphery.
$p'' =$ " " " hub.
$s =$ the assumed slip in a fraction of p.

Then $p = p'' + s$.

By these two pitches p and p'', the helixes, acb at the periphery and dcf at the hub are constructed as for common screws.

The actual pitch of the propeller at the centre of effort of the blades o is represented by p' at $r = 0{\cdot}725R$ from the centre; or the actual pitch $= 4p'$.

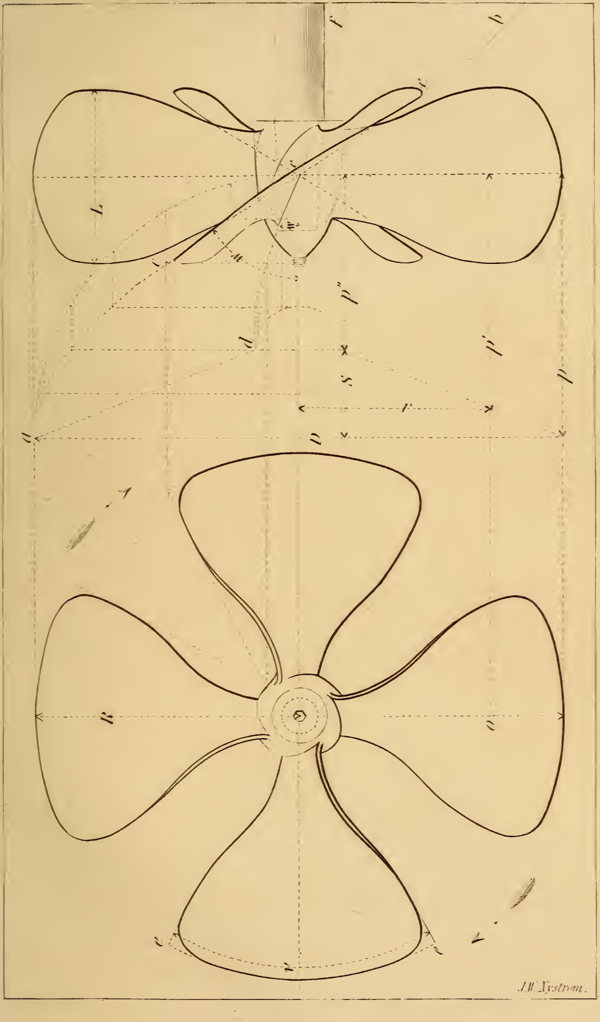
Letters Denote,

$P =$ pitch of the propeller } at the periphery.
$W =$ angle of the blades } at the periphery.
$D =$ diameter, $R =$ radius, extreme.
$L =$ length parallel with the centre-line.
$m =$ number of blades.
$b =$ extreme breadth of the propeller blades over the edge, between the corners e, e.
$e =$ circle arc in the angle v.
$v =$ the projected angle of the blades.
$\mathbf{a} =$ the projected area of the blades.
$A =$ the *true* inclined surface of the blades.
$\bigcirc =$ acting area of the propeller.
$H =$ horse-power required to drive the propeller n revolutions per minute.

Formulas for Screw Propellers,

$\cot. W = \frac{P}{\pi D}$, - - - 1,	$b = \sqrt{\frac{v^2 \pi^2 D^2}{129600} - L^2}$, 7,
$P = \cot. W \pi D$, - - 2,	$\mathbf{a} = \frac{0{\cdot}785 D^2 v m}{360}$, - - 8,
$P = \frac{360 L}{v}$, - - - - 3,	$A = \frac{R m}{2{\cdot}25}(b + L)$, - - - 9,
$P = \frac{\pi D L}{e}$, - - - 4,	$\bigcirc = \frac{2{\cdot}5 D^3}{\sqrt{P^2 + \pi^2 D^2}}$, - - 10,
$P = \frac{\pi D L}{\sqrt{b^2 - L^2}}$, - - 5,	$H = \frac{D^3 n^3}{480000}\left(L S \cos. W + 0{\cdot}11,\right)$ 11,
$v = \frac{360 L}{P}$, - - - - 6,	$n = \frac{78}{D} \sqrt[3]{\frac{H}{(L S \cos. W + 0{\cdot}11)}}$ 12.

SCREW PROPELLER. *Plate X.*



J.W. Nystrom.

Example 1. The diameter of a propeller is 10 feet 6 inches, and the angle $W = 58°$ at the periphery. Required the pitch $P =$ in feet?

$$P = \cot.58° \times 3{\cdot}14 \times 10{\cdot}5 = 20{\cdot}6 \text{ feet.}$$

Example 2. The propeller on Plate VII. is of dimensions $D = 15$ feet, $L = 5$ feet, $W = 57° \ 30'$, the slip is 38 per cent: or $S = 0{\cdot}38$. What power is required to drive it 40 revolutions per minute, $H = ?$

$$H = \frac{15^3 \times 40^3}{480000}\left(5 \times 0{\cdot}38 \times \cos.57° \ 30' + 0{\cdot}11\right) = 509 \text{ horses, nearly.}$$

Example 3. A Propeller of diameter $D = 12$ feet, angle $W = 64°$, and length $L = 3$ feet 6 inches, is to be driven by a steam engine of 450 horses, the slip $S = 0{\cdot}28$. How many revolutions will it make per minute, $n = ?$

$$n = \frac{78}{12}\sqrt[3]{\frac{450}{(3{\cdot}5 \times 0{\cdot}28 \times \cos{\cdot}64° + 0{\cdot}11)}} = 61 \text{ revolutions}$$

per minute.

Explanation of Tables.

Table I. is for finding the pitch and acting area of propellers; the column marked W contains the angle of the propeller blades, as marked on the drawing.

To Find the Pitch.

RULE. Multiply the diameter of the propeller by the tabular coefficient in the pitch column opposite the given angle, and the product is the pitch of the propeller.

Example. The diameter of a propeller is 12 feet, the angle $W = 60°$; diameter of the hub 1·5 feet, and the angle $w = 16°$.

$$\text{Required the} \begin{cases} \text{pitch at the periphery?} \\ \text{pitch at the hub?} \\ \text{pitch at the centre of effort?} \end{cases}$$

Pitch at the periphery $= 12 \times 1{\cdot}814 = 21{\cdot}768$ feet.
" " hub $= 1{\cdot}5 \times 10{\cdot}97 = 16{\cdot}455$ feet.

Let d and p be diameter and pitch of the hub.
$\mathfrak{p}$ = pitch at the centre of pressure. We shall have,

$$(P - p) : (\mathfrak{p} - p) = (D - d) : (0{\cdot}725D - d).$$

and

$$\mathfrak{p} = p + \frac{(P - p)(0{\cdot}725\,D - d)}{D - d},$$

$$\mathfrak{p} = 16{\cdot}455 + \frac{(21{\cdot}768 - 16{\cdot}455)(0{\cdot}725 \times 12 - 1{\cdot}5)}{12 - 1{\cdot}5} = 20{\cdot}097 \text{ feet.}$$

To Find the Acting Area.

RULE. Multiply the *square* of the diameter of the propeller by the tabular coefficient in the column ○ opposite the given angle, and the product is tho acting area of the propeller.

Example. The diameter of a propeller $D = 13$ feet, 3 inches, and the angle $W = 60°$. Required the acting area?

$$\bigcirc = 13{\cdot}25^2 \times 0{\cdot}679 = 119{\cdot}2 \text{ square feet.}$$

TABLE I.

Table for finding the Pitch and Acting Area of Propellers. D = 1.

Angle. W	Pitch. P	Act. Area O	Angle. W	Pitch. P	Act. Area. O
5°	36·	0·068	47°	2·930	0·573
6	30·	0·082	48	2·828	0·583
7	25·65	0·095	49	2·730	0·582
8	22·4	0·109	50	2·635	0·601
9	19·85	0·123	51	2·545	0·610
10	17·82	0·136	52	2·455	0·618
11	16·16	0·150	53	2·370	0·625
12	14·79	0·163	54	2·283	0·634
13	13·60	0·176	55	2·200	0·642
14	12·60	0·190	56	2·120	0·650
15	11·04	0·203	57	2·040	0·657
16	10·97	0·217	58	1·963	0·665
17	10·27	0·229	59	1·888	0·672
18	9·67	0·242	60	1·814	0·679
19	9·12	0·255	61	1·740	0·686
20	8·64	0·268	62	1·670	0·692
21	8·19	0·281	63	1·600	0·699
22	7·77	0·294	64	1·530	0·705
23	7·40	0·306	65	1·465	0·711
24	7·06	0·319	66	1·400	0·716
25	6·75	0·331	67	1·333	0·722
26	6·45	0·344	68	1·270	0·728
27	6·17	0·356	69	1·205	0·731
28	5·91	0·368	70	1·142	0·736
29	5·67	0·380	71	1·114	0·741
30	5·45	0·392	72	1·021	0·745
31	5·23	0·404	73	0·960	0·750
32	5·03	0·415	74	0·900	0·754
33	4·85	0·427	75	0·842	0·757
34	4·66	0·439	76	0·783	0·761
35	4·50	0·450	77	0·725	0·764
36	4·33	0·461	78	0·668	0·767
37	4·175	0·472	79	0·611	0·770
38	4·025	0·483	80	0·555	0·772
39	3·885	0·494	81	0·498	0·775
40	3·745	0·504	82	0·442	0·777
41	3·620	0·515	83	0·386	0·779
42	3·500	0·523	84	0·331	0·780
43	3·370	0·535	85	0·275	0·781
44	3·260	0·545	86	0·220	0·782
45	3·141	0·555	87	0·165	0 783
46	3·035	0·564	88	0·110	0·784

TABLE II.

Table for finding the Exponent and Coefficient of Vessels.

Full Lines.		*Hollow Lines.*	
Exponent x.	Coefficient k.	Exponent x.	Coefficient k.
1	0·000	0·68	1·71
0·95	0·024	0.67	1·77
0·90	0·228	0·66	1·84
0·88	0·326	0·65	1·90
0·86	0·432	0·64	1·96
0·84	0·558	0·63	2·00
0·82	0·692	0·62	1·97
0·80	0·836	0·61	1·93
0·79	0·902	0·60	1·88
0·78	0·978	0·59	1·82
0,77	1·050	0.58	1·77
0·76	1·12	0·57	1·72
0·75	1·20	0·56	1·67
0·74	1·28	0·55	1·61
0·73	1·35	0·54	1·55
0·72	1·43	0·53	1·50
0·71	1·51	0·52	1·44
0·70	1·59	0·51	1·38
0·69	1·64	0·50	1·32

TABLE III.

Table for finding the Slip and Acting Area. ○ = 1.

Slip. S. per cent.	Act. Area. ○	Slip. S. per cent.	Act. Area. ○	Slip. S. per cent.	Act. Area. ○	Slip. S. per cent.	Act. Area. ○
5	84·85	28	4·150	51	0·927	74	0·208
6	60·35	29	3·820	52	0·888	75	0·192
7	46·35	30	3·555	53	0·840	76	0·177
8	39·00	31	3·333	54	0·784	77	0·163
9	32·20	32	3·090	55	0·737	78	0·149
10	27·00	33	2·880	56	0·697	79	0·137
11	22·07	34	2·710	57	0·655	80	0·125
12	19·80	35	2·535	58	0·611	81	0·113
13	17·32	36	2·366	59	0·581	82	0·103
14	15·20	37	2·222	60	0·546	83	0·092
15	13·52	38	2·085	61	0·511	84	0·083
16	12·00	39	1·952	62	0·479	85	0·074
17	10·82	40	1·827	63	0·455	86	0·066
18	9·715	41	1·727	64	0·422	87	0·058
19	8·820	42	1·634	65	0·394	88	0·050
20	8·000	43	1·523	66	0·369	89	0·045
21	7·282	44	1·430	67	0·347	90	0·037
22	6·700	45	1·354	68	0·323	91	0·031
23	6·111	46	1·275	69	0·300	92	0·026
24	5·635	47	1·195	70	0·281	93	0·021
25	5·200	48	1·127	71	0·263	94	0·017
26	4·785	49	1·068	72	0·241	95	0·012
27	4·444	50	1·000	73	0·225		

STEAMSHIP PERFORMANCE.

Letters denote.

T = Displacement of the vessel in tons.
$\mathfrak{X}$ = greatest immersed section in square feet.
$\otimes$ = area of resistance in square feet.
l = length of the vessel in feet, in the loadline.
b = breadth of beam " "
F = resistance of the vessel in pounds, including friction of the immersed surface.
k = coefficient, x=exponent of the vessel. See Table II. preceding page.
H = Actual horse power required to propel the vessel.
M = nautical miles or knots per hour.

$$x=\frac{35\,T}{\mathfrak{X}\,l}, \quad - \quad - \quad - \quad - \quad 1 \qquad\qquad F=4\otimes M^2, \quad - \quad - \quad - \quad - \quad 3$$

$$\otimes=\mathfrak{X}\sqrt{\frac{b^2}{b^2+k\,l^2}}, \quad - \quad 2 \qquad\qquad H=\frac{\otimes M^3}{81}. \quad - \quad - \quad - \quad - \quad - \quad 4$$

Example 1. The U. S. steam Frigate Niagara is l=328·9 feet long, b=55 feet wide, greatest immersed section $\mathfrak{X}$= 855 square feet, displacement T=5000 tons. Required what horse power is necessary to propel her M=10 knots per hour in smooth water.

$$\text{Exponent } x=\frac{35\times5000}{855\times328\cdot9}=0\cdot63 \text{ nearly,}$$

for which the coefficient in Table II. preceding page is k=2. From formulæ 2 we have the area of resistance.

$$\otimes=855\sqrt{\frac{55^2}{55^2+2\times328\cdot9^2}}=104 \text{ square feet.}$$

$$\text{Actual power } H=\frac{104\times10^3}{81}=1284 \text{ horses.}$$

Example 2. A barge of l=165 feet long, b=28 feet wide, ofossection $\mathfrak{X}$=112 square feet, displacement T=410 tons. How many horses are required to draw the barge at a speed of M=2 miles per hour in a canal?

$$\text{Exponent } x=\frac{36^*\times410}{112\times165}=0\cdot8. \qquad \text{Coefficient } k=0\cdot836.$$

$$\text{Area of resistance } \otimes=112\sqrt{\frac{28^2}{28^2+0\cdot836\times165^2}}=65\cdot7 \text{ square feet.}$$

Tractive force $F=4\times65\cdot7\times2^2=1050$ pounds. Sea miles 2=2·3 statute miles. See page 123 for ability of a horse working d=5 hours continually.

$$F=\frac{375}{2\cdot3\sqrt{5}}=73 \text{ pounds.}$$

Number of horses 1050:73=14·36 or 15 horses are required to draw the barge 2·3 statute miles per hour.

The mechanical horse power performed will be,

$$H=\frac{65\cdot7\times2^3}{81}=6\cdot5 \text{ horses.}$$

If the breadth of the canal is less than three times that of the barge the resistance will be more, and require more horses.

For similar proportioned vessels the resistance is a function of $\mathfrak{X}\,M^2$, and the horse power a function of $\mathfrak{X}\,M^3$. The displacement of a vessel is a

* 35 for salt water, and 36 for fresh water.

function of the cube of any linear dimension of the same, and the greatest immersed section $\boxtimes$ a function of the square of any linear dimension of the displacement, consequently the $\sqrt[3]{T}$ is a function of any linear di-dimension, and $(\sqrt[3]{T})^2=\sqrt[3]{T^2}$ is a function of $\boxtimes$; therefore, the resistance is a function of $M^2\sqrt[3]{T^2}$, and the horse power a function of, $M\sqrt[3]{T^2}$;

thus we arrive at what is known as Mr. Atherton's formulæ $C=\dfrac{T^{\frac{2}{3}}M^3}{H}$, about which so much controversy has existed in English Journals.

The above formulas 1, 2, 3, and 4 will give the same result for similar vessels as that of Mr. Atherton's, and it will be found that they give different coefficients for different proportions of vessel as seen by the two examples where the quality of the performance is considered to be the same.

Ex. 1. $C=\dfrac{10^3\sqrt[3]{5000^2}}{1284}=228$ and *Ex.* 2. $C=\dfrac{2^3\sqrt[3]{410^2}}{6{\cdot}5}=68$ as coeffi.

These coefficients are very small compared with that of Mr. Atherton's on account of the different estimate of horse power; it can however in neither case be considered a measure of the quality of performance for different proportioned vessels; neither can it be considered a measure of commercial value, because the commercial effect produced will be,

Example 1. $\dfrac{10\times5000}{1284}=39.$ *Example* 2. $\dfrac{2\times410}{6{\cdot}5}=126$ effects,

which is quite the reverse of the coefficient.

The following Table is calculated from the formulas 1, 2, 3, and 4, for different sizes of vessels of similar proportions as that of Niagara. If those formulas are well understood it will be found that they trace a line of justice between the Engineer and ship builder, that when the performance is known it shows from where the fame or blame is due.

River steamers of light draft and flat bottom requires more power for the same sharpness of lines, as will be found by the formulas.

To Approximate the size of Steam Engines.

Example 3. It is required to build a river steamer of displacenent $T=1000$ tons, to run $M=16$ nautical miles per hour. Required the size of the cylinder for an ordinary overbeam engine? From the table of steamship performance will be found the required actual power $H=1798$ horses.

From the table of Nominal horse power select the approximate size of cylinder which may be $D=88$ inches, diameter of cylinder by $S=14$ feet stroke, which answers to $H=1866$ horses nominal. In this case the nominal horse power can be considered the same as the actual.

Example 4. A propeller steamer is to run $M=10$ nautical miles per hour, with a displacement $T=3400$ tons. Required the size of the cylinders?

From table of steamship performance $H=992$ horses, to be divided into two cylinders of 496 each. Select from table of Nominal horse power $D=60$ inches diameter of cylinders and $S=2'\ 9''$ stroke of piston, which answers to $H=504$, or $504\times2=1008$ horses of the two cylinders. After these approximations are made, make a careful calculation from the original formulas.

Example 5. Suppose the propeller for the steamer in the preceding example 4 makes $n=60$ revolutions per minute. Required the diameter of the propeller shaft? See Table, page 176 for wrought iron shafts, for 500 horses and 60 revolutions, the shaft should be 10·1 inches. Multiply this by the cuberoot of 2, or $10{\cdot}1\times1{\cdot}26=12{\cdot}926$, say 13 inches the diameter required.

Example 6. A steamer of $T=2500$ tons is to run $M=9$ nautical miles per hour with an indicated steam pressure of 20 lbs., or $P=35$ lbs. per square inch, expanded ½. Required the consumption of fuel in tons per 24 hours?

Table of steamship performance $H=585$ horses.

Table V., page 258 consumption of fuel, 3·44 tons.

The required consumption will be $5{\cdot}85\times3{\cdot}44=20{\cdot}124$ tons per 24 hours, steaming.

Displacement.	Nautical Miles or Knots per Hour.							
	5	6	7	8	9	10	11	12
T	H	H	H	H	H	H	H	H
100	11·8	19·4	32·4	48·4	68·5	94·5	126	163
200	18·8	32·5	51·5	76·9	110	150	200	260
300	24·5	42·4	67·5	100	142	196	262	340
400	29·8	51·4	81·7	122	172	238	317	412
500	34·6	59·6	94·3	141	200	276	368	478
600	39·0	67·2	107	160	226	313	415	540
700	43·3	74·6	119	177	250	377	460	599
800	47·3	81·5	130	194	274	378	503	654
900	51·1	88·1	140	210	296	409	545	708
1000	54·9	94·6	150	225	318	439	585	759
1100	58·4	100	160	239	338	467	622	806
1200	62·0	107	170	254	359	495	660	858
1300	65·3	112	179	267	378	523	696	903
1400	68·7	119	189	281	398	549	732	950
1500	71·9	124	197	295	417	575	766	995
1600	75·0	130	206	307	435	600	800	1038
1700	78·1	135	215	320	453	625	833	1083
1800	81·2	140	224	332	470	649	864	1123
1900	84·2	145	231	345	488	673	897	1166
2000	87·0	150	239	356	504	696	927	1205
2100	90·0	155	247	369	521	720	958	1247
2200	92·7	160	255	380	537	741	988	1284
2300	95·6	165	262	391	554	764	1017	1324
2400	98·4	170	270	402	569	786	1047	1360
2500	101	174	277	414	585	808	1077	1400
2600	104	179	285	424	600	828	1102	1435
2700	106	184	292	436	616	850	1131	1473
2800	109	188	299	446	631	871	1160	1508
2900	111	192	306	457	646	893	1189	1545
3000	114	197	313	467	660	913	1215	1582
3100	117	201	320	478	676	933	1242	1614
3200	119	205	327	488	690	952	1268	1648
3300	121	209	334	498	704	972	1296	1683
3400	124	214	340	508	718	992	1320	1717
3500	127	218	347	518	733	1010	1347	1750
3600	129	222	354	528	746	1025	1373	1783
3700	131	226	360	538	759	1049	1398	1815
3800	133	230	367	548	774	1070	1422	1848
3900	135	234	373	558	787	1087	1446	1880
4000	138	238	380	567	801	1105	1473	1912
4100	140	242	386	577	814	1122	1497	1944
4200	142	246	392	586	827	1141	1520	1975
4300	145	250	398	595	840	1160	1545	2008
4400	147	254	404	604	853	1179	1568	2037
4500	150	258	410	613	866	1198	1593	2070
4600	152	261	416	622	879	1216	1614	2100
4800	156	270	423	640	904	1248	1663	2160
5000	160	277	440	658	929	1282	1708	2220
5500	171	295	469	700	990	1367	1822	2365
6000	181	303	497	742	1050	1448	1930	2507

Displacement.	Nautical Miles or Knots per Hour.							
	13	14	15	16	17	18	19	20
T	H	H	H	H	H	H	H	H
100	207	259	318	387	464	551	648	756
200	329	412	506	615	737	875	1027	1201
300	432	540	662	806	966	1146	1347	1573
400	522	654	803	976	1170	1402	1632	1907
500	607	759	932	1131	1358	1611	1896	2213
600	684	856	1036	1280	1532	1820	2140	2500
700	759	938	1166	1417	1700	2016	2373	2770
800	830	1038	1274	1548	1857	2206	2593	3026
900	898	1123	1380	1675	2009	2385	2803	3274
1000	963	1206	1480	1798	2157	2560	3008	3514
1100	1024	1284	1574	1913	2295	2723	3203	3736
1200	1090	1360	1670	2030	2435	2890	3400	3967
1300	1147	1432	1758	2136	2564	3043	3576	4178
1400	1204	1508	1850	2248	2697	3200	3762	4394
1500	1264	1580	1938	2355	2825	3352	3943	4605
1600	1317	1648	2020	2458	2948	3500	4113	4803
1700	1374	1718	2107	2561	3072	3646	4286	5006
1800	1422	1784	2188	2660	3190	3785	4448	5195
1900	1479	1850	2270	2760	3310	3928	4615	5390
2000	1527	1913	2345	2854	3420	4060	4770	5570
2100	1582	1979	2382	2948	3535	4195	4935	5762
2200	1628	2037	2500	3038	3642	4325	5084	5935
2300	1680	2102	2578	3134	3755	4460	5241	6120
2400	1723	2160	2646	3220	3860	4580	5386	6290
2500	1777	2222	2725	3313	3970	4715	5542	6470
2600	1820	2280	2796	3400	4075	4835	5655	6637
2700	1870	2338	2868	3486	4180	4960	5832	6813
2800	1911	2395	2935	3568	4280	5076	5970	6970
2900	1960	2452	3010	3655	4385	5200	6115	7142
3000	2000	2508	3075	3740	4485	5318	6255	7300
3100	2048	2565	3145	3822	4585	5440	6394	7470
3200	2092	2616	3210	3905	4680	5550	6525	7622
3300	2134	2671	3280	3985	4775	5670	6666	7781
3400	2178	2725	3343	4063	4870	5784	6784	7936
3500	2220	2779	3408	4143	4965	5893	6936	8090
3600	2261	2830	3475	4222	5060	6010	7061	8250
3700	2303	2881	3534	4300	5155	6115	7184	8400
3800	2348	2941	3606	4385	5250	6238	7333	8563
3900	2385	2986	3660	4453	5340	6336	7444	8695
4000	2427	3038	3725	4530	5430	6444	7580	8847
4100	2468	3086	3785	4610	5520	6550	7700	8988
4200	2507	3137	3850	4680	5610	6655	7830	9141
4300	2546	3186	3910	4750	5700	6761	7950	9285
4400	2585	3238	3970	4825	5790	6865	8072	9432
4500	2624	3286	4025	4900	5875	6970	8195	9572
4600	2664	3333	4087	4970	5960	7070	8320	9710
4800	2740	3431	4202	5113	6130	7275	8555	9990
5000	2817	3525	4321	5253	6300	7475	8792	10250
5500	3000	3755	4608	5600	6715	7972	9370	10953
6000	3180	3981	4880	5935	7120	8446	9935	11586

Speed of Steamboats.

Screw Propellers.		*Paddle-Wheels.*	
$M = \frac{P\,n}{100}(1 - S)$,	1,	$M = \frac{R\,n\cos.\frac{1}{3}W}{16}(1 - S)$,	4,
$n = \frac{100M}{P(1 - S)}$,	2,	$n = \frac{16M}{R\cos.\frac{1}{3}W(1 - S)}$,	5,
$S = 1 - \frac{100M}{P\,n}$,	3,	$S = 1 - \frac{16M}{n\,R\cos.\frac{1}{3}W}$,	6,

Example. The pitch of a propeller is $P = 30$ feet, and makes 40 revolutions per minute; the slip is $S = 0{\cdot}4$. Required the speed of the vessel?

$$M = \frac{30 \times 40}{100}(1 - 0{\cdot}4) = 7{\cdot}2 \text{ miles per hour.}$$

Explanation of Table III.

To find the Slip.

RULE. Divide the acting area of the propeller, or paddle-wheels by the area of resistance of the vessel; find the quotient in the columns of *acting area*, and opposite, in the slip column, is the *slip per cent.*

TONNAGE OF VESSELS.

THE United States Custom House measurement of tonnage of vessels is expressed by the formula.

$$T = \frac{b\,d}{95}(l - \tfrac{3}{5}\,b),$$

Letters Denote,

T = *tonnage* of the vessel.
b = extreme *beam* in feet, taken above the main-wales.
d = *depth* of the vessel in feet. In double-decked vessels half the beam b is taken as the depth. For single decked vessels, the depth is taken from the under side of the deck plank to the ceiling of the hold.
l = *length* of the vessel in feet, from the fore-part of the steam to the after part of the stern-post, measured on the upper deck.

Example. The dimensions of a vessel are $l = 186$ feet, $b = 30$, and $d = 15$ feet, for a double-decked vessel. Required her tonnage?

$$T = \frac{30 \times 15}{95}\left(186 - \tfrac{3}{5}\,30\right) = 795{\cdot}77 \text{ tons.}$$

To Approximate the Tonnage of the Displacement.

$$Q = \mathfrak{B}l \times \begin{cases} 0{\cdot}015 \text{ very sharp vessels.} \\ 0{\cdot}022 \text{ very full vessels.} \end{cases}$$

FRESH WATER CONDENSOR.

Fig. 1.

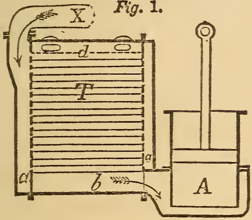


Fig. 2.

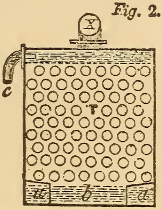


Fig. 1, is a longitudinal, and Fig. 2, a transverse section of a fresh water condensor with horizontal tubes.

A, air-pump. *a*, fresh water. *X*, exhaust-pipe. *b*, hot well. *T*, tubes. *c*, injection pipe. *d*, strainer.

The tubes are of copper one inch outside diameter, thickness of metal, No. 22 or 24 wire guage, weighs $5\frac{1}{4}$ ounces per foot. The space occupied by the tubes should be about cubical, that is, the sides of the tube-plate should be about the length of the tubes.

Between the injection and the tubes is a horizontal strainer, to spread the cold water uniformly over the tubes. Steam inside the tubes.

Letters Denote.

A = condensing area of all the tubes in square inches.
l = length of tubes, height and breadth of tube-plate in inches.
N = number of tubes in the condensor.
D = diameter of steam cylinder. S = stroke of piston, in inches.
n = number of revolutions of engine per minute.
T = temperature of exhaust steam } see steam table, page 248.
k = voluum coefficient of steam }

$$A = \frac{D^2 S \sqrt{n}}{5{\cdot}5\,k}\left(940 + T\right), \quad l = 0{\cdot}853\sqrt[3]{A}, \quad N = 0{\cdot}5128\,l^2, \quad A = 1{\cdot}61\,l^3.$$

Example. A fresh water condensor is to be constructed for an engine of $D = 62$ in. $S = 76$ in. making $n = 34$ revolutions per minute; $T = 230°$, $k = 1225$.

Required the condensing area of tubes, A = ?

$$A = \frac{62^2 \times 76 \times \sqrt{34}}{5{\cdot}5 \times 1225}\left(940 + 230°\right) = 295{,}709 \text{ square inches.}$$

Required the length of tubes, and sides of tube plate, l = ?

$$l = 0.853\sqrt[3]{295{,}769} = 57 \text{ inches nearly.}$$

Required the number of tubes in the condensor, N = ?

$$N = 0{\cdot}5128 \times 57^2 = 1666 \text{ tubes.}$$

Number of tubes in the top row $\frac{l}{1{\cdot}5} = \frac{57}{1{\cdot}5} = 38$, side row $= \frac{57}{1{\cdot}3} = 44$ tubes.

The tubes to be placed zigzag, as shown in Fig. 2. Should the location for the condensor not permit the cubical form, we have,

Length tubes $l = \frac{A}{1{\cdot}61bh}$, Breadth of tube plate $b = \frac{A}{1{\cdot}61lh}$, Height $h = \frac{A}{1{\cdot}61lb}$

Fresh water produced $G = \frac{D^2 S n}{164\,k}$, gallons per minute, or about 75 per cent of the feed water.

Temperature in the hot well about 110° to 115°.
" cf fresh water 120° to 130°.

For fresh water condensors the capacity of the air pump should be about 10 per cent larger than by the rules on page 236.

STEAM BOILERS.

The accompanying proportions are averages of a great number of good marine boilers.

Letters denote.

D = diameter of the steam-cylinder in inches.
S = stroke of piston under which steam is fully admitted, in inches.
n = number of double strokes, or revolutions per minute.
w = pounds of water evaporated per pound of coal, per hour.
k = volum coefficient from the steam table.
$\boxminus$ = fire grate in square feet, for each cylinder, and with natural draft.

To Find the Area of Fire Grate.

$$\boxminus = \frac{D^2 S n}{4{\cdot}66\, w k}. \qquad n = \frac{4{\cdot}66\, w k \boxminus}{D^2 S}. \quad \text{- - - - -} \quad 1, 2.$$

Example 1. A steam engine of $D = 54$ inches diameter of the cylinder, and stroke of piston 96 inches, cut off at $\frac{1}{2}$, S=48 inches; is to make 22 revolutions per minute. Anthracite coal to be used, that evaporates $w = 7$ pounds of water per pound of coal, and to carry 27 pounds of steam per square inch, $k = 649$. Required the area of fire grate $\boxminus$ = ? in square feet.

$$\boxminus = \frac{54^2 \times 48 \times 22}{4{\cdot}66 \times 7 \times 649} = 145{\cdot}34 \text{ square feet.}$$

Example 2. A steamboiler of $\boxminus$ = 128 square feet, is to be used for an engine of $D = 36$ inches diameter, and 64 inches stroke,—cut off the steam at $\frac{2}{3}$ then $S = 42{\cdot}66$ inches. Steam pressure to be kept at 25 pounds per square inch $k = 679$. $w = 6{\cdot}5$. Required for how many revolutions per minutes can the steam be kept at 25 pounds?

$$n = \frac{4{\cdot}66 \times 6{\cdot}5 \times 679 \times 128}{36^2 \times 42{\cdot}66} = 47{\cdot}6 \text{ revolutions.}$$

Horse Power of the Fire Grate.

H = horse power of the fire grate.
P = pressure in the boiler in pounds per square inch, excluding the atmosphere.
p = vacuum in the condensor in pounds per square inch.

$$\boxminus = \frac{Hx}{k w (P + 0.8 p)}, \qquad H = \frac{\boxminus\, k w (P + 0.8 p)}{x}. \quad 3\ 4.$$

Cut off the steam at	$\frac{1}{4}$ the stroke.	$x = 27700.$	saves	55	per cent of fuel.
	$\frac{1}{3}$ „ „	$x = 31400.$	„	49	
	$\frac{1}{2}$ „ „	$x = 38400.$	„	38	
	$\frac{2}{3}$ „ „	$x = 45500.$	„	26	
	$\frac{3}{4}$ „ „	$x = 49100.$	„	20	
Steam admitted throughout the stroke		$x = 61700.$	„	0 per cent.	

Example 3. Steamboilers are to be constructed for an engine of 650 horses, the steam to be cut off at $\frac{1}{2}$ the stroke, and $P = 36$ pounds per square inch, $k = 544$, $w = 7{\cdot}5$ pounds of water evaporated per pound of coal. Required the fire grate in the boilers $\boxminus$ = ? in square feet.

$$\boxminus = \frac{650 \times 38400}{554 \times 7.5\,(36 + 0.8 \times 11)} = 136 \text{ square feet.}$$

Example 4. Required the horse-power of a fire grate $\boxminus = 112$ square feet, to carry 18 pounds steam, and cut off at $\frac{2}{3}$ the stroke? $k = 810$, $w = 7$ pounds.

$$H = \frac{112 \times 18 \times 810 \times 7}{45500} = 251{\cdot}2 \text{ horses.}$$

Consumption of Coal.

$C =$ coal consumed in pounds per hour.

$$C = \frac{3\,D^2\,S\,n}{w\,k}. \qquad C = \frac{14\,H\,x}{k\,w\,(P+0{\cdot}8p)} \quad . \quad . \quad 5,\ 6.$$

Example 5 A steam engine of $D = 42$ inches diameter, and 48 inches stroke, cut off the steam at $\frac{1}{3}$ $S = 16$ inches, is to make $n = 65$ revolutions per minute with a pressure of 34 pounds per square inch, $k = 564$, and $w = 6$ pounds. Required the consumption of coal in pounds per hour $C = ?$

$$C = \frac{3 \times 42^2 \times 16 \times 65}{6 \times 564} = 1625 \text{ pounds per hour.}$$

Example 6. A pair of steam engines of $H = 260$ horses are to be worked with $P = 28$ pounds per square inch, cut off at $\frac{1}{2}$ the stroke, $k = 635$, the coal to evaporate $w = 6{\cdot}5$ pounds of water per pound of coal. Required the consumption of coal in pounds per hour $C = ?$

$$C = \frac{14 \times 260 \times 31400}{630 \times 6.5(28 + 0.8 \times 10)} = 775 \text{ pounds per hour.}$$

It will be observed in the formulas 4 and 6, that the higher steam used, the less fuel and fire grate is required for the same power,—the proportion of fuel will be nearly as the square root of the steam pressure; and still more fuel is saved by cutting off the steam at an early part of the stroke.

Fire Surface.

1. In the common single returning flue boilers, the whole amount of fire surface from the grate to the water line should be about 25 $\boxminus$.

2. In flue and returning tubular boilers the whole amount of fire surface 30 $\boxminus$.

3. Boilers with vertical flues, (2 inches diameter,) fire outside and water inside, fire surface 35 $\boxminus$.

Area of Flues. (Calorimeter.)

In the common single returning flue boilers, the area of the first row should be about 0.18 $\boxminus$.

Returning row, (flue or tube) 0.13 $\boxminus$

Area of chimneys 0.16 $\boxminus$.

Height of Chimney.

$$C = 2 \boxminus \sqrt{2+h}, \qquad h = \frac{C^2}{4 \boxminus^2} - 2.$$

Example. Required the height of a chimney for the consumption of 15 lbs. of coal per square foot of grate.

$$\text{height, } h = \frac{15^2}{4 \times 1^2} - 2 = 54 \text{ feet,}$$

Properties of Fuel.

Kind of Fuel.	lbs. of water evaporated per lb. of coal.	Per cent of Carbon.	Cubic feet of air req. for one lb. of coal.	Weight per cubic foot.	Cubic feet to stow a ton.
Bituminous coal, - - - -	7 to 9	80	265	50	44
Anthracite coal, - - - - - -	8 to 10	92	282	54	40
Coke, - - - - - - - - - -	8 to 10	86	245	31	72
Coke, nat. Virginia, - - - -	8 to 9	80	260	48	48
Coke Cumberland, - - - -	8 to 10	80	250	32	70
Charcoal, - - - - - - - -	5 to 6	96	265	24	104
Dry wood, - - - - - - - -	4 to 5	44	147	20	100
Wood with 20 per ct. water, -	4	34	115	25	100
Turf dry, - - - - - - - -	6	51	165	28	80
Turf 20 per ct. water, - - -	5	40	132	30	75
Illuminating gas, - - - - - -	13·8		194	0·037	29800
Oil, Wax, Tallow, - - - -	14	77	200	59	37
Alcohol (from market.) - - -	9·56	58	154	52	42

Chemically one pound of carbon burnt to carbonic acid requires the oxygen of 153 cubic feet of atmospheric air.

Morris Tasker & Co., Pascal Iron Works, Philada. Jan. 1861. Lap-welded American Charcoal Iron Boiler Flues.

Cut to specific lengths to suit purchasers.

Outside diameter.	Thickness of iron.		Heating surface per ft.	Weight per ft.	Price per ft.	Extra Price, Each safe end collar or shoulder	Extra Price, Screw on one end collar other.	cross flues each.
inches.	Wg.	inches.	Sq. ft.	pounds.	$. c.	$. c.	$. c.	$. c.
1·25	15	0·072	0·3273	1·12	18	16	82	1·45
1·5	14	0·083	0·3926	1·40	22	16	82	1·75
1·75	13	0·095	0·4589	1·60	24	18	86	
2	13	0·095	0·5236	1·95	28	18	86	2·50
2·25	13	0·095	0.5890	2·16	31	20	90	2·75
2·5	12	0·109	0·6545	2·60	33	23	96	3·00
2·75	12	0·109	0·7200	2·98	36	26	1·02	
3	12	0·109	0·7853	3·16	41	29	1·08	4·00
3·25	11	0·120	0·8508	3·78	46	32	Made to order.	Made to order.
3·5	11	0·120	0·9163	4·21	60	35		
3·75	11	0·120	0·9817	4·90	70	38		
4·	10	0·134	1·0472	5·25	75	40		
4·5	10	0·134	1·179	5·54	1·00	43		
5	9	0·148	1·3680	6·48	1·15	45		
6	8	0·165	1·5708	10·0	1·65	50		
7	8	0·165	1·8326	12	2·00	60		
8	8	0·165	2·0944	14	2·75	75		
9	7	0·180	2.3562	17	3·50			
10·	6	0·203	2·5347	20	4·25			

"*Safe Ends*" of thicker iron welded to one or both ends of flues to order, gives increased strength to the flue for calking in the tube-plate. The additional thickness of the safe end is placed in or outside of the Flue, as may be specified in the order.

Combustion is the rapid chemical combination of substances with oxygen. Carbon C and hydrogen H, are the substances most generally employed for generating heat. Carbon is fully consumed when combined with oxygen O, to form carbonic acid gas CO_2, and partly consumed when in the form of carbonic oxide gas CO or smoke. h=units of heat generated of one pound of fuel. The heat necessary to raise one pound of water one degree Fah. is one unit of heat. w=pounds of water at 212° evaporated per pound of fuel. A=volume in cubic feet and a=weight in pounds of atmospheric air required for the perfect combustion of one pound of fuel. C, O, and H are in the four first formulas, fractions in one pound of the compound fuel.

Perfect Combustion.

$$A=149\left[C+3\left(H-\frac{O}{8}\right)\right], \quad - \quad - \quad 1$$

$$a=12\left[C+3\left(H-\frac{O}{8}\right)\right], \quad - \quad - \quad 2$$

$$h=14500\left[C+4{\cdot}28H-\frac{O}{8}\right)\right], \quad 3$$

$$w=\frac{h}{966}=15\left[C+4{\cdot}28\left(H-\frac{O}{8}\right)\right]$$

Imperfect Combustion.

$$(CO)=\frac{56C-21O}{12}, \quad - \quad - \quad - \quad 5$$

$$(CO_2)=\frac{33O-44C}{12}, \quad - \quad - \quad - \quad 6$$

$$h=3960(CO_2)+1650(CO), \quad - \quad 7$$

$$h=8002{\cdot}5O-6820C, \quad - \quad - \quad 8$$

When oxygen is supplied to carbon in a proportion between CO and CO_2, both the gases will be formed separately in the proportion of the formulas 5 and 6, when the heat generated will be as formulas 7 and 8, in which C, O, CO and CO_2 are expressed in pounds, for instance: O=20 lbs. of oxygen united with C=12 lbs. of carbon will form

$$(CO)=\frac{56\times12-21\times20}{12}=21 \text{ lbs. carbonic oxide or smoke,}$$

$$(CO_2)=\frac{33\times20-44\times12}{12}=11 \text{ lbs. carbonic acid, and will}$$

generate $h=8002{\cdot}5\times20-6820\times12=78210$ units of heat.

One unit of heat=772 foot pounds, if generated per second will be H= 1.4 horses, of which we in our days practice utilizes about one-twentieth. The following table will show how important it is to fully consume the combustibles to acid. One pound of carbon consumed to oxide will generate only 1·72 horses, instead of 5·66 when consumed to acid.

Properties of Combustion, per Hour.

C	CO	CO_2	O	a	A	h	w	H
lbs.	lbs.	lbs.	lbs.	lbs.	cub. ft.	heat.	lbs.	horses.
1		3·666	2·666	12	149	14500	15	5·660
1	2·666		1·333	6	74·50	4400	4·55	1·720
0·433	1		0·566	2·550	31·65	1650	5·633	1·200
0·272		1	0·727	3·275	40·56	3960	4·100	1·545
	1·750	1·375	1	3·500	43·33	5440	5·633	2·125
	0·445	0·392	0.222	1	12·38	1210	1·250	0·472
	·0358	·0246	·0231	·0808	1	97·3	0·100	0·038
	0·584	0·244	0·170	0·800	9·920	966	1	0·378
	1·550	0·651	0·470	2·120	26·30	2558	2·645	1

To Approximate the Weight of Steam Boilers.

The area of fire grate gives a nearer approximation to the weight of Marine boilers, than the heating surface.

Letters denote.

⊟ = total fire grate in square feet.
W = weight of the boiler in pounds, including fire bars, doors, smoke pipe, fire tools and appendages, but without water. $W=800$ ⊟.

Example. Required the weight $W=?$ of a steam boiler of ⊟ $=250$ square feet, grate surface.

$$W=800\times250=200{,}000 \text{ lbs.}$$

Weight of the water is about three-fourths of W or of the total weight of boilers.

Weight of rivets, braces or stays, doors and fire bars, is about one quarter of W or of the total weight of boilers.

To Approximate the Weight of Engines.

Letters denote

D = diameter, S = stroke } of cylinder in inches.

W = weight of engine in pounds, including engine room tools, oil and tallow tanks, wheels, propeller and shafts.

	coefficient k.
Trunk and oscillating engines, - - - - - - - -	4
Direct action paddle wheel engines, - - - - - - -	4·25
Horizontal direct action propeller engine, - - - - -	4·5
Geared propeller engines, - - - - - - - - -	5·
American overhead beam engines, - - - - - - -	5·5
Side lever engines, - - - - - - - - - - -	6·
Horizontal direct action high pressure, - - - - - -	3·5

$$W=k\,D^2\sqrt{S}.$$

Example. Require the weight $W=?$ of a pair of Horizontal direct action propeller engines of $D=72$, $S=36$ inches, $k=4·5$.

$W=4·5\times72^2\sqrt{36}=139968$ lbs. for one cylinder, multiplied by $2=279936$ lbs. the weight required.

For trunk engines must be taken the largest diameter.

Punching Iron Plates.

To punch iron plates of from $\frac{1}{4}$ to 1 inch thick requires 24 tons per square inch of metal cut; that is, the circumference of the hole multiplied by the thickness of the plate is the area cut through.

Letters denote.

d = diameter of the punch or hole.
D = diameter of the hole in the die.
t = thickness of the iron plate.
All dimensions in 16ths of an inch.
W = weight or force in pounds required to punch the hole.
$W = 660\, t\, d$. $D = d + 0{\cdot}2\, t$.

Example 1. An iron plate of $t=6$ sixteenths of an inch thick, and the hole to be $d=12$ sixteenths in diameter. Required the force $W=?$

$$W = 660 \times 6 \times 12 = 47520 \text{ lbs., the answer.}$$

Example 2. Under the same conditions require the diameter $D=?$ of the die.

$$D = 12 + 0{\cdot}2 \times 6 = 13{\cdot}2 \text{ sixteenths.}$$

Example 3. Required the diameter of piston for a direction action steam punch, for the plate and hole as in example 1, pressure of steam to be 50 lbs. per square inch.

Force $47520 = A \times 50$ of which area of piston will be $A = \frac{47520}{50} = 950{\cdot}4$ square inches, which answers to a diameter of 34·8, say 36 inches.

Shearing Iron Plates.

It requires the same force per section cut, for shearing as for punching, namely, 20 to 24 tons per square inch. If the shears are good, sharp, and well adjusted, 16 tons may be sufficient.

When the cutters in the shears are inclined to one another, the area cut, will be the square of the thickness of the plate multiplied by half the cotagent for the angle of the shears. Let v=angle of the shears, W and t same as for punching.

$$W = 88\, t^2 \cot.v.$$

Example 4. What force is required to cut a half inch plate $t=8$ sixteenths with a pair of shears forming an angle of $v=12°?$ Cot.12°=4·7.

$$W = 88 \times 8^2 \times 4{\cdot}7 = 26470 \text{ lbs.}$$

Atmospheric Columns.

Water=33·95 feet.	2·3 feet for 1 lbs.	per square	inch.
Seawater=33·05 ft.	2·23	"	"
Mercury at 60°=30 inches.	2·05 inches,		"
Atm. air=28183 feet.	1912 feet,	"	"

Atmospheric air Required for each.

Blacksmith's forge,	100 to 200	Cubic feet per minute.
Charcoal forge,	400 to 500	
Finery forge,	800 to 1000	
Charcoal furnace,	1000 to 3000	
Anthracite furnace	2000 to 5000	

Cupola.

In a cupola of 3 feet 4 inches diameter, and 10 feet high, can be melted down 1000 lbs. of cast iron, 200 lbs. of bitumninous coal per hour, with a blowing machine of 4·5 horses making 1700 cubic feet of air per minute into a pressure of 2·25 inches of mercury at which the temperature of the blast will be about 70° Fah.

BLOWING MACHINES.

Letters denote.

D = diameter in inches, S = stroke in feet, } of blowing cylinder double acting.

l = part of the stroke S under which the air compresses from the atmospheric density to that in the reservoir.

F = mean resistance in pounds per square inch of the air on the cylinder piston.

P = pressure in pounds per square inch of the blast in the reservoir.

C = cubic feet of air of atmospheric density, delivered from the blowing cylinder to the reservoir per minute.

H = actual horse power required to work the blowing engine, including 13 per cent. for friction.

d = diameter of blast pipe in inches.

n = number of revolutions or double stroke per minute.

A = area of supply valve to the blowing cylinder in square inches, at each end of cylinder.

p = vacuum in pounds per square inch, on the supply side of the cylinder piston, which should not exceed 0·1 lbs.

V = velocity of the blast through the tuyeres in feet per second.

v = velocity of the air through the supply valve A, in feet per second, which should not exceed 100 feet.

a = area of the orifice or tuyeres in square inches.

h = height of mercury in inches, in the gauge on the blast reservoir.

L = length of the blast pipe in feet from the receiver to the tuyeres.

k = volume coefficient, see Table.

t = temperature Fah. of the blast caused by compression or heating.

Example 1. *Formulæ* 8. For an Anthracite blast furnace is required 4000 cubic feet of air per minute, under a pressure of 6 inches mercury. Required the horse power necessary for the blowing machine? The effectual resistance $F=2{\cdot}365$ lbs. see Table. Assume the vacuum to be $v=0{\cdot}09$ lbs.

$$\text{We have } H=\frac{4000\,(2{\cdot}365+0{\cdot}09)}{198}=49{\cdot}6 \text{ horses.}$$

Example 2. *Formulæ* 10. Suppose the blast cylinder to be $D=144$ inches diameter with $S=15$ feet stroke. Required the number of double strokes per minute $n=$?

$$n=\frac{96\times4000}{144^2\times15}=12{\cdot}3 \text{ the answer.}$$

Example 3. *Formulæ* 9. Under the above conditions, require the area of the supply valves $A=$? when the velocity $v=105$ feet, per second.

$$A=\frac{144^2\times15\times12{\cdot}3}{40\times105}=911 \text{ square inches.}$$

Capacity of Blast Reservoir should not be less than the following proportions, but more is better.

For one single acting cylinder, 20 }
For one double acting cylinder, 10 } times the capacity of one cylind'r.
Two double act. cyl. cranks at 90° 5 }

One double acting cylinder, same as two single acting. The more cylinders the less capacity required for blast reservoir.

$$V=246\sqrt{h\,(1+0{\cdot}00208\,t)}, \qquad t=32+493\,(k-1),$$

$$P=14{\cdot}7\,(k-1), \qquad t=33{\cdot}55\,P+32,$$

$$P=\frac{t-32}{33{\cdot}55}, \qquad k=\frac{P}{14{\cdot}7},$$

Formulas for Blowing Machines.

$$l=\frac{S\,h}{30+h}, \quad 1$$

$$P=0{\cdot}49h, \quad 2$$

$$C=\frac{D^2\,S\,n}{96}, \quad 3$$

$$C=\frac{198\,H}{F+p}, \quad 4$$

$$C=\frac{a\,k\,V}{26}, \quad 5$$

$$C=1{:}83ah(30+h), \quad 6$$

$$H=\frac{D^2\,S\,n(F+p)}{19000}, \quad 7$$

$$H=\frac{C\,(F+p)}{198}, \quad 8$$

$$A=\frac{D^2\,S\,n}{40v}, \quad 9$$

$$n=\frac{96\,C}{D^2\,S}, \quad 10$$

$$d=\frac{\sqrt{C+10\,L}}{3} \quad 11$$

$$v=350\sqrt{p} \quad 12$$

$$v=\frac{D^2\,S\,n}{4\,A}, \quad 13$$

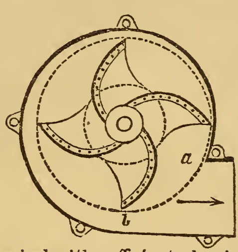
$$p=\frac{D^4\,S^4\,n^2}{180000000\,A^2}, \quad 14$$

$$k=\frac{30+h}{30}, \quad 15$$

Table for Blast and Blowing Machines.

Volume and temperat.		Guage in inches.		Pressure lbs. sq inch.		Stroke.	Velocity.
k	t	water.	h	P	F	l	V
1·002	33°	1	0·073	0·036	0·032	0·0024	72
1·005	34·5	2	0·147	0·079	0·063	0·0049	102
1·007	35·5	3	0·220	0·108	0·095	0·0073	125
1·010	37	4	0·294	0·144	0·128	0·0097	144
1·012	38	5	0·368	0·180	0·159	0·0121	161
1·015	39·5	6	0·441	0·216	0·191	0·0145	176
1·020	42	8	0·588	0·288	0·253	0·0192	204
1·025	44·5	10	0·736	0·360	0·309	0·0239	228
1·030	47	12	0·884	0·432	0·379	0·0287	249
1·035	49·5	14	1·030	0·503	0·437	0·0334	269
1·043	53·5	17	1·250	0·612	0·531	0·0400	297
1·051	57·5	20	1·470	0·719	0·623	0·0467	322
1·062	63	24	1·766	0·863	0·745	0·0556	352
1·074	69	28	2·060	1·008	0·865	0·0643	381
1·082	73	31	2·281	1·116	0·955	0·0706	401
1·091	77.3	34	2·501	1·223	1·043	0·0769	420
1·100	82	37	2·720	1·332	1·130	0·0833	438
1·109	86·5	46	3·000	1·470	1·205	0·0908	460
1·116	90	47·5	3·500	1·715	1·431	0·1045	496
1·132	98	54·3	4·000	1·961	1·636	0·1178	530
1·165	114·5	67·7	5·000	2·450	2·010	0·1431	593
1·200	132	81·4	6·000	2·941	2·365	0·1667	650
1·265	164·5	108·5	8·000	3·925	3·088	0·2105	751
1·400	232	163	12·00	5·900	4·389	0·2859	918
1·500	282	203·7	15·00	7·375	6·875	0·3333	1077
1·625	344·5	254·6	18·75	9·217	8·831	0·3846	1393
1·750	407	305·5	22·49	11·06	10·67	0·4285	1590
1·875	469·5	356·4	26·24	13·90	11·64	0·4666	1760
2·000	532	407·4	30·00	14·75	12·50	0·5000	1955

FAN OR VENTILATOR.



Fans constructed as the accompanying figure have been found by the Author who has made several of them, to be the most effective.

The vanes are each one quarter of an arithmetical spiral with a pitch twice the diameter of the fan, that is, each vane should be constructed in an angle of 90° from centre to tip. Length of fan to be from $\frac{1}{3}$ to $\frac{1}{4}$ the diameter. Inlet to be half the diameter of the fan. Number of vanes to be not more than six, and not less than four. Six vanes work softer and better, but they give no better effect than four.

The housing should be an arithmetical spiral with sufficient clearing for the fan at a, and leaving a space at b about $\frac{1}{8}$ of the diameter. Fans of this construction make no noise.

Letters denote.

$d =$ diameter, $l =$ length, of fan in inches.
$A =$ area in sq. in., $L =$ length in feet, of blast pipe, to be as straight as possible.
$a =$ area in sq. in. tuyeres or outlet.
$C =$ cubic feet of air delivered per minute.
$h =$ inches of mercury.
$v =$ velocity in feet per second through a.
$k =$ volume coefficient, see Table, page 287.
$n =$ revolutions of fan per minute.
$H =$ actual horse power required to drive the fan.

Formulas for Fans.

$$h = \frac{d\,n^2}{50000000}\sqrt{\frac{d\,l}{25\,a + d\,l}}, \qquad 1$$

$$H = \frac{d\,l\,h\,n}{24000}, \qquad 2$$

$$h = \frac{24000\,H}{d\,l\,n}, \qquad 3$$

$$v = 244\sqrt{h} \qquad 4$$

$$A = a\sqrt{L} \qquad 5$$

$$v = \frac{n\sqrt{d}}{28{\cdot}86}\sqrt[4]{\frac{d\,l}{25\,a + d\,l}}, \qquad 6$$

$$n = \frac{24000\,H}{d\,l\,h}, \qquad 7$$

$$C = \frac{v\,a\,k}{2{\cdot}6}, \qquad 8$$

$$C = 94\,a\,k\sqrt{h} \qquad 9$$

$$A = \frac{C}{94\,k}\sqrt{\frac{L}{h}}. \qquad 10$$

Example 1. A fan of $d=36$ inches diameter, $l=12$ inches, making $n=725$ revolutions per minute, area of tuyere being $a=25$ square inches. Required the density of the blast in inches of mercury $h=?$

Formulæ 1. $h = \dfrac{36 \times 725^2}{50000000}\sqrt{\dfrac{36 \times 12}{25 \times 25 + 36 \times 12}} = 0{\cdot}242$ inches.

Example 2. Under the same conditions require the cubic containt of air delivered per minute, $C=?$ $k=1{\cdot}01$ the nearest in the Table.

Formulæ 9. $C = 94 \times 25 \times 1{\cdot}01\sqrt{0{\cdot}242} = 1167{\cdot}7$ cubic feet. Required the horse power $H=?$

Formulæ 2. $H = \dfrac{36 \times 12 \times 0{\cdot}242 \times 725}{24000} = 3{\cdot}16$ horses.

BLAST OR IRON FURNACES.

It is almost impossible to set up the many variable circumstances connected with the performance of Blast Furnaces, into a table form. The datas herein given are deduced to an average from the performances of a great many furnaces both in America and Europe.

The accompanying Tables are so arranged that the numbers in Table I., multiplied by the numbers in Table II., gives the corresponding charge of Iron ore, lime stone, coal, and the produce of pig iron in pounds per 24 hours, with the consumption of air in cubic feet per minute.

Table II. contains the effective capacity of blast furnaces in cubic yards.

Example. It is required to construct a blowing machine for an Anthracite blast furnace of 12 feet diameter of boshes, height of stack 45 feet, to be worked with hot blast. Required the produce of pig iron per 24 hours, cubic feet of air per minute and actual horse power of the blowing engine?

Produce of pig iron 155 Table I.×123 Table II.=19065 lbs. or 8·5 tons per 24 hours.

Consumption of air 20×123=2460 cubic feet per minute. Suppose the blast to be blown into the furnace at a pressure of $P=2{\cdot}94$ lbs., vacuum in the supply side in cylinder to be $p=0{\cdot}07$ lbs. we shall have the required actual power.

Forumlæ 8, p. 287. $$H=\frac{2460\ (2{\cdot}36+0{\cdot}07)}{198}=30{\cdot}2 \text{ horses.}$$

Table I. Iron or Blast Furnaces.

The unit being the capacity of the Furnace in cubic yards.		Charge and produce per 24 hours. Iron Ore.	Pig Iron.	Lime Stone.	Coal.	Air per minute.
		lbs.	lbs.	lbs.	lbs.	cub. feet.
Soft charcoal	Cold blast,	535	215	196	400	24
	Warm blast,	700	292	256	350	19
Hard charcoal	Cold blast,	670	270	245	400	24
	Warm blast,	875	365	320	350	19
Coke	Cold blast,	268	108	98	515	26
	Warm blast,	350	146	128	397	20
Bituminous	Cold blast,	252	101	92	515	24
	Warm blast,	327	136	120	397	19
Anthracite	Cold blast,	287	115	105	515	26
	Warm blast,	373	155	137	597	20

Table II. Capacity and Dimensions of Iron Furnaces.

Diameter of Boshes in ft.	Height of stack in feet. 25	30	35	40	45	50	55	60
8	40	44	47	51	54	58	62	65
9	50	55	60	64	69	73	78	83
10	62	68	74	79	75	91	96	102
11	75	82	89	96	103	110	117	123
12	90	98	106	114	123	130	139	147
13	105	115	125	134	144	153	163	172
14	121	133	145	155	167	178	189	200
15	140	153	166	178	191	204	217	230
16	160	174	189	203	217	232	247	261
17	280	197	213	229	245	262	279	295
18	202	220	239	257	275	293	312	330

Sixty-two Simple Substances, with their Symbols, Equivalents and Specific Gravities.

Name.	Symbol.	Equiv.	Sp. Grav.	Name.	Symbol.	Equiv.	Sp. Grav.
Aluminium,	Al	13·7	2·56	Molybedenum,	Mo	47·9	8.60
Antimony, (Stib.)	Sb	64·6	6·70	Nickel,	Ni	29·5	8.80
Arsenic,	As	37·7	5·70	Niobium,	Nr		
Barium,	Ba	68·6	4·70	Nitrogen,	N	14·2	0·972
Bismuth,	Bi	71·5	9·82	Norium,	No		
Boron,	B	11·0	2·00	Osmium,	Os	99·7	10·00
Bromine,	Br	78·4	3·00	Oxygen,	O	8·0	1·102
Cadmium,	Cd	55·8	8·65	Palladium,	Pd	53·3	11·35
Calcium,	Ca	20·5	1·58	Pelopium,	Pe		
Carbon,	C	6·1	3·50	Phosphorus,	P	15·9	1·77
Cerium,	Ce	46·0		Platinum,	Pt	98·8	21·50
Chlorine,	Cl	35·5	2.44	Potassium, (Kal.)	K	39·2	0·865
Chromium,	Cr	26·2	5.90	Rhodium,	R	52·2	11·00
Cobalt,	Co	29·5	8.53	Ruthenium,	Ru	52·1	8·60
Columbium, (Ta.)	Ta	184·8	6.00	Selenium,	Se	40·0	4·5
Copper, (Copr'm)	Cu	31·7	8·80	Silicon,	Si	22·0	
Didymium,	D	48·0		Silver, (Argent'm)	Ag	108·3	10·5
Erbium,	E			Sodium, (Natr.)	Na	23·5	0·972
Fluorine,	F	18·7	1·32	Strontium,	Sr	43·8	2·54
Glucinum,	G	6·9		Sulphur,	S	16·1	1·99
Gold, (Aurum)	Au	196·6	19·30	Tellurium,	Te	64·2	6·30
Hydrogen,	H	1	0·069	Terbium,	Tb		
Iodine,	I	126·5	4·94	Thorium,	Th	60·0	
Iridium,	Ir	98·5	18·68	Tin, (Stannum)	Sn	58·9	7·29
Iron, (Ferrum)	Fe	28·0	7·75	Titanium,	Ti	24·5	5·28
Lantanium,	Ln	48·0		Tungsten, (Wolf.)	W	92·0	17·00
Lead, (Plumbum)	Pb	103·7	11·35	Uranium,	U	60·0	10·15
Lithium,	L	7·0	0·59	Vanadium,	V	68·5	
Magnesium,	Mg	12·7	1·75	Yttrium,	Y	32·0	
Manganese,	Mn	26·0	8·00	Zinc,	Zn	32·3	7·00
Mercury (Hydra.)	Hg	200.0	13·50	Zirconium,	Zr	34·0	

PROPORTIONS OF COMPOUNDS.

Names.	*Carbon.* C	*Hydrogen.* H	*Oxygen.* O	*Nitrogen.* N
Olive oil, by weight, - -	772	133	95	
Spermaceti oil, " - -	780	118	102	
Castor oil, " - - -	740	103	157	
Linseed oil, " - - -	760	113	127	
Alcohol, " - - -	527	129	344	
Sugar, " - - -	432	68	500	
Atmosp. air, " - - -	- -	- - -	770	230
" air by volume, - -	- -	- - -	210	790
Water, fresh, by weight, - -	- -	1	8	
" " " volume, -	- -	2	1	
India rubber by weight, - -	853	147		
Blood by wt. 66·6 iron.	665	53	110·4	104

Gunpowder, 75 Nitre, 13 charcoal, 12 Sulphur.

Binary Compounds, with their Formulas and Equivalents,

Name of Compound.	Formula.	Equiv.	Name of Compound.	Formula.	Equiv.
Water,	HO	9·0	Binoxide of Lead,	$Pb\ O_2$	119·7
Binoxide of Hydro.,	HO_2	17.0	Dinoxide of Copper,	$Cu_2\ O$	71·4
Protoxide of Nitro.,	NO	22.2	Protoxide of Copper,	$Cu\ O$	39 7
Binoxide of Nitrog.,	NO_2	30·2	Chloride of Copper,	$Cu\ Cl$	67·2
Hyponitrous Acid,	NO_3	38·2	Oxide of Zinc,	$Zn\ O$	40·3
Nitrous Acid,	NO_4	46·2	Sesquioxide of Ant.,	$Sb_2\ O_3$	153·2
Nitric Acid,	NO_5	54·2	Antimonious Acid,	$Sb_2\ O_4$	161·2
Ammonia,	NH_3	17·2	Antimonic Acid,	$Sb_2\ O_5$	169·2
Cyanogen,	NC_2	26·4	Protoxide of Tin,	$Sn\ O$	66·9
Sulphurous Acid,	SO_2	32·1	Binoxide of Tin,	$Sn\ O_2$	74·9
Sulphuric Acid,	SO_3	40·1	Bisulphide of Tin,	$Sn\ S_2$	91·1
Carbonic Oxide,	CO	14·1	Chloride of Tin,	$Sn\ Cl$	94·4
Carbonic Acid,	CO_2	22·1	Bichloride of Tin,	$Sn\ Cl_2$	129·9
Light Carb'ett'd Hy.	$H_2\ C$	8·1	Oxide of Bismuth,	$Bi\ O$	79·5
Olefiant Gas,	$H_2\ C_2$	14·2	Chloride of Bismuth	$Bi\ Cl$	107·0
Bisulphide of Carb.,	CS_2	38·3	Protoxide Mangan.,	$Mn\ O$	34·0
Boracic Acid,	BO_3	35·0	Sesquioxide Manga.	$Mn_2\ O_3$	76·0
Chlorous Acid,	$Cl\ O_4$	67·5	Red Ox. Manganese	$Mn_3\ O_4$	110·0
Chloric Acid,	$Cl\ O_5$	75·5	BinoxideManganese	$Mn\ O_2$	42·0
Hydrochloric Acid,	$H\ Cl$	36·5	Protoxide of Cobalt,	$Co\ O$	37·5
Quadrochloride Nit.,	$N\ Cl_4$	156·2	Peroxide of Cobalt,	$Co_2\ O_3$	83·0
Iodic Acid,	IO_5	166·5	Protoxide of Nickel,	$Ni\ O$	37·5
Hydroiodic Acid,	HI	127·5	Peroxide of Nickel,	$Ni_2\ O_3$	83·0
Teriodide of Nit'gn,	NI_3	393·7	Arsenious Acid,	$As_2\ O_3$	99·4
Hydrofluoric Acid,	$H\ Fl$	19·7	Arsenic Acid,	$As_2\ O_5$	115·4
Phosphorous Acid,	$P_2\ O_3$	55·4	Arseniuretted Hydr.	$H_3\ As_2$	78·4
Phosphoric Acid,	$P_2\ O_5$	71·4	SesquisulphideArse.	$As_2\ S_3$	123·7
Phosphoret'd Hydr.,	$H_3\ P_2$	34·4	Protoxide Mercury,	$Hg\ O$	208·0
Selenious Acid,	$Se\ O_2$	56·0	Peroxide of Mercury	$Hg\ O_2$	216·0
Selenic Acid,	$Se\ O_3$	64·0	Bisulphide of Merc.,	$Hg\ S_2$	232·2
Seleniuret'd Hydro.,	$H\ Se$	41·0	Chloride of Mercury	$Hg\ Cl$	235·5
Sulphurr'd Hydro.,	HS	17·1	Bichloride of Merc.,	$Hg\ Cl_2$	271·8
Protoxide of Iron,	$Fe\ O$	36·0	Oxide of Silver,	$Ag\ O$	116·3
Peroxide of Iron,	$Fe_2\ O_3$	80·0	Chloride of Silver,	$Ag\ Cl$	143·8
Dinoxide of Lead,	$Pb\ O$	111·7	Teroxide of Gold,	$Au\ O_3$	220·6
Protoxide of Lead,	$Pb_2\ O$	215·4	Terchloride of Gold,	$Au\ Cl_3$	303·1
Quadrotrisoxi'e Le'd	$Pb_3\ O_4$	343·1	Bichloride Platin'm.	$Pt\ Cl_2$	169·8

To Transform Chemical Formulas into a Mathematical Expression.

Rule. Multiply together the equivalent, (equiv.) and the exponent, (exp.) of each substance and the product is the proportion in the compound by weight. Divide this by its specific gravity gives the proportions by bulk or volume.

Example 1. The chemical formulæ for common alcohol is $C_4\ H_6\ O_2$. Required its proportioned parts by weight in 1000?

$$\begin{array}{lll} & \text{equiv, exp.} & \\ \text{Carbon} & C_4, = 6{\cdot}12 \times 4 = 24{\cdot}48 & \\ \text{Hydrogen} & H_6, = \quad 1 \times 6 = \ 6 & \\ \text{Oxygen} & O_2, = \quad 8 \times 2 = 16 & \end{array} \Bigg\} \times 21{\cdot}5 \left\{ \begin{array}{c} \text{proportions.} \\ 527 \\ 129 \\ 344 \end{array} \right\} \text{by weight.}$$

$$1000 : 46{\cdot}48 = 21{\cdot}5. \qquad 1000$$

OPTICS.

Optics is that branch of philosophy which treats of the property and motion of light.

Mirrors.

Example 1. Fig. 307. Before a *concave mirror* of $r = 6$ feet radius, is placed an object $O = 1$, at $d = 1{\cdot}75$ feet from the vertex. Required the size of the image $I = ?$

$$\text{image } I = \frac{O\ r}{r - 2\ d} = \frac{1 \times 6}{6 - 2 \times 1{\cdot}75} = 2{\cdot}4$$

Example 2. Fig. 308. Before a *concave mirror* of $r = 5{\cdot}25$ feet radius, is placed an object $O = 1$, at $D = 4{\cdot}5$ feet from the vertex. Required the size of the inverted image $I = ?$

$$\text{image } I = \frac{O\ r}{2\ D - r} = \frac{1 \times 5{\cdot}25}{2 \times 4{\cdot}5 - 5{\cdot}25} = 1{\cdot}4$$

Example 3. Fig. 309. Before a *convex mirror* of $r = 1{\cdot}8$ feet radius, is placed an object $O = 1$, at $D = 3{\cdot}15$ feet from the vertex. Required the size of the image $I = ?$, and the distance in the mirror $d = ?$

$$\text{image } I = \frac{1 \times 1{\cdot}8}{2 \times 3{\cdot}15 + 1{\cdot}8} = 0{\cdot}222 \quad \text{distance } d = \frac{3{\cdot}15 \times 1{\cdot}8}{2 \times 3{\cdot}15 + 1{\cdot}8} = 0{\cdot}699 \text{ ft.}$$

Example 4. Fig. 310. A *parabolic mirror* is $h = 1{\cdot}31$ feet high, and $d = 2{\cdot}15$ feet in diameter. Required the focal distance $f = ?$ from the vertex.

$$\text{focal distance } f = \frac{d^2}{16\ h} = \frac{2{\cdot}15^2 \times 12}{16 \times 1{\cdot}31} = 2{\cdot}646 \text{ inches.}$$

Optical Lenses.

Example 5. Fig. 316. A double *convex lens*, of *crown glass*, having its radii $R = r = 6$ inches. Required its principal focal distance $f = ?$

For crown glass the index of refraction is $m = 1{\cdot}52$. See table.

$$f = \frac{6}{2(1{\cdot}52 - 1)} = 5{\cdot}768 \text{ inches.}$$

Microscope.

Letters denote.

$\mathfrak{p}$ = magnifying power of a lens.

$\mathfrak{D}$ = limit of distinct vision.

$\mathfrak{a}$ = limit of distinct sight, which for *long-sighted* eyes is about 10 or 12 inches, and *near-sighted* 6 to 8 inches. For common eyes take $\mathfrak{a} = 10$ inches.

$\mathfrak{d}$ = limit distance of the object from the optical centre at distinct vision.

Example 6. Fig. 322. Required the magnifying power of a single microscope with principal focal distance, $f = 4{\cdot}3$ inches?

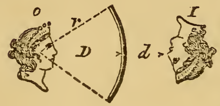
$$\text{Mag. power } \mathfrak{p} = \frac{\mathfrak{a} + f}{f} = \frac{10 + 4{\cdot}3}{4{\cdot}3} = 3{\cdot}325 \text{ times.}$$

307 *Spherical Concave Mirror.*

r = radius, and $f=\frac{1}{2}r$, focal distance of the mirror.

$$I=\frac{O\,r}{r-2\,d}.\quad D=\frac{d\,r}{r-2\,d}.$$

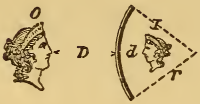
The image disappears when $d=f=\frac{1}{2}r$.



308 *Spherical Concave Mirror.*

$$I=\frac{O\,r}{2\,D-r}.\qquad d=\frac{D\,r}{2\,D-r}.$$

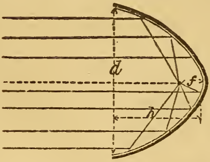
When the object is beyond the focal distance the image will be inverted.



309

Spherical Convex Mirror.

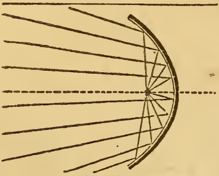
$$I=\frac{O\,r}{2\,D+r}\qquad d=\frac{D\,r}{2\,D+r}.$$



310

Parabolic Concave Mirror.

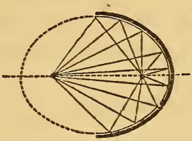
$$f=\frac{d^2}{16\,h}.\qquad h=\frac{d^2}{16\,f}.$$



311

Hyperbolic Concave Mirror.

Heat, Light, or Sound emanating from the foci of a hyperbola will be reflected diverged, from the concave surface.



312

Eliptic Concave Mirror.

Emanating rays from either of the two foci in an elipse, will be refracted by the convex surface to the other foci.

Astronomical Telescopes and Opera Glasses.

Example 7. Fig. 325. The principal focal distance $f = 0{\cdot}65$ inches of the ocular or eye-lens. $F = 58$ inches the principal focal distance of the objective-lens. Required the magnifying power of the telescope $I = ?$

$$\text{image } I = \frac{O\,F}{f} = \frac{1 \times 58}{0{\cdot}65} = 89{\cdot}23 \text{ times the object.}$$

The telescope is to be used at the limited distance $D = 1380$ feet and $D = \infty$. Required the proper lengths $l = ?$ and micrometrical motion of the ocular or eye-lens? when the limit of distinct sight $a = 10$in. $F = 58 : 12 = 4{\cdot}833$ feet. $f = 0{\cdot}65 : 12 = 0{\cdot}05416$ feet.

$$l = \frac{1380 \times 4{\cdot}833}{1380 - 4{\cdot}833} + \frac{10 \times 0{\cdot}05416}{10 + 0{\cdot}05416} = \frac{4{\cdot}89035}{0{\cdot}05386}$$

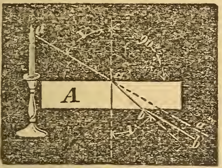
When $D = 1380$ feet, the length $l = 4{\cdot}94421$ feet.

When $D = \infty$, $l = 4{\cdot}8333 + 0{\cdot}05386 = 4{\cdot}88719$ "

Micrometrical motion of eye lens $\begin{cases} 0{\cdot}05702 \text{ "} \\ 0{\cdot}68424 \text{ inches.} \\ \frac{11}{16} \text{ nearly.} \end{cases}$

Table of Refractive Indices.

Substances.	Index. m.	Substances.	Index. m.
Cromate of Lead	2·97 2·50	Quartz	1.54
		Muriatic Acid	1.40
Realgar	2·55	Water	1.33
Diamond	2·45	Ice	1.30
Glass, flint	1·57	Hydrogen	1.000138
Glass, crown	1·52	Oxygen	1.000272
Oil of Cassia	1·63	Atmospheric air	1.000294
Oil of Olives	1·47		

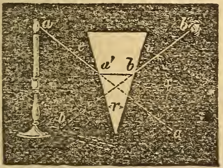


314 *Prism.*

An emergent rays of light $a\ a'$ falling upon a transparent medium A (say a glass prism) will be transmitted through in the direction $a\ b$, and delivered in the direction $b\ b'$, parallel to $a\ a'\ a''$.

V = angle of incident, v = angle of refraction.

$$\text{Indix of refraction } m = \frac{\sin. V}{\sin. v}.$$



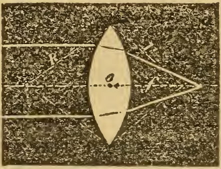
315 Given the direction of the emergent rays $a\ a'$, angles e and r.—to find the angles u and x,—or the direction of the rays $b\ b'$.

$$\cos. z = \frac{\cos. e}{m}, \quad \cos. u = m \cos. (180 - z - r).$$

$$x = 180 - (e + r + u).$$

When $e = u$, the angle x is smallest.

An eye in b' will see the candle in the direction $b'\ b\ b''$.

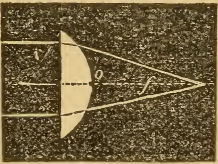


316 *Double Convex Lens.*

$$f = +\frac{R\,r}{(m-1)(R+r)}.$$ the principal focal distance.

$$f = \frac{r}{2(m-1)}.$$ when $R = r$

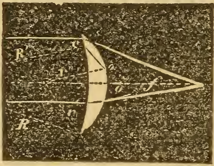
o = optical centre of the lens.



317 *Plano Convex Lens.*

$$f = +\frac{r}{m-1}.$$

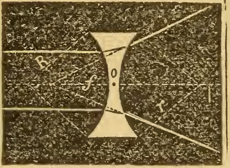
The optical centre is in the convex surface.



318 *Convex-concave Lens (Meniscus.)*

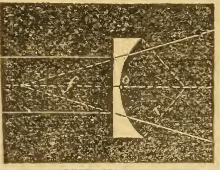
$$f = +\frac{R\,r}{(m-1)(R-r)}.$$

Draw the radii R' and r' parallel to one another.—Draw $n\ o$, then o is the optical centre.



319 *Double Concave Lens.*

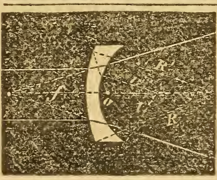
$$f = -\frac{R\,r}{(m-1)(R+r)}.$$



320 *Plano Concave Lens.*

$$f = -\frac{r}{m-1}.$$

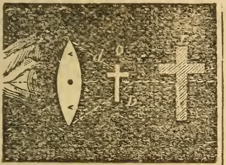
The optical centre is in the concave surface.



321 *Concavo-convex Lens.*

$$f = -\frac{R\,r}{(m-1)(R-r)}.$$

Draw R and r' parallel to one another. Draw $n\ o$, then o is the optical centre.

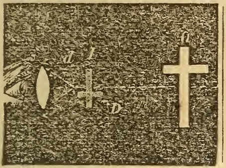


322

Single Microscope.

$$I:O=f:f-d,\quad I=\frac{Of}{f-d},\quad D=\frac{df}{f-d},$$

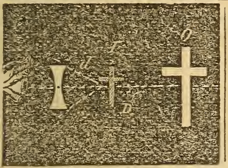
$$\mathfrak{P}=\frac{\mathfrak{a}+f}{f},\quad \mathfrak{D}=\frac{\mathfrak{d}f}{f-\mathfrak{d}},\quad \mathfrak{d}=\frac{\mathfrak{a}f}{\mathfrak{a}+f}.$$



323

When the object O is beyond the focal distance the image I will be inverted.

$$I:O=f:D-f,\quad I=\frac{Of}{d-f},\quad d=\frac{Df}{D-f}.$$

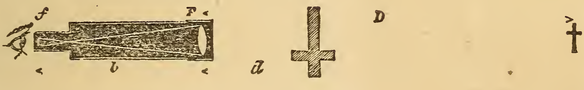


324 *Diminishing Power of a Double Concave Lens.*

$$I:O=f:f+D\qquad I=\frac{Of}{f+D},$$

$$D=\frac{f(O-I)}{I},\qquad d=\frac{Df}{D+f}.$$

325 *Astronomical Telescope.*



$$I:O=F:f,\qquad I=\frac{OF}{f},\qquad d=\frac{Df}{F}.$$

$$l=\frac{DF}{D-F}\overset{*}{\pm}\frac{\mathfrak{a}f}{\mathfrak{a}+f}.\qquad \left(\text{if } D=\infty,\quad l=F+\frac{\mathfrak{a}f}{\mathfrak{a}+f}\right)$$

* + for astronomical telescope, — for opera-glasses.

326 *Opera Glass.*

Formulas are the same as for Astronomical Telescope.

GEOGRAPHY.

THE *Earth* on which we live, is a round ball or sphere, with a mean diameter of 7914 statute miles. The whole surface of the earth is 196800000 square miles, of which only one fourth or nearly 50,000000 square miles is land, and about 150,000000 square miles water.

Table of Area and Population of the whole Earth.

Divisions of the Earth.	*Area in Square Miles.*	*Population.*	*Proportion to Square Mile.*
America, - - - - - - - -	14,491,000	60,000:000	4·1
Europe, - - - - - - - - -	3,760,000	272,000,000	72
Asia, - - - - - - - - - -	16,313,000	730,000.000	44
Africa, - - - - - - - - -	10,936,000	200,000,000	18
Oceanica, - - - - - - - -	4,500,000	27,000,000	6.7
Total, - - - - - - - - - -	50,000,000	1,289,000,000	26

About $\frac{1}{30}$ th of the whole population are born every year, and nearly an equal number die in the same time; making about one born and one dead per second.

The Earth is not a perfect sphere, it is flatted at the Poles. The following are her true dimensions in statute miles of 5280 feet.

Dimensions of the Earth.

Diameter	7898·8809	miles	at the Poles.
	7911·92	"	mean, or in 45° lat.
	7924·911	"	at the Equator.
Difference - - -	26·0302	"	Poles and Equator.
Flatted - - - -	13·015	"	at each Pole.
Circumference	24802·486	"	round the Poles.
	24851·640	"	Mean, or in 45° lat.
	24884·22	"	round the Equator.

To Find the radius of the Earth in any given Latitude

$$R = 3955{\cdot}96(1 + 0{\cdot}00164 \cos.2l), \text{ statute miles.}$$

Definitions,

Axis of the Earth is an imaginary diameter around which the earth revolves.

Poles of the Earth are the two extremities of the axis, and are called North and South.

Equator of the Earth is the great circle at equo-distances from the Poles; it divides the Earth into the Northern and Southern Hemispheres

Meridian is any great circle of the Earth drawn through the Poles; hence the Meridian runs north and south, and are at right-angles to the Equator.

The *Equator* and *Meridians* are divided into degrees, minutes, and seconds.

Latitude is the degrees on a Meridian counted from the Equator.

Longitude is the degrees on the Equator or on circles parallel with the Equator, counted at right angles from a Meridian.

Parallels are circles drawn through equal Latitudes; they are parallel and concentric with the Equator, and at right-angles to the Meridians.

East and **West** is the direction of the Equator and Parallels, or at right-angles to north and south. Turn your face towards the south, the east is on the left hand, and the west on the right.

The time in which the earth makes one revolution, is divided into 24 hours,

$$\text{and} \qquad \frac{360°}{24} = 15° \text{ per hour.}$$

To Reduce Longitude into Time.

RULE. Divide the number of degrees, minutes, and seconds by 15, and the quotient will be the time.

Example 1. Longitude 74° 48′ 15″, what is it in time?

$4h\ 59^m 13^s$ the answer.

To Find the Difference in Time between two Places.

RULE. Divide the difference in longitude by 15, and the quotient is the difference in time.

Example 2. Required the difference in time between New York and Cincinnati?

Longitude of Cincinnatti	-	84° 27′ *W*
" " New York -	-	74 07 *W*
Difference in longitude -	-	10° 20′

$$\frac{10 \times 60 + 20}{15} = 41 \text{ minutes } 20 \text{ seconds,}$$

the difference in time. When it is 12 o'clock in Cincinnati, it is $12h\ 41'\ 20''$ in New York.

Example 3. Required the difference in time between Philadelphia and Paris?

Longitude of Philadelphia	75° 10′ *W*
" " Paris - -	2 20 *E*
Difference in longitude	77° 30°

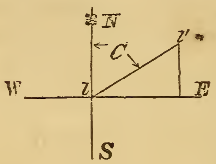
divided by 15 will be $5h\ 10^m$ the difference in time. When it is 12 o'clock in Philadelphia, it is $5h\ 10^m$ o'clock in Paris.

Example 4. A vessel sails from New York for Liverpool, after she has been at sea about one week, her difference in time with New York was found to be $2h\ 7^m 45^s$. Required her longitude from New York?

$15(2h\ 7\ \ 45\ ) = 31°\ 58'\ 15''$ from New York.

NAVIGATION.

To navigate a vessel upon the supposition that the earth is a level plane, on which the meridians are drawn north and south, parallel with each other; and the parallels east and west, at right-angles to the former.



The line NS represents a meridian north and south; the line WE represents a parallel east and west.

A ship in l sailing in the direction of $l\,l'$, and having reached l', it is required to know her position to the point l, which is measured by the line $l\,l'$, and the angle $N\,l\,l'$; and imagined by the lines $l\,a$ and $a\,l'$

While the vessel is running from l to l', the distance is measured by the log and time; and the course $N\,l\,l'$ is measured by the compass commonly expressed in points.

These four quantities bear the following names.

$d = l\,l'$, *distance* from l to l' in miles.
$C = N\,l\,l'$, *course*, or points from the meridian.
$\mathfrak{d} = l\,a$, *departure* or difference in longitudes, in miles.
$u = a\,l'$, *difference in latitudes*, in miles.
$l =$ *latitude* in degrees.
$L =$ difference in longitude, in degrees or time.

Formulas for Plane Sailing.

$\mathfrak{d} = d \sin.C$, - - 1,

$\mathfrak{d} = u \tan.C$, - - 2,

$\mathfrak{d} = 60 \cos.l\, L$, - 3,

$\mathfrak{d} = \sqrt{d^2 - u^2}$, - 4,

$u = d \cos.C$, - - 5,

$u = \mathfrak{d} \cot.C$, - - 6,

$u = \dfrac{60L \cos.l}{\tan.C}$, - 7,

$u = \sqrt{d^2 - \mathfrak{d}^2}$, - 8,

$d = \dfrac{\mathfrak{d}}{\sin.C}$, - - 9,

$d = \dfrac{u}{\cos.C}$, - - 10,

$d = \dfrac{60L \cos.l}{\sin.C}$, - 11,

$d = \sqrt{\mathfrak{d}^2 + u^2}$, - 12,

$\cos.l = \dfrac{\mathfrak{d}}{60L}$, - - 13,

$\cos.l = \dfrac{d \sin.C}{60L}$, - 14,

$\cos.l = \dfrac{u \tan.C}{60L}$, - 15,

$L = \dfrac{\mathfrak{d}}{60\cos.l}$, - 16,

$L = \dfrac{d \sin.C}{60\cos.l}$, - - 17,

$L = \dfrac{u \tan.C}{60\cos.l}$, - 18,

$\cos.C = \dfrac{u}{d}$, - - 19,

$\sin.C = \dfrac{\mathfrak{d}}{d}$, - - 20,

$\tan.C = \dfrac{\mathfrak{d}}{u}$, - - 21,

$\sin.C = \dfrac{60L \cos.l}{d}$, 22,

$\tan.C = \dfrac{60L \cos.l}{u}$, 23,

See Table of Formulas for Plane Sailing.

Example 1. A vessel sails *east north east* (6 points,) 236 miles. Required her departure $\mathfrak{D}$? and difference in latitude u?

Formula 1. $\mathfrak{D} = d \sin.C = 236 \times \sin.6$ points $= 218$ miles departure, and $u = d \cos.c = 236 \times \cos.6$ points $= 90{\cdot}3$ miles difference in latitude.

Example 2. A ship sails in north latitude in a course $C = ESE\tfrac{3}{4}E = 6\tfrac{3}{4}$ points, at a distance of 132 miles she made a difference in longitude of $L = 3^\circ\ 34'$. What latitude is she in?

Formula 14. $$\cos.l = \frac{d \sin.C}{60L} = \frac{132 \times \sin.6\tfrac{3}{4}}{60 \times 3 + 34} = 0{\cdot}59832,$$

or $l = 53^\circ\ 15'$ the latitude.

In high latitudes and very long distances, the preceding formulas will not give such correct results as may be desired, because they are set up with the supposition that the earth is a level plane; but by the aid of spherical trigonometry, we are enabled to ascertain courses and distances *correctly*, from and between any known points on the earth.

Spherical Distances.

For the spherical formulas, letters will denote.

l = lower latitude, in degrees from the equator.
l' = highest latitude, " " "
C = course, from the latitude l to l'.
C' = course, from " l' to l.
d = shortest distance between l and l' in degrees of the great circle.
L = difference in longitude between l and l', in degrees, or time.
$\tan.m = \cot.l' \cos.L$.
$n = 90 \mp l - m$.
$-\,l$, when l and l' are on one side of the equator.
$+\,l$, when l is on one side and l' on the other. Then

$$\cos.d = \frac{\sin.l' \cos.n}{\cos.m}, \quad - \quad - \quad 1,$$

$$\sin.C = \frac{\sin.L \cos.l'}{\sin.d}, \quad - \quad - \quad 2,$$

$$\sin.C' = \frac{\sin.L \cos.l}{\sin.d}, \quad - \quad - \quad 3,$$

Example. Required the shortest distance and course from New York to Liverpool?

$$\left.\begin{array}{l} l = 40^\circ\ 42' \text{ N. latitude} \\ \quad\ 74^\circ \text{ `` W. longitude} \end{array}\right\} \text{New York.}$$

$$\left.\begin{array}{l} l' = 53^\circ\ 22' \text{ N. latitude} \\ \quad\ 2^\circ\ 52' \text{ W. longitude} \end{array}\right\} \text{Liverpool.}$$

$L = 71^\circ\ 8'$ difference in longitude.

$\tan.m = \cot 53^\circ\ 22' \times \cos.71^\circ\ 8' = 13^\circ\ 31'$.

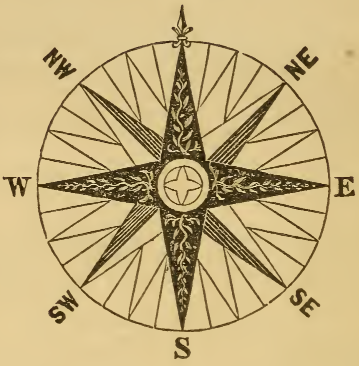
$n = 90^\circ - 13^\circ\ 31' - 40^\circ\ 42' = 35^\circ\ 47'$.

For. 1. $$\cos.d = \frac{\sin.53^\circ\ 22' \times \cos.35^\circ\ 47'}{\cos.13^\circ\ 31''} = 47^\circ\ 58'.$$

Shortest distance $= 47^\circ \times 60 + 58 = 2878$ geographical miles.

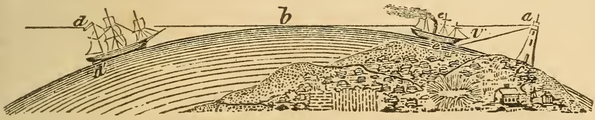
$$\sin.C = \frac{\sin.71^\circ\ 8' \times \cos.53^\circ\ 22'}{\sin.47^\circ\ 58'} = 49^\circ\ 23' = 4\tfrac{3}{8} \text{ points}$$

course from New York $NE\tfrac{3}{8}E$.



North.	South.	Points.	Degrees.	*sineC.*	*Cos.C.*	*tan.C.*
N.	S.	¼	2° 49′	·0491	·9988	·0492
		½	5 37	·0979	·9952	·0983
		¾	8 26	·1544	·9880	·1982
N. by E. and N. by W.	S. by E. and S. by W.	1	11 15	·1936	·9811	·1989
		1¼	14 4	·2430	·9700	·2505
		1½	16 52	·2901	·9570	·3032
		1¾	19 41	·3368	·9416	·3577
N. N. E. and N. N. W.	S. S. E. and S. S. W.	2	22 30	·3827	·9239	·4142
		2¼	25 19	·4276	·9039	·4730
		2½	28 7	·4713	·8820	·5343
		2¾	30 56	·5140	·8577	·5993
N. E. by N. and N. W. by N.	S. E. by S. and S. W. by S.	3	33 45	·5555	·8314	·6883
		3¼	36 44	·5981	·8014	·7463
		3½	39 22	·6343	·7731	·8204
		3¾	42 11	·6715	·7410	·9062
N. E. and N. W.	S. E. and S. W	4	45 0	·7071	·7071	1·000
		4¼	47 49	·7410	·6715	1·103
		4½	50 37	·7731	·6345	1·218
		4¾	53 26	·8014	·5981	1·348
N. E. by E. and N. W. by W.	S. E. by E. and S. W. by W.	5	56 15	·8314	·5555	1·496
		5¼	59 4	·8577	·5140	1·668
		5½	61 52	·8820	·4713	1·870
		5¾	64 41	·9039	4276	2 114
E. N. E and W. N. W.	E. S. E. and W. S. W.	6	67 30	·9239	·3827	2·414
		6¼	70 19	·9416	·3368	2·795
		6½	73 7	·9570	·2901	3 295
		6¾	75 56	·9700	·2430	3·991
E. by N. and W. by N.	E. by S. and W. by S.	7	78 45	·9811	·1936	5·027
		7¼	81 34	·9880	·1544	6·744
		7½	84 22	·9952	·0979	11·14
		7¾	87 11	·9988	·0491	20·32
East or	West . .	8	90°	1 000	0.000	∞

To Find the Distances of Objects at Sea.



Height in feet.	Distance in miles.	Height in feet.	Distance in miles.	Height in feet.	Distance in miles.	Height in feet.	Distance in miles.
0·582	1	11	4·39	30	7·25	200	18·72
1	1·31	12	4·58	35	7·83	300	22·91
2	1·87	13	4·77	40	8·37	400	26·46
3	2·29	14	4·95	45	8·87	500	29·58
4	2·63	15	5·12	50	9·35	1000	32·41
5	2·96	16	5·29	60	10·25	2000	59·20
6	3·24	17	5·45	70	11·07	3000	72·50
7	3·49	18	5·61	80	11·83	4000	83·7
8	3·73	19	5·77	90	12·55	5000	93·5
9	3·96	20	5·92	100	13·23	1 mile.	96·1
10	4·18	25	6·61	150	16·22		

The distance being the tangent *a b* in statute miles, at the elevation *a c*, in feet.

Example 1. The light-house at *a* is 100 feet above the level of the sea. Required the distance *a b*.

Height 100 feet = 13·23 miles.

Example 2. The flag of a ship is seen from *a* in *d*. Required the distance *a*, *d*, when the flag is known to be 50 feet above the level *d′* of the sea?

Height of the light 100	= 13·23 miles	*a*, *b*
Height of the flag 50	= 9·35 "	*b*, *d*,
Distance to the ship	= 22·58 miles	*a*, *d*.

Example 3. A steamer is seen at *e*, the horizon *b* seen in the masts is assumed to be 16 feet above the level *e′*. Required the distance to the ship?

Height of the light 100	= 13·23 miles	*a b*,
The assumed height 16	= 5·29 "	*e b*,
Distance to the ship	= 7·94 miles	*a e*,

To Find the Distance by an Observed Angle v.

Letters denote.

d = distance in statute miles ($a\,e'$) to the object observed.
t = the tangent ($a\,b$) in statute miles, or distance to the horizon.
v = the observed angle $e\,a\,e'$, of the horizon and the loadline of the object.
r = radius of the earth.
w = the angle $b\,a\,c$.

$$\cos. w = \frac{t}{\sqrt{t^2+r^2}},$$

$$d = \cos.(w - v)\sqrt{t^2 - r^2} - \sqrt{\cos.^2(w - v)(t^2+r^2) - t^2}.$$

POPULATION OF COUNTRIES AND CITIES IN THE WORLD.

Names.	Year	*Populat'n*
NORTH AMERICA	1861	60,000,000
U. S. of America	1861	31,429,891
New York - - - -	1853	850,000
Philadelphia - - -	"	575,000
Baltimore - - - -	"	195,000
Cincinnati - - - -	"	160,186
New Orleans - - -	"	145,449
Boston - - - - -	1850	136,880
Pittsburg - - - -	1853	110,241
St. Louis - - - -	"	100,000
Chicago - - - - -	"	60,625
Buffalo - - - - -	"	60,000
Louisville - - - -	"	51,726
Albany - - - - -	"	50,763
Providence - - - -	"	47,500
Newark, N. J. - - -	"	45,500
Charleston - - - -	1850	42,985
Washington - - -	"	40,001
Rochester - - - -	1853	40,000
Troy - - - - - -	1850	28,785
Richmond - - - -	"	27,570
Savannah - - - -	1853	23,458
San Francisco - -	"	60,000
British America	"	3,634,850
Montreal - - - -	1851	57,715
Toronto - - - - -	1855	50,000
Quebec - - - - -	1852	42,052
Halifax - - - - -	1851	33,582
St. John - - - - -	1852	22,745
Cuba	1853	1,009,060
Havana - - - - -	"	147,360
Santiago - - - -	"	85,242
Matanzas - - - -	"	81,397
Puerto Princip - -	"	46,532
Hayti	"	943,000
Portau Prince - -	"	20,000
St. Domingo - - -	"	10,000
Jamaica		377,433
Kingston - - - -		35,000
Mexico		7,853,394
Mexico City - - -		200,000
Guadalaxara - - -		70,000
La Puebla - - - -		50,000
San Luis Potosi - -		40,000
Central America		2,146,000
New Guatimala - -		50,000
SOUTH AMERICA		16,000,000
Brazil		6,065,000
Rio Janeiro - - -	1853	400,000
Bahia		120,000
Bolivia		1,030,000
La Paz - - - -		20,000
Equador		500,000
Quito - - - - - -		60,000
Plata		820,000
Buenos Ayres -	1854	85,000
Cordova - - - -	"	13,000
Paraguay		1,000,000

Names.	Year.	*Population*
Patagonia		1,200,000
Chili		1,200,000
Santiago - - - -		80,000
Valparaiso - - -	1848	60,000
New Grenada		2,363,054
Bogota - - - - -		40,000
Péru	1851	2,279,085
Lima - - - - - - -	1850	100,000
Venezuela	1854	1,419,289
Caraccas - - - -	1853	63,000
Great Britain & Ireland		27,686,609
London - - - -	1856	2,500,000
Manchester - - -	1851	401,321
Liverpool - - - -	"	376,065
Glasgow - - - - -	"	347,001
Dublin - - - - -	"	254,850
Edinburg - - - -	"	158,015
France		35,779,222
Paris - - - - - -	1851	1,053,262
Marseilles - - - -	1852	192,527
Lyons - - - - - -	"	156,169
Spain	1849	13,936,218
Madrid - - - - -	1850	260,000
Barcelona - - - -		121,815
Portugal	1850	3,471,203
Lisbon - - - - -	"	455,217
Belgium	1849	4,359,090
Brussels - - - - -	1846	123,874
Holland		3,962,290
Amsterdam - - -	1852	228,800
Denmark		2,412,926
Copenhagen - - -	1852	133,140
Hamburg Free City		200,690
Bremen ditto - - -		53,156
Sweden & Norway	1850	4,810,812
Stockholm - - - -	1855	100,000
Gottenburg - - -	"	30,000
Christiania - - - -		26,500
Prussia	1856	17,178,091
Berlin - - - - -	1852	441,931
Austria	1850	36,514,466
Vienna - - - - -	1846	407,980
Italy		24,733,385
Rome - - - - - -	1856	177,461
Naples - - - - -	1851	416,475
Palermo - - - - -	1850	167,222
Turkey		35,360,000
Constantinople - -		786,990
Russia	1851	60,098,821
St. Petersburg - -	1852	533,241
Moscow - - - - -	1840	349,068
Odessa - - - - -	1850	71,392
Sevastopol - - - -	1855	40,000
China		387,632,907
Pekin - - - - -		2,000,000
Canton - - - - -		1,000,000

Names of Places.	Latitude.	Longitude.	West from Greenwich,
N. AMERICA AND WEST INDIES.			
Quebec, - -	46° 49′ *N*	71 16	
Halifax, - -	44 38 ,,	63 35	
Portland *light*	43 36 ,,	70 12	
Buffalo, -	42 53 ,,	78 55	
Chicago, -	42 0 ,,	87 35	
Newburyport *light*,	42 48 ,,	70 49	
Boston *State House*,	42 21 ,,	71 4	
Nantucket *light*,	41 23 ,,	70 3	
Newport *Custom*,	41 29 ,,	71 19	
New York,	40 42 ,,	74 00·7	
Philadelphia,	39 57 ,,	75 10	
Cape Henlopen,	38 46 ,,	75 4	
Cincinnati, -	39 6 ,,	84 27	
St. Louis, -	38 36 ,,	89 36	
Richmond, -	37 32 ,,	77 27	
Washington *City*,	38 53 ,,	77 0·3	
Baltimore, -	39 18 ,,	76 37	
Cape Hatteras,	35 14 ,,	75 30	
Charleston *light*,	32 42 ,,	79 54	
Savannah, -	32 5 ,,	81 8	
Cape Florida *light*,	25 41 ,,	80 5	
Pensacola, -	30 24 ,,	87 10	
Mobile, -	30 42 ,,	87 59	
New Orleans,	29 57 ,,	90 0	
Porto Rico, -	18 29 ,,	66 7	
Cape Hayti's City,	19 46 ,,	72 11	
Havana, - -	23 9 ,,	82 22	
Vera Cruz, -	19 12 ,,	96 9	
Mexico, - -	19 26 ,,	99 5	
Porto Bello, -	9 34 ,,	79 40	
Porto Cabello,	10 28 ,,	68 7	
Cape St. August'e,	8 21 *S*	34 57	
Rio Janeiro, -	22 56 *S*	43 9	
Buenos Ayres,	34 36 ,,	58 22	
Cape Horn, -	55 59 ,,	67 16	
Valparaiso Fort,	33 2 *N*	71 41	
Panama Ft. N.E.,	8 57 ,,	79 31	
San Francisco,	37 47 ,,	122 21	
ENGLAND.			
London, -	51 31 "	0 6	“
Liverpool, -	53 22 "	2 52	“
Greenwich,	51 29 "	0 0	0
Glasgow, -	55 52 "	4 16	*W*
Dublin, -	53 23 "	6 20	,,
Edinburgh,	55 57 "	3 12	,,
Bristol, - -	51 27 "	2 35	,,
Dover, - -	51 8 "	1 19	*E*
FRANCE.			
Paris Observatory,	48 50 "	2 20	,,
Havre de Grace,	49 29 "	0 6	,,
Cherbourgh, -	49 38 "	1 37	,,
Marseilles Observ.	43 18 "	5 22	,,
Antwerp, -	51 13 "	4 24	,,
Calais, - -	50 58 "	1 51	,,
ITALY.			,,
Florence, -	43° 46 "	11 16	,,
Leghorn, -	43 32 "	10 18	,,
Rome, *St. Peter's*,	41 54 "	12 27	,,
Naples, *light*, -	40 50 "	14 16	,,
Ancona, *light*,-	43 38 "	13 30	,,

Names of Places.	Latitude.	Longitude.	East from Greenwich,
GERMANY.			
Berlin, - -	52 31 *N*	13 24	
Bern, - -	46 57 “	7 25	
Rotterdam, -	51 54 “	4 28	
Antwerp, -	51 13 “	4 24	
Amsterdam, -	52 22 “	4 51	
Bremen, -	53 5 “	8 49	
Hague, - -	52 4 “	4 16	
Hamburg, -	53 33 “	9 56	
Lubeck, - -	53 52 “	10 49	
AUSTRIA.			
Vienna, - -	48 13 “	16 23	
Venice, - -	45 26 “	12 21	
Trieste *castle*, -	45 39 “	13 46	
TURKEY.			
Ragusa, *mole*,	42 38 “	18 7	
Athens *Philopa.*,	37 58 “	23 44	
Salonica, -	40 39 “	22 57	
Constantinople,	41 1 “	28 59	
SWEDEN AND NORWAY.			
Stockholm, -	59 21 “	18 4	
Gothenburg, -	57 42 “	11 57	
Christiania, -	59 55 “	10 52	
Bergen, - -	60 24 “	5 20	
Wisby *Gotland*,	57 39 “	18 17	
DENMARK.			
Copenhagen, -	55 41 “	12 34	
Elsineur, -	56 2 “	12 37	
RUSSIA.			
St. Petersburg,	59 56 “	30 19	
Moscow, - -	55 46 “	35 33	
Revel, - -	59 26 “	24 46	
Riga, - -	56 51 “	23 57	
Cronstadt, -	59 58 “	29 51	
Abo, - -	60 27 “	22 15	
Odessa, - -	46 27 “	30 42	
SPAIN.			
Madrid, - -	40 25 “	3 42	*W*
Barcelona, -	41 23 “	2 11	*E*
Algiers *light*, -	36 49 “	3 1	“
Gibralter, -	36 6 “	5 20	*W*
Carthagena obser.,	37 36 “	1 1	“
PORTUGAL.			
Oporto, - -	41 11 “	8 38	“
Lisbon, - -	38 42 “	9 9	“
Cape St. Vincent,	37 3 “	9 2	“
SICILY.			
Messina, -	38 12 “	15 35	*E*
Palermo, -	38 8 “	13 22	“
Malta, - -	35 54 “	14 13	“
CHINA.			
Peking, - -	39 54 “	116 28	“
Canton, - -	23 7 “	113 14	“
			“
Cape of Good Hope.	34 22 *S*	18 30	“
Sidney, *Australia*,	34 0 ,,	151 23	“
Jerusalem, *Pales.*,	31 48 *N*	37 20	“

Distances by Sea

between the most important places on the earth: in Geographical miles, 60 per degree.

* Round Cape of Good Hope.
† Round Cape Horn.

From San Francisco 1980 to Sandwich Islands.
From Canton, 4500 " "

	Batavia	Cape Good Hope	Constantinople	Rome	Gibraltar	Lisbon	Liverpool	London	Paris	Antwerp	Hamburg	Gottenburg	Copenhagen	Stockholm	St. Petersburg	New York	New Orleans	Cape Horn	San Francisco	Canton
San Francisco																				5820
Cape Horn																			6100	9780
New Orleans																		7620	13800†	17400†
New York																	2129	7200	13380†	16980†
St. Petersburg																4516	6076	8837	15936†	15796*
Stockholm															377	4311	5871	8632	15731†	15591*
Copenhagen														495	700	3816	5376	8137	15236†	15096*
Gottenburg													139	634	839	3727	5287	8047	15147†	15007*
Hamburg												360	488	983	1188	3576	5136	7896	14076†	14836*
Antwerp											348	582	721	1216	1421	3348	4908	7668	13848†	14628*
Paris										518	726	897	986	1481	1686	3290	4850	7610	13790†	14570*
London									410	258	451	617	706	1201	1407	3240	4800	7560	13740†	14520*
Liverpool								710	760	818	1256	1197	1286	1781	1986	3000	4560	7320	13500†	14280*
Lisbon							900	1060	1110	1168	1370	1547	1636	2131	2336	2980	4300	6460	1254†	13020*
Gibraltar						330	1200	1400	1380	1438	1680	1817	1906	2405	2606	3220	4500	6460	12640	13080*
Rome					1018	1348	2218	2418	2398	2456	2698	2835	2924	3419	3624	4238	5518	7578	13658	14091*
Constantinople				1150	2040	2340	3240	3440	3490	3548	3720	3927	4016	4511	4716	5260	6540	8460	14680†	15000*
Cape Good Hope			7320	6418	5400	5340	6600	6840	6840	6838	7080	7277	7366	7861	8066	6900	7380	3600	9780†	7680
Batavia		5180	12500*	11598*	10580*	10520*	11780*	12020*	12060*	12018*	12260*	12457*	12544*	13039*	13244*	12080*	12560*	9100	7860	2580
Port Jackson	3860	6120	13560*	12538*	11520*	11460*	12720*	12960*	1301*	13068*	13296*	41347*	13536*	14031*	14236*	12300†	12720	5280	6480	4620

Distances in Statute Miles.

Nearest travelling distances between Cities in Europe.

	Copenhagen	Stockholm	St. Petersburg	Moscow	Odessa	Constantinople	Warsaw	Triest	Rome	Munich	Vienna	Turin	Berne	Berlin	Hamburg	Antwerp	Lisbon	Madrid	Paris	Liverpool	London
Liverpool																					212
Paris																				509	297
Madrid																			625	1002	790
Lisbon																		265	890	1267	1055
Antwerp																	1142	877	252	544	332
Hamburg																391	1560	1295	670	936	724
Berlin															175	509	1515	1250	625	984	772
Berne														608	605	541	1150	885	442	951	739
Turin													193	803	901	736	895	630	445	954	742
Vienna												662	574	500	679	751	1724	1459	790	1295	1083
Munich											270	508	281	454	600	513	1431	1166	551	1045	833
Rome										507	745	328	521	963	1109	1022	1223	958	773	1373	1161
Triest									407	342	338	456	584	796	942	855	1351	1086	1107	1399	1187
Warsaw								798	1069	563	460	1034	836	536	711	1045	2051	1786	1297	1589	1377
Constantinople							1200	1000	1050	1679	1409	1456	1584	1909	2084	2160	2350	2086	2412	2704	2492
Odessa						750	805	1400	1907	1369	1099	1856	2000	1336	1511	1836	2750	2485	1900	2450	2238
Moscow					910	1660	792	2751	3158	2683	2413	3191	2964	2489	2664	3173	4004	3739	3114	3717	3505
St. Petersburg				403	1200	1950	733	1506	1774	1271	1162	1742	1544	969	1144	1478	2484	2219	1594	1953	1741
Stockholm			440	835	1953	2775	1148	1633	1928	1291	1370	1799	1606	866	691	1082	2251	1986	1334	1626	1413
Copenhagen		435	875	1270	1772	2340	967	1198	1493	856	935	1364	1171	431	256	647	1816	1551	899	1191	979
Lubeck	234	669	1109	1504	1514	2123	709	748	1146	639	718	849	643	214	39	352	1599	1334	710	896	684

Sailing distances

between the most important seaports in the United States, in Geographical miles, 60 per Degree.

	Vera Cruz	New Orleans	Havana	Key West	Savannah	Charleston	Cape Hatteras	Richmond	Norfolk	Washington	Baltimore	Cape Henry	Philadelphia	Cape Henlopen	New York	Sandy Hook	Nantucket light	Boston	Halifax
Boston.																			390
Nantucket light.																		107	360
Sandy Hook.																	193	284	648
New York.																18	211	302	666
Cape Henlopen.															149	131	274	380	650
Philadelphia.														103	252	234	377	483	753
Cape Henry.													238	135	276	258	403	509	765
Baltimore.												152	390	287	428	410	555	661	917
Washington.											174	158	396	293	434	416	561	667	923
Norfolk.										178	172	30	268	165	306	288	433	539	795
Richmond.									90	248	242	100	338	235	375	358	503	609	865
Cape Hatteras.								214	144	272	266	114	321	218	340	322	426	624	786
Charleston.							281	495	425	553	547	395	602	499	621	603	707	905	1065
Savannah.						95	376	590	520	648	612	490	697	594	716	698	802	1000	1162
Key West.					542	622	768	1348	2288	1406	1410	1258	1465	1362	1484	1466	1570	1768	1930
Havana.				85	562	592	738	1328	1258	1376	1380	1228	1435	1332	1454	1436	1540	1738	1900
New Orleans.			640	640	1187	1267	1413	2003	1933	2051	2055	1903	2110	2007	2129	2111	2215	2413	2685
Vera Cruz.		850	823	870	1412	1492	1638	2228	1158	2276	2280	2128	2335	2232	2354	2336	2440	2638	2800
Porto Bello	1407	1370	1200	1070	1612	1692	1838	2428	2358	2476	2480	2328	2535	2432	2554	2536	2640	2738	2635

Distances in Statute Miles

between the most important places in the United States by Railroads and Rivers. The distances are given by the shortest routes.

*Mississippi River

																				Montreal.
																			Boston.	326
																		Albany.	200	253
																	New York.	144	236	396
																Philadelphia.	87	231	323	483
															Baltimore.	98	185	329	421	581
														Washington.	39	137	224	368	460	610
													Richmond.	131	170	268	355	499	591	741
												Wilmington.	247	378	417	515	602	746	838	988
											Charleston.	262	509	640	679	777	864	308	1100	1250
										Savannah.	112	374	621	732	791	889	976	1088	1212	1462
									Atlanta.	292	308	619	866	977	1036	1134	1221	1333	1487	1754
								Montgomery.	175	467	483	794	1041	1152	1211	1309	1396	1508	1632	1929
							New Orleans.	496	671	963	979	1290	1537	1648	1707	1805	1892	2004	2128	2425
						St. Louis.	1201*	726	901	1018	1034	1392	1121	990	951	967	1293	1143	1490	1069
					Chicago.	310	1618*	1493	1318	1611	1441	1937	1011	880	841	865	1048	908	1073	918
				Cincinnati.	273	697	1548*	1148	973	1266	1436	1109	862	731	692	663	847	693	993	813
			Pittsburgh.	310	344	1174	2025*	1625	1450	1169	1057	795	533	402	294	353	470	545	693	595
		Harrisburg.	245	684	1094	904	1790*	1294	1119	874	762	500	253	122	82	106	194	289	430	489
	Buffalo.	664	220	468	543	1165	2016*	1937	1441	1446	1334	1072	825	694	655	557	470	325	525	375
Cleveland.	183	395	150	255	360	511	1803*	1403	1327	1235	1123	861	614	483	444	608	636	508	708	558

ASTRONOMY.

ASTRONOMY is that branch of Philosophy which treats of the properties of heavenly bodies.

The mean Solar day is divided into 24 hours.

* A Sidereal year is - - - - - - - -	365d 6h 9m 9·6s.
† An Anomalistic year is - - - - - - -	365d 6h 13m 49·3s.
✕ A Tropical year is - - - - - - - -	365d 6h 13m 49·3s.

* *Sidereal year* is the time in which the Earth makes one revolution round the Sun, in reference to a fixed star.

† *Anomalistic* year is the time which the Earth occupies between each perihelion to the sun.

✕ *Tropical year* is the periodical return of seasons.

Mean distance from the Earth to the Sun 95000000 miles or 11992 diameters of the Earth.

Inclination of the Ecliptic to the Equinoctial 23° 28′ 40″.

The Sun subtends an angle from the Earth of 32′ 3″.

Horizontal paralax of the Sun 8·6″ seconds.

Velocity of a point at the Equator by the rotation of the Earth 152·5 feet per second.

Cycle of the Sun is the period of 28 years at which the days of the week return to the same days at the month.

The Moon.

Distance from the Earth to the Moon is 237000 miles, = 30 diameters of the Earth, = about 0·25 the diameter of the Sun, or the diameter of the Sun is twice the diameter of the Moon's orbit around the Earth.

Diameter of the Moon is 2160 miles, or about 0·2729 of the Earth's diameter.

Volume of the Moon compared with the Earth is 0·02024.

Density of the Moon compared with the Earth is 0·5657.

Mass of the Moon compared with the Earth is 0·011399.

Inclination of the Moon's orbit to the Ecliptic 5° 8′ 48″.

The Moon subtends an angle from the Earth of 31′ 7″.

Mean Sidereal revolution of the Moon 27·32166 days.

Mean Synodical revolution of the Moon 29·5305887 days.

The Moon passes the meridian in periods of 24·814 hours, or 48m 50s later every day.

Moon's Age is the number of days from the last new-moon.

Number of Months for the Moon's Age.

January,	0,	April,	2,	July,	5,	October,	8,
February,	2,	May,	3,	August.	6,	November,	10,
March,	1,	June,	4,	September,	8,	December,	10.

To Find the Moon's Age on any given day.

RULE. Add together the day of the month, Epact for the year, and the Tabular number for the month, the sum will be the moon's age. If it exceed 30, reject 30's, and the remainder will be the moon's age.

Example. Find out if it will be moonlight at Christmas, 1855?

Number of day = 25 in December
Epact for 1855 (see Table) = 12
Tabular number of December = 10

47 — 30 = 17 the moon's age

or in its third quarter, consequently moon-light at mid-night.

Golden Number or Lunar Cycle

is the period of 19 years at which the changes of the moon fall on the same days of the month.

To find the Golden number.

RULE. Add one to the given year, divide the sum by 19, and the remainder will be the Golden number.

Yrs.	*Days.*	*Dom. let-ter.*	*Epact.*	*Yrs.*	*Days.*	*Dom. let-ter.*	*Epact.*	*Yrs.*	*Days.*	*Dom. let-ter.*	*Epact.*
1800	Saturd.*	FE	4	1834	Saturd.	E	20	1868	Sund.*	ED	6
1801	Sunday.	D	15	1835	Sunday.	D	1	1869	Monday.	C	17
1802	Monday.	C	26	1836	Tuesd.*	CB	12	1870	Tuesd.	B	28
1803	Tuesday	B	7	1837	Wedsd.	A	23	1871	Wedsd.	A	9
1804	Thursd*	AG	18	1838	Thursd.	G	4	1872	Friday*.	GF	20
1805	Friday.	F	29	1839	Friday.	F	15	1873	Saturd.	E	1
1806	Saturd.	E	11	1840	Sund.*	ED	26	1874	Sunday.	D	12
1807	Sunday.	D	22	1841	Monday.	C	7	1875	Monday.	C	23
1808	Tuesd.*	CB	3	1842	Tuesd.	B	18	1876	Wedsd.*	BA	4
1809	Wedns.	A	14	1843	Wedsd.	A	29	1877	Thursd.	G	15
1810	Thursd.	G	25	1844	Friday.*	GF	11	1878	Friday.	F	26
1811	Friday.	F	6	1845	Saturd.	E	22	1879	Saturd.	E	7
1812	Sunday*	ED	17	1846	Sunday.	D	3	1880	Mond.*	DC	18
1813	Monday.	C	28	1847	Monday.	C	14	1881	Tuesd.	B	29
1814	Tuesd.	B	9	1848	Wedsd*.	BA	25	1882	Wedsd.	A	11
1815	Wedns.	A	20	1849	Thursd.	G	6	1883	Thurs.	G	22
1816	Friday*.	GF	1	1850	Friday.	F	17	1884	Saturd.*	FE	3
1817	Saturd.	E	12	1851	Saturd.	E	28	1885	Sunday.	D	14
1818	Sunday.	D	23	1852	Mond.*	DC	9	1886	Monday.	C	25
1819	Monday.	C	4	1853	Tuesd.	B	20	1887	Tuesd.	B	6
1820	Weds.*	BA	15	1854	Wedsd.	A	1	1888	Thurs.*	AG	17
1821	Thursd.	G	26	1855	Thursd.	G	12	1889	Friday.	F	28
1822	Friday.	F	7	1856	Saturd.*	FE	23	1890	Saturd.	E	9
1823	Saturd.	E	18	1857	Sunday.	D	4	1891	Sunday.	D	20
1824	Monda*.	DC	29	1858	Monday.	C	15	1892	Tuesd.*	CB	1
1825	Tuesd.	B	11	1859	Tuesd.	B	26	1893	Wedsd.	A	12
1826	Wedsd.	A	22	1860	Thurs.*	AG	7	1894	Thursd.	G	23
1827	Thursd.	G	3	1861	Friday.	F	18	1895	Friday,	F	4
1828	Saturd.*	FE	14	1862	Saturd.	E	29	1896	Sund.*	ED	15
1829	Sunday.	D	25	1863	Sunday.	D	11	1897	Monday.	C	26
1830	Monday.	C	6	1864	Tuesd*.	CB	22	1898	Tuesd.	B	7
1831	Tuesd.	B	17	1865	Wedsd.	A	3	1899	Thurs.	A	18
1832	Thurs.*	AG	28	1866	Thursd.	G	14	1900	Friday*.	GF	29
1833	Friday.	F	9	1867	Friday.	F	25				

In Leap years take January, February.**

February, March, November.	*February* August.*	*May.*	*January October.*	*January,* April, July.*	*September December.*	*June.*
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Example. On what day in the week will the fourth of July fall in the year 1868?

See Table 1868 = Sunday*. In the Table of months, July column, Sunday is on the 5th; consequently the fourth falls on Saturday.

To Find the Latitude of a place by the Meridian Altitude of the Sun.

Letters denote.

A = meridian altitude of the sun's centre above the horizon, in degrees and minutes. (At sea the sun's lower limb is generally observed, then for corrections of semi-diameter dip of horizon, parallex, add 12 minutes to the observed altitude, and the sum will be the centre altitude very nearly.)

D = declination of the sun at the time of observation, to be found on page 314.

l = latitude of the place of observation.

$$l = 90 - A \pm D, \quad - \quad - \quad 1,$$
$$A = 90 - l \pm D, \quad - \quad - \quad - \quad 2,$$
$$D = \mp 90 \pm A \pm l, \quad - \quad - \quad - \quad 3,$$

Where the quantities have double signs, *plus* and *minus*, use the upper one when the latitude and declination are of *equal names;* and the lower one when the latitude and declination are of *different names.*

Example. On the 25th day of October, 1853, in north latitude was observed the sun's meridian centre altitude to be $A = 37° \, 53'$; the declination on that day was $D = 12° \, 10'$ south. Required the latitude?

$l = 90 - 37° \, 53' - 12° \, 10' = 39° \, 57'$ the latitude of Philadelphia.

To Find the Time when the Sun Rises or Sets.

Let v be the angle of time before or after six o'clock when the sun rises or sets; this angle divided by 15 and added to, or subtracted from six o'clock will be the true time when the sun rises or sets.

$$\sin.v = \tan.D \tan.l, \quad - \quad - \quad - \quad 4,$$

Example. What time does the sun rise and set, on the 27th day of July, 1854, in north latitude $l = 42° \, 6'$?

$$\text{Sun's declination} \left\{ \begin{array}{l} D = 19° \, 12' \text{ in the morning.} \\ D = 19 \quad 5' \text{ in the evening.} \end{array} \right\} \text{North.}$$

$\sin.v = \tan.19° \, 12' \times \tan.42° \, 6' = 0{\cdot}31454$ or $v = 18° \, 20'$.

$18° \, 20' : 15 = 1^h \, 13^m \, 20^s$ subtract from 6^h
$5^h \, 59^m \, 60^s$
Sun rises at $4^h \, 46^m \, 40^s$ in the morning.

$\sin.v = \tan.19° \, 5' \times \tan.42° \, 6' = 0{\cdot}312611$ or $v = 18° \, 13'$.

6^h
$18° \, 13' : 15 = 1 \;\; 12 \;\; 56$ add to 6^h
Sun sets at $7^h \, 12^m \, 56^s$

To Find the Length of Day and Night.

RULE. Double the time when the sun sets is the length of the day.
RULE. Double the time when the sun rises is the length of the night.

To Find the apparent Time by an Altitude of the Sun.

Let L be the angle of time from 12 o'clock, (noon,) when the sun's altitude a is observed,

$a =$ the observed altitude of the sun, (if the sun's lower limb is observed add 12′ for corrections),

A and v, same as in the preceding examples.

$$\cos.L = \frac{\sin.a(1 \pm \sin.v)}{\sin.A} \mp \sin.v, \quad - \quad - \quad 5,$$

The sign + or — is to be used as before described.

Example. On the 11th of May, 1853, the sun's altitude in the afternoon was observed to be $a = 42° \, 30'$, in the latitude $l = 33° \, 40'$; the sun's declination at the time of observation was $D = 18° \, N$. Required the apparent time.

$$\sin.v = \tan.18° \times \tan.33° \, 40' = 0{\cdot}21642, \quad - \quad 4,$$
$$A = 90 - 33° \, 40' + 18 = 74° \, 20' \quad - \quad - \quad 2,$$

$$\cos.L = \frac{\sin.42° \, 30'(1 + 0{\cdot}21642)}{\sin. \, 74° \, 20'} - 0{\cdot}21642 = 0{\cdot}63709, \quad 5,$$

or $50° \, 25'$ is the angle L, $\frac{50° \, 25}{15} = 3^h \, 21^m \, 40^s$, the apparent time of observation.

If the altitude is taken in the forenoon, subtract the obtained time from 12^h and the remainder is the apparent time.

Planets.	Signs.	Mean distance from the Sun.	Mean Sidereal period in mean Solar days.	Eccentricity in parts of the semi-axis.	Velocity in the orbit in miles per second.	Diameter in miles.	Density.	Mass.	Volume.	Time of rotations in hours.
Sun,	☉					882000	0·25	344000	1378000	607h 48m
Mercury,	☿	0·3870981	87·969258	0·2055149	30·4	3140	1·12	0·06966	0·06218	24 5
Venus,	♀	0·7233316	224·70078	0·0068607	22·3	7800	0·92	0·877	0·9531	23 21
Earth,	⊕	1·0000000	365·25636	0·0167836	18·9	7926	1·00	1·000	1·000	24 0
Mars,	♂	1·5236293	686·97964	0·0933070	15·33	4100	0·95	0·1313	0·1384	24 37
Flora,		2·2016870	1193·249	0·1565570	12·77	250 ?			0·000031	
Vesta,	⚶	2·3610810	1325·147	0·0895694	12·30					
Iris,		2·3806240	1341·636	0·2299424	12·25					
Metis,		2·3856070	1345·85	0·1202532	12·25					
Juno,	⚵	2·6708370	1594·296	0·2548847	11·57	79 ?				27 0 ?
Ceres,	⚳	2·7680510	1682·125	0·0766523	11·8	163			0·000012	
Pallas,	⚴	2·7728580	1686·51	0·2398150	11·35					
Jupiter,	♃	5·202776	4332·5848	0·0481621	8·31	87000	0·24	317·5	1322·5	9 56
Saturn,	♄	9·5387861	10759·2198	0·0561505	6·14	79160	0·14	139·5	996·2	10 29
Uranus,	♅	19·18239	30686·8208	0·0466794	4·33	34500	0·24	198·0	82·47	9 30
Neptune.		30·0368	60126·71	0·0087195	3·45	41500	0·14	20·	143·5	

SUN'S DECLINATION.

Days.	Jan. Sou. ° ′	Feb. Sou. ° ′	Mar. Sou. ° ′	Apr. Nor. ° ′	May Nor. ° ′	June Nor. ° ′	July Nor. ° ′	Aug. Nor. ° ′	Sep. Nor. ° ′	Oct. Sou. ° ′	Nov. Sou. ° ′	Dec. Sou. ° ′	Days.
1	22 59	16 59	7 26	4 41	15 11	22 7	23 6	17 58	8 10	3 19	14 34	21 53	1
2	22 53	16 42	7 3	5 4	15 29	22 15	23 2	17 42	7 48	3 43	14 53	22 2	2
3	22 47	16 24	6 40	5 27	15 47	22 22	22 57	17 27	7 26	4 6	15 11	22 11	3
4	22 41	16 5	6 17	5 50	16 4	22 29	22 52	17 11	7 4	4 29	15 30	22 19	4
5	22 34	15 48	5 54	6 13	16 21	22 36	22 46	16 55	6 42	4 52	15 48	22 27	5
6	22 27	15 29	5 30	6 36	16 38	22 42	22 40	16 38	6 20	5 15	16 6	22 34	6
7	22 19	15 11	5 7	6 58	16 55	22 48	22 34	16 21	5 57	5 38	16 24	22 41	7
8	22 11	14 52	4 44	7 21	17 11	22 53	22 27	16 4	5 35	9 1	16 41	22 47	8
9	22 3	14 32	4 20	7 43	17 27	22 59	22 20	15 47	5 12	6 24	16 59	22 53	9
10	21 54	14 13	3 57	8 5	17 43	23 3	22 13	15 29	4 49	6 47	17 16	22 58	10
11	21 44	13 53	3 33	8 27	17 58	23 7	22 5	15 12	4 26	7 10	17 32	23 3	11
12	21 35	13 33	3 10	8 49	18 13	23 11	21 56	14 54	4 3	7 32	17 48	23 8	12
13	21 24	13 13	2 46	9 11	18 28	23 15	21 48	14 35	3 40	7 55	18 4	23 12	13
14	21 14	12 53	2 22	9 32	18 43	23 18	21 39	14 17	3 17	8 17	18 20	23 15	14
15	21 3	12 32	1 59	9 54	18 57	23 20	21 29	13 58	2 54	8 39	18 36	23 18	15
16	20 51	12 11	1 35	10 15	19 11	23 23	21 20	13 39	2 31	9 1	18 51	23 21	16
17	20 39	11 50	1 11	10 36	19 24	23 24	21 9	13 20	2 8	9 23	19 5	23 23	17
18	20 27	11 29	0 48	10 57	19 37	23 26	20 59	13 1	1 45	9 45	19 20	23 25	18
19	20 15	11 8	0 24	11 18	19 50	23 27	20 48	12 41	1 21	10 7	19 34	23 26	19
20	20 2	10 46	0 0N	11 38	20 03	23 27	20 37	12 21	0 58	10 29	19 47	23 27	20
21	19 49	10 24	0 23	11 59	20 15	23 27	20 25	12 1	0 35	10 50	20 1	23 27	21
22	19 35	10 3	0 47	12 19	20 27	23 27	20 13	11 41	0 11	11 11	20 13	23 27	22
23	19 21	9 41	1 11	12 39	20 40	23 26	20 2	11 21	0 12S	11 32	20 26	23 27	23
24	19 7	9 18	1 34	12 59	20 51	23 25	19 48	11 0	0 36	11 53	20 38	23 25	24
25	18 52	8 56	1 58	13 18	21 1	23 24	19 37	10 40	0 59	12 14	20 50	23 24	25
26	18 37	8 34	2 21	13 38	21 12	23 22	19 24	10 19	1 22	12 35	21 1	23 22	26
27	18 21	8 11	2 45	13 57	21 22	23 20	19 10	9 58	1 46	12 55	21 12	23 19	27
28	18 6	7 49	3 8	14 16	21 32	23 17	18 56	9 37	2 9	13 15	21 24	23 16	28
29	17 49		3 32	14 34	21 41	23 14	18 42	9 15	2 33	13 35	21 34	23 13	29
30	17 33		3 55	14 53	21 50	23 10	18 28	8 54	2 56	13 55	21 44	23 9	30
31	17 16		4 18		21 59		18 13	8 32		14 14		23 5	31

Declination of the Sun and equation of time for the years

Leap years 1852,	·56,	·60,	·64,	in New York at	6h a.m.
1853,	·57,	·61,	1865,	„ „	apparent noon.
1854,	·58,	·62,	&c.	„ „	6h p.m.
1855,	·59,	·63,	&c.	„ „	12h midnight.

Exampe 1. Required the sun's declination in New York at 10 o'clock a.m. on the 13th of April, 1856?

From the table April 13	declin.	9° 11′ N.
„ „ „ „ 14	„	9 32
Difference		21
	Dec.	9° 12′
Correction 21 (10—6) : 24 =		3½ add.
Required declination		9° 15′ 30″ the answer.

Note. In leap years the declination and equation of time must be taken one day earlier in the tables for January, and February; for instance, declination on the 20th of February 1856 is 7° 49′.

EQUATION OF TIME.

Days.	JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEP.	OCT.	NOV.	DEC.	Days.
	Add	Add	Add	Add	Sub.	Sub.	Add	Add	Sub.	Sub.	Sub.	Sub.	
	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	m. s.	
1	4 4	13 58	12 31	3 52	3 4	2 28	3 31	6 0	0 13	10 24	16 16	10 37	1
2	4 33	14 5	12 20	3 34	3 11	2 19	3 42	5 56	0 32	10 43	16 17	10 14	2
3	4 59	14 11	12 7	3 17	3 18	2 9	3 53	5 52	0 51	11 2	16 17	9 50	3
4	5 27	14 17	11 54	2 59	3 24	1 59	4 4	5 47	1 11	11 20	16 16	9 25	4
5	5 54	14 22	11 40	2 41	3 29	1 49	4 15	5 41	1 30	11 37	16 14	9 0	5
6	6 20	14 26	11 26	2 24	3 34	1 38	4 25	5 35	1 50	11 55	16 11	8 35	6
7	6 46	14 29	11 12	2 7	3 38	1 27	4 35	5 28	2 10	12 12	16 8	8 9	7
8	7 12	14 31	10 57	1 50	3 42	1 16	4 44	5 20	2 31	12 29	16 4	7 43	8
9	7 37	14 33	10 41	1 32	3 45	1 4	4 53	5 12	2 51	12 45	15 58	7 16	9
10	8 1	14 33	10 26	1 16	3 48	0 53	5 2	5 3	3 12	13 1	15 52	6 48	10
11	8 25	14 33	10 10	1 0	3 50	0 41	5 10	4 54	3 33	13 16	15 46	6 21	11
12	8 48	14 32	9 53	0 42	3 51	0 29	5 17	4 44	3 54	13 31	15 38	5 53	12
13	9 10	14 31	9 37	0 26	3 52	0 16	5 25	4 34	4 15	13 45	15 30	5 24	13
14	9 32	14 28	9 20	0 11	3 53	0 4	5 31	4 23	4 36	13 59	15 20	4 56	14
15	9 53	14 25	9 3	0S 2	3 53	0A 9	5 38	4 11	4 57	14 12	15 10	4 27	15
16	10 14	14 21	8 45	0 17	3 52	0 21	5 43	3 59	5 18	14 25	14 59	3 57	16
17	10 33	14 16	8 25	0 31	3 51	0 34	5 48	3 46	5 40	14 37	14 47	3 28	17
18	10 52	14 11	8 10	0 45	3 49	0 47	5 53	3 33	6 1	14 48	14 34	2 58	18
19	11 11	14 5	7 52	0 58	3 46	1 0	5 57	3 20	6 22	14 59	14 21	2 28	19
20	11 28	13 58	7 33	1 12	3 44	1 13	6 1	3 6	6 43	15 9	14 6	1 58	20
21	11 45	13 51	7 15	1 24	3 40	1 26	6 4	2 51	7 4	15 20	13 51	1 28	21
22	12 1	13 43	6 57	1 37	3 36	1 39	6 6	2 36	7 25	15 29	13 35	0 58	22
23	12 16	13 35	6 38	1 48	3 31	1 52	6 8	2 21	7 45	15 37	13 18	0 28	23
24	12 30	13 25	6 20	2 0	3 26	2 4	6 10	2 5	8 6	15 44	13 0	A0 4	24
25	12 44	13 16	6 1	2 10	3 22	2 17	6 11	1 49	8 26	15 51	12 42	0 32	25
26	12 57	13 5	5 43	2 21	3 15	2 30	6 11	1 33	8 47	15 57	12 22	1 2	26
27	13 10	12 55	5 24	2 30	3 9	2 42	6 11	1 16	9 7	16 2	12 2	1 32	27
28	13 21	12 43	5 6	2 40	3 2	2 55	6 10	0 58	9 26	16 6	11 42	2 1	28
29	13 31		4 47	2 48	2 54	3 7	6 8	0 41	9 46	16 10	11 20	2 31	29
30	13 41		4 29	2 56	2 46	3 19	6 6	0 23	10 5	16 13	10 58	3 0	30
31	13 50		4 11		2 37		6 4	0 5		16 15		3 28	31

Example 2. Required the sun's declination in San Francisco at 4h 46m p.m., on the 5th of Aug. 1855.

From the table Aug. 5 declin. 16° 55′ N.
" " " " 6 " 16 38

Difference 17

Long. of San Francisco 122°
" New York 74

48 : 15 = 3h 12m difference in time.
Time in S. F 4 46
Midnight 12h — 8h = 4h × 17 : 24 = 3′ nearly.
16° 45′
Correction add 3

Required declination 16° 48′ the answer.

Example 3. On the 5th day of Nov. 1857, at 2h 21m 56s apparent time p.m. Required the mean time?

Apparent time 2h 21m 56s
Equation of time, sub. 16 14

Mean time 2h 5m 42s the answer.

NOTE. Add the equation of time to or subtract from the apparent time, is the mean time.

EPACT OF THE YEAR.

d. h.	d. h.	d. h.	d. h.	d. h.	d. h.	d. h.	d h.	d. h.	d. h.	d. h.	d. h.
1850	1851	1852	1853	1854	1855	1856	1857	1858	1859	1860	1861
17 17	28 8	9 10	20 1	1 3	11 18	23 10	4 12	15 3	25 17	7 21	18 12
1862	1863	1864	1865	1866	1867	1868	1869	1870	1871	1872	1873
29 3	10 6	21 21	2 23	13 15	24 6	6 8	16 23	27 18	8 20	19 15	0 17

EPACT OF THE MONTH.

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
0 0	1 17	0 4	1 16	2 3	3 14	4 2	5 13	7	7 11	8 23	9 10

To the time of high water, or the time when the moon passes the meridian, add two minutes for every exceeding hour of the moon's age, for corrections.

MOON'S POSITION.

Q'rt'r	Face	Age d.	South. h. m.	High water in N. York. h. m.
New		0	12 0p.m.	8 37a.m.
		1	12 49 „	9 21 „
		2	1 38 „	10 2 „
1st quarter		3	2 26 „	10 40 „
		4	3 26 „	11 16 „
		5	4 4 „	11 54 „
		6	4 55 „	12 36p.m.
Half		7	6 42 „	1 23 „
		8	6 30 „	2 16 „
		9	7 19 „	3 16 „
2d quarter		10	8 8 „	4 14 „
		11	8 57 „	5 9 „
		12	9 46 „	6 0 „
		13	10 34 „	6 47 „
Full		14	11 23 „	7 29 „
		15	12 12a.m.	8 7 „
		16	1 1 „	8 43 „
		17	1 50 „	9 21 „
3d quarter		18	2 38 „	10 4 „
		19	3 27 „	10 51 „
		20	4 16 „	11 42 „
		21	5 5 „	12 37a.m.
Half		22	5 54 „	1 37 „
		23	6 42 „	2 39 „
		24	7 31 „	3 42 „
		25	8 20 „	4 43 „
4th quarter		26	9 9 „	5 41 „
		27	9 58 „	6 36 „
		28	10 46 „	7 27 „
		29	11 35 „	8 14 „
		29½	12 „	8 37 „

HIGH WATER IN DIFFERENT PLACES.

Add to, or subtract from the time of high water in N. Y. is the time of high water in the desired place.			Rise Ft.
New York		0h 0m	5
Quebec	add	8 49	17
Halifax	sub.	1 0	8
Boston	add	3 54	12
New Haven	add	2 32	17
Portsmouth	add	3 4	10
Providence	sub.	0 41	
Albany	add	6 34	
Amboy	sub.	0 39	
Sandy Hook	sub.	1 8	6
Philadelphia	add	5 15	6
Cape Henlopen	add	0 35	5
Baltimore	sub.	4 14	12
Cape Henry	add	0 57	4
Washington	sub.	4 8	
Norfolk	sub.	0 52	7
Charleston	sub.	0 22	5
Key West	add	1 16	2
Havana	add	1 35	3
Rio Janeiro	sub.	6 35	6
Buenos Ayres		0 0	
Cape Horn	sub.	3 57	9
Valparaiso	add	0 55	5
San Francisco	add	2 23	6
Liverpool	add	2 39	25
London	sub.	6 30	18
Hull	sub.	2 37	18
Bremen	add	2 37	
Lisbon	sub.	4 37	
Cape Good Hope	sub.	5 37	

Moon's Age. High Water. Moon South.

Example 1. Required the moon's age on the 25th day of September, 1855?

Date in September	25d	add
Epact of September	7d	
Epact of 1855	11d 18h	
Reject 30	13d 18h	is the moon's age at noon in New York.

Example 2. Required the time of high water in New York, and at what time the moon is south, on the same date as in preceding example?

In the annexed table we have given the moon's

	Age.	South.		High water.
	13 days	10h 34m		6h 47m
Correction add 18h×2=		36m		36m
Moon south at		11h 10m,	and	7h 23m is the time of high water in New York.

Example 3. Find the time of high water in San Francisco on the 18th of July, 1856.

Date in July	18d	add
Epact of July	3d 26h	
Epact of 1856	23d 10h	
Reject 30	15d 12h	the moon's age.

In the table	Age.	High water.
	15	8h 7m
Correction add 12h×2		24m
For San Francisco add		2h 23m

Time of high water in San Francisco 10h 54m, July 18, 1856.

The strength and direction of the wind sometimes accelerates and sometimes retards the tide, in consequence the most careful and scientific calculations may differ anhour from the time of high water.

SOUNDINGS,

To Reduce Soundings to Low Water.

Letters denote.

T = time in hours between high and low water.

t = time in hours from low water to the time when the soundings are taken.

H = vertical rise of tide in feet from high to low water.

h = reduction of the sounding taken at the time, t, in feet.

$$v = \frac{180\,t}{T}, \qquad \text{and } h = \tfrac{1}{2} H (1 \mp \cos. v),$$

$$- \cos. v \text{ when } v < 90$$
$$+ \cos. v \text{ when } v > 90$$

Example.	High water at	10h 15m	p.m.	
	Low water at	3h 45m	,,	
	Time T =	6h 30m	,,	
	The sounding taken at	5h 30m	,,	was 16 feet 6 inches
	Time t =	1h 45m		
	Vertical rise H =	9·75 feet.		

Required the reduction h = ? and true sounding at low water.

$$v = \frac{180 \times 1{\cdot}75}{6{\cdot}5} = 48^\circ\ 30\,, \quad \cos. v = 0{\cdot}66262.$$

Reduction $h = \frac{1}{2} \times 9{\cdot}75\ (1 - 0{\cdot}66262) = 1{\cdot}6447$ feet.

Sounding taken at 5h 30m was 16·5 feet.

Reduction subtract	h =	1·6447
True sounding at low water		14·8553 feet.

PARABOLIC CONSTRUCTION OF SHIPS.

In this kind of construction, the load water-line and greatest immersed section of a vessel are parabolas with the vertex at o, Figs. 1 and 2; also, the square root of the areas of immersed cross-sections, taken at any distance fore or abaft from ⊗, are subordinates in a parabola of the formula $y = \sqrt[n]{px}$, where, in the conic parabola, the exponent $n = 2$. In the accompanying tables I and II, ordinates and cross-sections are calculated for different exponents from 2 to 16, and the corresponding lines for the exponents 2, 3, 4, 6, and 10, are illustrated on plates XI and XII, from which any desired sharpness of a vessel can be selected. The highest exponent makes the fullest lines.

Letters denote.—D = disp. in cub. ft.; T = disp. in tons of salt water; ⊗ = greatest immersed cross-section; ⊗ = ordinate cross-section; L = length; l = length from ⊗ to stern or bow; B = beam; b = half the beam; d = load draft of water, omitting the depth of keel; δ = any draft corresponding to the displacement t; e = depth of centre of gravity of displacement under water-line; x = abscissa; y = ordinate; n = exponent for the parabola in the water-line, n' = exponent for ⊗, and n'' = exponent for the areas of the ordinate cross-sections; r = index for displacement; **a** = area of load water-line; ⊗ = $b\ d$ multiplied with the same coefficient as for **a**; k = coefficient for speed and horse-power; M = nautical miles or knots per hour; H = horse-power required for the speed M; m = height of metacenter above the center of gravity of displacement; A = area of the hull of the vessel; **a**$'$ = area of the upper deck, or any horizontal section of the hull at d' feet from the keel; ⊗$'$ = area of greatest cross-section from the keel to **a**$'$, e' = depth of center of gravity of the hull from the top of d', supposing the hull to be of uniform thickness; $\mathfrak{d}\delta$ = increment of draft of water; $\mathfrak{d}t$ = increment of displacement; $\mathfrak{d}H$ = increment of horse-power; $\mathfrak{d}M$ = increment of speed; C = steamship performance—coefficient. See page 275; N = coefficient for displacement, table IV; O = tabular number in table V. All linear dimensions are in feet, and areas in square feet.

Hollow lines.—Let i, Fig. 1, be the point where the hollow line is to commence; draw through i a line parallel to the centre line; draw the ordinate z, find the ordinate $z' = z$, make $a = a'$, then e is the stem of the boat. Draw equal number of ordinates on a and a', by which the line $e'\ i$ is transferred to $i\ e$, and forms the hollow part of the water-line. Table III contains ordinates for hollow water-lines of $z = \frac{1}{3}\ b$. All the ordinate water-lines are parabolas of different orders, and the ordinate for any point in them can be calculated by the accompanying formulas. l' = the whole length from ⊗ to e, which must be divided into 8 equal parts for the ordinate cross-sections of the exponent n''.

Fig. 3 is a scale of displacement at different drafts of water, and for different exponents of vessels.

Fig. 4 is a diagram for laying out the ordinates in the water-line and cross-section. One of this should be constructed for each exponent n. Fig. 4 is constructed with the exponent 2, the line $g\,h = b$, Figs. 1 and 2, and the ordinates in the inner parabola correspond with the distances from g in the diagram.

Fig. 5 represents the spring of beams; the length is divided into 8 parts from each end, numbered as shown by the figure. The spring $b = 1$; the ordinates are calculated from the line b, exponent 2, in Table I.

Table V is calculated from the formula 20, for the elliptic form of the stern rail and deck. The origin of the ellipse being at o, Fig. 8; the abscissa x in the centre line, and the ordinate y in the breadths.

Table VI is calculated from formula 19, and contains the coefficient N.

Table VII contains the coefficient O. Suppose the size of a vessel is given in tons of the displacement, select the coefficient N in table VI as to the object and condition of the vessel, find in column N, table VII, the nearest number to the selected coefficient, and along that line select the coefficient O, which, multiplied by the cube root of the displacement, gives the required length of the vessel; divide this length by the number of the column noted at the top, and it gives the draft of water; the beam is calculated from formula 30.

Practical Formulas.

$$y=l\sqrt[n]{\frac{x}{b}}, \tag{1}$$

$$x=\frac{y^n b}{l^n}, \tag{2}$$

$$\mathbf{a}=\frac{n B L}{n+1}, \tag{3}$$

$$n=\frac{\mathbf{a}}{B L-\mathbf{a}}, \tag{4}$$

$$\mathfrak{X}=\frac{n' B d}{n'+1}, \tag{5}$$

$$n'=\frac{\mathfrak{X}}{B d-\mathfrak{X}}, \tag{6}$$

$$\delta=d\sqrt[r]{\frac{t}{T}}, \tag{7}$$

$$t=\frac{\delta^r T}{d^r}, \tag{8}$$

$$\mathfrak{d}t=\frac{r T \delta^{r-1}\mathfrak{d}\delta}{d^r} \tag{9}$$

$$\mathfrak{d}\delta=\frac{d^r.\mathfrak{d}t}{r T \delta^{r-1}}, \tag{10}$$

$$n=\frac{\log.b-\log.x}{\log.l-\log.y}, \tag{11}$$

$$D=\frac{\mathfrak{X} L\, 2 n^{2\prime\prime}}{2 n^{2\prime\prime}+3 n''+1}, \tag{12}$$

$$e=\frac{d}{2}\left(\frac{n'+1}{n'+2}\right)\sqrt{\frac{n''+1}{n+2}}, \tag{13}$$

$$r=\frac{(n'+1)(n+1)}{n' n''}, \tag{14}$$

$$n''=\frac{\log.\ b\sqrt{\frac{\otimes}{\mathfrak{X}}}}{\log.l-\log.y}, \tag{15}$$

$$n''=\frac{\sqrt{8 D(\mathfrak{X} L-D)^2+3 D}}{4(\mathfrak{X} L-D)}, \tag{16}$$

$$m=\frac{B^3}{12\,\mathfrak{X}}\sqrt[3]{\frac{D}{\mathfrak{X} L}}, \tag{17}$$

$$\otimes=\mathfrak{X}\left(1-\frac{y^{n''}}{l^{n''}}\right)^3, \tag{18}$$

$$N=\frac{2 n' n^{2\prime\prime}}{(n'+1)(2 n^{2\prime\prime}+3 n''+1)} \tag{19}$$

$$y=\sqrt[n]{b^n-\left(\frac{b x}{l}\right)^n}, \tag{20}$$

$$H=\frac{M^3\,\mathfrak{X}}{k L}, \tag{21}$$

$$M=\sqrt[3]{\frac{k H L}{\mathfrak{X}}}, \tag{22}$$

$$H=\frac{M^3\sqrt[3]{T^2}}{C}, \tag{23}$$

$$M=\frac{C H}{\sqrt[3]{T^2}} \tag{24}$$

$$\mathfrak{d}H=\frac{3 H.\mathfrak{d}M}{M} \tag{25}$$

$$\mathfrak{d}M=\frac{M.\mathfrak{d}H}{3 H}, \tag{26}$$

$$N=\frac{35\ T}{B L D}, \tag{27}$$

$$T=\frac{B L d N}{35}, \tag{28}$$

$$L=O\sqrt[3]{T}, \tag{29}$$

$$B=\frac{35\ T}{L d N}, \tag{30}$$

Formulas for Hollow Lines.

$$l'=l\left(2\sqrt[n]{\frac{b-z}{b}}-\sqrt[n]{\frac{b-2 z}{b}}\right) \tag{31}$$

$$l=\frac{l'}{2\sqrt[n]{\frac{b-z}{b}}-\sqrt[n]{\frac{b-2 z}{b}}}, \tag{32}$$

$$c=l\left(1-\sqrt[n]{\frac{b-z}{b}}\right), \tag{33}$$

$$a=l'+c-l, \tag{34}$$

$$A=2\sqrt{\frac{n''}{n+1}}\,(\mathfrak{X}'+d' L)+\mathbf{a}', \tag{35}$$

$$e'=\frac{d'(d' L+\mathbf{a}+\mathfrak{X}')}{2 d' L+\mathbf{a}+2\,\mathfrak{X}'}\left(\frac{n'+1}{n'+2}\right)\sqrt{\frac{n''+1}{n+2}}, \tag{36}$$

Length of a Parabola.

$$2\ b\sqrt{\frac{l^n}{4\ b^n}+1}+\frac{l^n}{4\ b}\ \text{hyp. log.}\ \frac{2\ b}{l}\left(1+\sqrt{\frac{l^n}{4\ l^n}+1}\right), \tag{37}$$

Expo. n or n'	WATER-LINE OR CROSS-SECTION. $b=1$. 1	2	3	4	5	6	7	a=BL×. $\mathfrak{M}$=Bd×	k
2	·2345	·4375	·6094	·7500	·8593	·9375	·9844	·6666	1·94
2·25	·2595	·4766	·6527	·7897	·8899	·9558	·9907	·6923	1·98
2·5	·2838	·5129	·6912	·8232	·9139	·9687	·9944	·7142	2·00
2·75	·3073	·5466	·7254	·8513	·9326	·9779	·9967	·7333	1·98
3·	·3301	·5781	·7558	·8750	·9472	·9844	·9980	·7500	1·94
3·25	·3521	·6074	·7829	·8944	·9587	·9889	·9988	·7647	1·91
3·5	·3733	·6346	·8070	·9116	·9677	·9922	·9993	·7777	1·88
3·75	·3939	·6600	·8284	·9256	·9747	·9944	·9995	·7894	1·85
4·	·4138	·6836	·8474	·9375	·9802	·9961	·9997	·8000	1·82
4·5	·4517	·7260	·8794	·9557	·9878	·9978	·9998	·8181	1·76
5·	·4871	·7627	·9046	·9687	·9926	·9990	·9999	·8333	1·70
5·5	·5202	·7945	·9246	·9779	·9954	·9994	·9999	·8461	1·64
6·	·5512	·8220	·9404	·9843	·9972	·9997	1·000	·8571	1·58
6·5	·5802	·8459	·9528	·9889	·9983	·9998	1·000	·8666	1·52
7·	·6073	·8665	·9627	·9922	·9989	·9998	1·000	·8750	1·46
8·	·6564	·8989	·9767	·9960	·9996	·9999	1·000	·8888	1·34
9·	·6993	·9249	·9854	·9985	·9998	·9999	1·000	·9000	1·28
10·	·7369	·9437	·9909	·9990	·9999	1·000	1·000	·9090	1·18
12·	·7963	·9683	·9976	·9998	1·000	1·000	1·000	·9231	1·08
16·	·8819	·9991	·9999	1·000	1·000	1·000	1·000	·9412	1·00

TABLE II. DISPLACEMENT FOR THE EXPONENT n''.

Expo. n''	ORDINATE CROSS-SECTIONS $\mathfrak{O}$. $\mathfrak{M}=1$. 1	2	3	4	5	6	7	Displacement. D=$\mathfrak{M}$L×	T=$\mathfrak{M}$L×
2·	·0545	·1914	·3713	·5625	·7384	·8909	·9688	·5333	·0152
2·25	·0673	·1795	·4260	·6236	·7919	·9135	·9815	·5663	·0162
2·5	·0805	·2512	·4772	·6777	·8352	·9383	·9889	·5952	·0170
2·75	·0944	·2987	·5262	·7247	·8697	·9563	·9934	·6204	·0177
3·	·1090	·3342	·5713	·7667	·8972	·9691	·9960	·6429	·0184
3·25	·1239	·3689	·6129	·7999	·9191	·9779	·9976	·6629	·0189
3·5	·1394	·4027	·6512	·8310	·9365	·9845	·9986	·6806	·0194
3·75	·1551	·4356	·6862	·8567	·9500	·9888	·9990	·6968	·0199
4·	·1712	·4673	·7181	·8789	·9608	·9922	·9994	·7111	·0203
4·5	·2039	·5270	·7736	·9146	·9751	·9956	·9996	·7364	·0211
5·	·2373	·5817	·8183	·9384	·9853	·9980	·9998	·7575	·0216
5·5	·2706	·6312	·8548	·9563	·9908	·9988	·9998	·7755	·0221
6·	·3038	·6757	·8844	·9688	·9944	·9995	·9999	·7924	·0227
6·5	·3366	·7155	·9078	·9780	·9966	·9996	·9999	·8047	·0230
7·	·3688	·7508	·9268	·9845	·9978	·9996	·9999	·8170	·0233
8·	·4309	·8080	·9540	·9920	·9992	·9998	1·000	·8366	·0239
9·	·4890	·8554	·9710	·9970	·9996	·9998	1·000	·8521	·0244
10·	·5430	·8906	·9819	·9980	·9998	·9999	1·000	·8656	·0248
12·	·6341	·9375	·9934	·9996	·9999	·9999	1·000	·8861	·0253
16·	·7777	·9991	·9998	·9999	·9999	1·000	1·000	·9126	·0261

Parabolic Construction of Ships

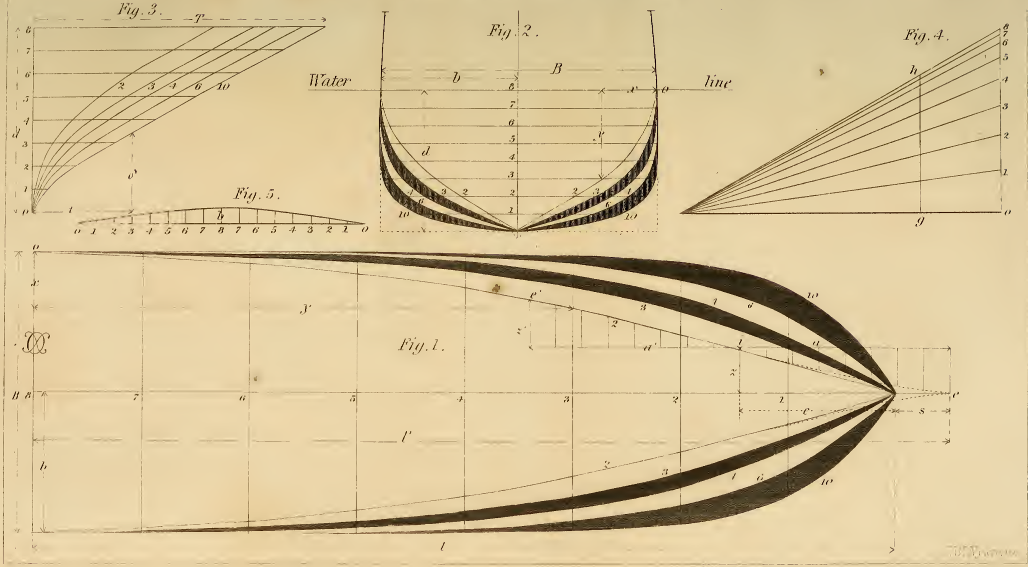
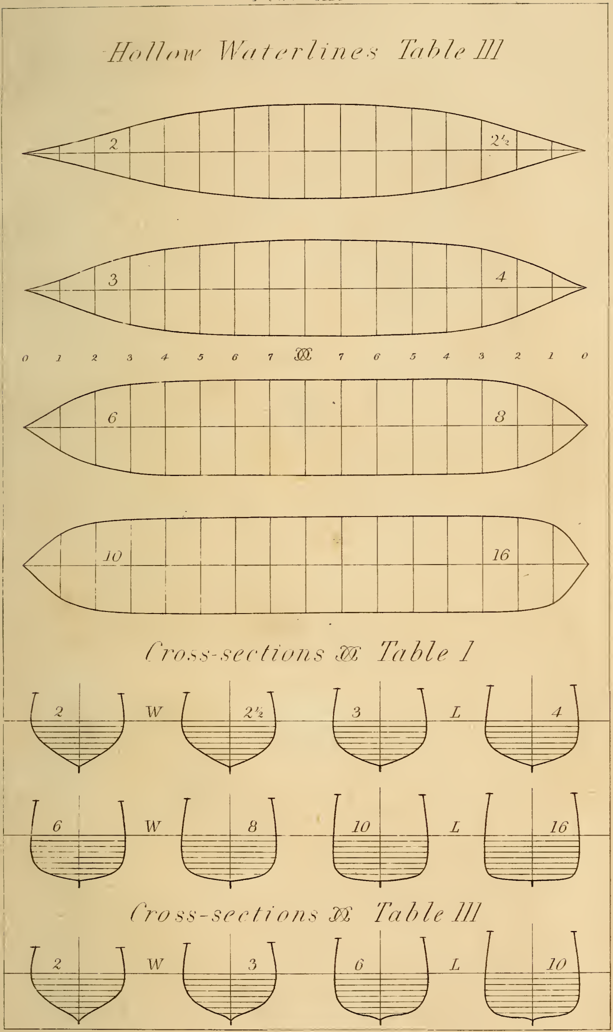


Plate XI.


Hollow Waterlines Table III
2
2½
3
4
0 1 2 3 4 5 6 7 ✠ 7 6 5 4 3 2 1 0
6
8
10
16
Cross-sections ✠ Table I
2
W
2½
3
L
4
6
W
8
10
L
16
Cross-sections ✠ Table III
2
W
3
6
L
10

Exp.	ORDINATES OF HOLLOW WATER LINES. $z = \frac{1}{3}b$.							AREA.	$l'-l$
n	1	2	3	4	5	6	7	a=BL×	$=s$
2	·1697	·3732	·5647	·7214	·8433	·9303	·9825	·6491	·0527
2·25	·1944	·4074	·6068	·7620	·8754	·9500	·9895	·6730	·0535
2·5	·1960	·4415	·6460	·7443	·9013	·9642	·9943	·6939	·0532
2.75	·2127	·4744	·6817	·8277	·9219	·9744	·9962	·7124	·0523
3	·2284	.5063	·7143	·8537	·9383	·9817	·9977	·7287	·0510
3·25	·2445	·5368	·7439	·8760	·9513	·9870	·9986	·7433	.0496
3.5	·2596	·5657	·7706	·9895	·9617	·9907	·9992	·7654	·0482
3.75	·2779	·5947	·7954	·9114	·9699	.9924	·9995	·7682	·0463
4	·2962	·6193	·8164	·9247	·9762	·9953	·9997	·7789	·0452
4·5	·3337	·6670	·8534	·9463	·9853	·9976	·9998	·7975	·0424
5	·3744	·7105	·8839	·9619	·9909	·9988	·9998	·8139	·0390
5·5	·4081	·7465	·9070	·9727	·9944	·9994	·9999	·8269	·0375
6	·4428	·7790	·9260	·9806	·9965	·9997	·9999	·8385	·0354
6.5	·4703	·8266	·9405	·9860	·9978	·9998	1·000	·8479	·0352
7	·5081	·8328	·9533	·9902	·9988	·9999	1·000	·8582	·0327
8	·5861	·8794	·9719	·9953	·9994	1·000	1·000	·8730	·0286
9	·6186	·9047	·9815	·9975	·9998	1·000	1·000	·8852	·0261
10	·6606	·9274	·9883	·9992	·9999	1·000	1·000	·8942	·0237
12	·7415	·9601	·9954	·9997	1·000	1·000	1·000	·9110	·0206
16	·8481	·9871	·9993	·9999	1·000	1·000	1·000	·9319	·0156

When the exponents for the water line and cross section are selected, multiply half the beam with the numbers in the tables, which gives the corresponding distances from the centre line; for instance, if the exponent for the water line is $n=4$ first column, table III, beam $b=16$ feet, the third ordinate will be $16\times0·8164=13·0624$ feet. Suppose the exponent for the cross section to be $n'=3·25$, $b=16$ feet, the fifth ordinate will be $16\times0·9587=15·3392$ feet. Area of the water line $\mathfrak{a}$ is calculated from formula 3, and cross section $\boxtimes$ from formula 5, or both are obtained by the coefficient in the column next to the last table. Required the area of the water line of a vessel of $L=256$ feet, $B=32$ feet and $n=3·75$?

$\mathfrak{a}=256\times32\times0·7894=6472·7648$ square feet the answer. Required the cross section $\boxtimes$ of a vessel of $B=27$ feet drafted$=12$ feet, and the exponent $n'=2·5$?

$\boxtimes=27\times12\times0·7142=231·4$ square feet the answer.

The area of the ordinate cross sections or the immersed area of any frame $\otimes$ is calculated from formula 18 or by the numbers in table II. Required the 4th cross section $\otimes=$? in a vessel of $B=42$ ft., $d=18$ ft., and the exponent $n'=4·5$ and $n''=5$. $\boxtimes=42\times18\times0·8181=618·4836$ square feet, and $\otimes=618·48\times0·9384=580·38$ square feet.

The Displacement is calculated from formula 12 or by the coefficients in the two last columns, table II. Required the displacement of a vessel of dimensions as in the preceeding example when $L=325$ feet?

$D=618·48\times325\times0·7575=152172$ cubic feet or $T=618·48\times325\times0·0216=4341·729$ tons, the answer.

When the exponents for the cross section n' and for the displacement n'' are selected, the displacement in cubic feet is obtained by multiplying together the length L, beam B, and draft of water d, and the product by the number in table VI.

Example. A vessel of $T=3450$ tons displacement is constructed with the exponents $n=3$, $n'=6$, and $n''=3·25$, drawing $d=16$ feet water. Required the draft of water when the vessel and all on board weighs $t=2160$ tons?

$$\textit{Formula 14.}\quad r=\frac{(6+1)\ (3+1)}{6\times3·25}=1·435 \text{ the indix.}$$

$$\textit{Formula 7.}\quad \delta=16\sqrt[r]{\frac{2160}{3450}}=11·55 \text{ feet the answer.}$$

$$\text{or } \log\delta=\log.16-\frac{\log.3450-\log.2160}{1.435}=1.0624.$$

Required how much $\mathfrak{d}t=$? the same vessel can be loadded per inch $\mathfrak{d}\delta$ of additional draft, at a draft of $\delta=12$ feet.

$$\textit{Formula 9.}\quad \mathfrak{d}t=\frac{1·435\times3450\times12^{0·435}\times\frac{1}{12}}{16^{1·435}}=64·14 \text{ tons, the answer.}$$

The exponent for any displacement is found by formulas 15 and 16.

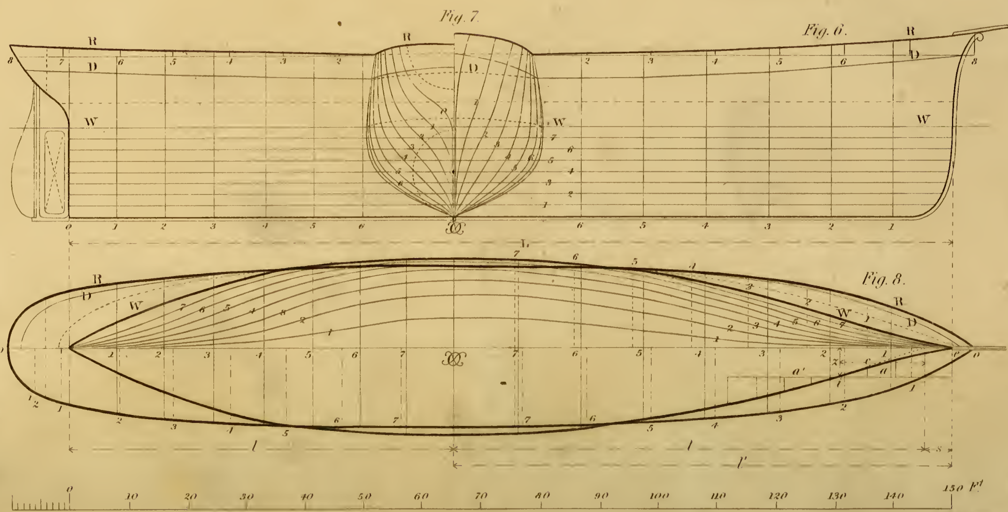
Required the centre of gravity of displacement $e=$? of the vessel in the preceding examples, when loaded to $d=16$ feet.

$$\text{Formula 13.}\quad e=\frac{16}{2}\left(\frac{6+1}{6+2}\right)\sqrt{\frac{3.25+1}{3+2}}=6·43 \text{ feet, the answer.}$$

Required the area $A=$? of the immersed hull of the same vessel, when $\mathfrak{a}=10242$ square feet.

$$\textit{Formula 35.}\quad A=2\sqrt{\frac{3·25}{6+1}}\left(618.48+16\times325\right)+10242=18112 \text{ square feet,}$$
answer.

Nystrom's Parabolic Construction of Ships.



For light draft and speed, the exponents should be selected towards the corner ·490, Table IV; and for freight and light draft towards the corner ·797. For heavy freight and light draft, the proportions of the vessel should be selected towards the corner 36·0, Table V; sailing yachts for deep water towards the corner 14·9, and ordinary vessels for deep water in the middle of column 12.

The exponent for the displacement n'' generally varies very little from that of the water-line n; that when selecting a desired sharpness from fig. 1 we can consider $n'' = n$. The proper relation between n and n'' depends on taste and judgment of the ship-builder.

CONSTRUCTION OF A PROPELLER STEAMER.

Figs. 6, 7, and 8, Plate XIV, are constructions of a steam propeller of the following dimensions: $L = 150$ ft., $B = 30$ ft., $d = 15$ ft. The scale is 1-32 of an inch to the foot. The exponents are selected as follows: $n = 2$ of the forward water-line, with hollow lines table III; for the aft water-line, $n = 2\frac{1}{2}$, full lines table I; ⊗ is located 1-16 of the length abaft of the middle of L, making 84·375 ft. forward, and 65·625 ft. abaft of ⊗. Exponent for the cross-section is $n' = 3$, when the area will be ⊗ = 337·5 sq. ft. Exponent for the displacement $n'' = 2$, when, from Formula 12, $D = 26998{\cdot}3$ cub. ft., and $T = 713{\cdot}8$ tons. From Formula 32, $l = 79{\cdot}9$ ft., which makes the parabola cut the water-line at 84·375—79·9 = 4·475 feet from the stem. Exponent for the forward parabolic rail is $n = 4$, and for the elliptic rail abaft $n = 3$. The sheer is 3 ft. forward, 1½ ft. abaft, with 45° curve. The following table contains the principal dimensions for the form of the vessel, which table should always be calculated before the construction is commenced.

Ordinates.	Cross-section. $n' = 3$ ⊗	Displ't. Tab. II. $n'' = 2$ Half ⊗	WATER-LINE. Tab. III. $n = 2$ Forward	WATER-LINE. Table I. $n = 2\frac{1}{2}$ Abaft.	RAIL. Table I. $n = 4$ Forward	RAIL. Table V. $n = 3$ Abaft.	SHEER. 45° For'rd	SHEER. 45° Abaft.
1	4·951	9·211	2·547	4·257	5·591	9·34	0·047	0·023
2	8·671	32·28	5·798	7·693	9·230	11·24	0·161	0·081
3	11.33	62·63	8·470	10·36	11·44	12·18	0·366	0·183
4	13·12	94·89	10·82	12·33	12·66	12·92	0·645	0·322
5	14·21	124·5	12·65	13·71	13·24	13·26	1·055	0·527
6	14·76	150·2	13·95	14·53	13·44	13·42	1·570	0·785
7	14·97	164·4	14·74	14·91	13·49	13·49	2·195	1·097
8	15 ft.	168·7	15 ft.	15 ft.	13·5 ft.	13·5 ft.	3 ft.	1·5 ft

The centre of gravity of displacement will be found by Formula 13, when the medium of $n = 2\frac{1}{4}$, $e = \frac{15}{2}\left(\frac{3+1}{3+2}\right)\sqrt{\frac{2+1}{2\frac{1}{4}+2}} = 5{\cdot}04$ feet under the water-line, and $\frac{34{\cdot}375 - 75}{2} = 4{\cdot}6875$ feet forward of ⊗.

Her launching weight $t = 310$ tons. Required, her launching draft, δ?

Form 14. $r = \frac{(3+1)(2\frac{1}{4}+1)}{3 \times 2} = 2{\cdot}16.$ | Form. 7. $\delta = 15\sqrt{\frac{310}{792}} = 9{\cdot}716$ feet.

For further explanations, with examples, see *Nystrom's Treatise on Parabolic Shipbuilding and Marine Engineering Subjects*, where the Parabolic Construction can be acquired, and the tables and formulas in this book will serve as a memorandum. J. B. Lippincott & Co., Philadelphia—Trubner, London.

TABLE V.—*For Elliptic Stern of Vessels.*

Expot. n	Ordinates for ellipses of different order. ½	1	2	3	4	5	6	7
2	·3398	·4840	·6616	·7808	·8660	·9204	·9682	·9922
2¼	·4108	·5490	·7147	·8274	·9004	·9495	·9801	·9958
2½	·4670	·5537	·7657	·8627	·9252	·9546	·9873	·9978
2¾	·5174	·6514	·8029	·8901	·9434	·9749	·9932	·9989
3	·5604	·6911	·8331	·9019	·9565	·9821	·9948	·9994
3¼	·5991	·7252	·8578	·9275	·9664	·9871	·9973	·9996
3½	·6333	·7548	·8782	·9406	·9740	·9907	·9978	·9998
4	·6906	·8021	·9003	·9595	·9840	·9950	·9995	·9999
Sheer	30°	·0149	·0582	·1321	·2374	·3740	·5449	·7531
of	45°	·0157	·0539	·1221	·2152	·3517	·5227	·7313
vessels.	60°	·0160	·0474	·1086	·1972	·3190	·4794	·6946

TABLE VI.—*To Approximate Size and Shape of Vessels.*

d	𝔛 n'	Exponent for displacement n''. 2	2·5	3	3·5	4	5	6	8	10
Draft of water. Deep.	2	·356	·397	·429	·453	·474	·500	·528	·558	·577
	2.5	·381	·425	·459	·486	·508	·541	·566	·597	·620
	3	·400	·447	·482	·510	·533	·563	·594	·627	·650
Middling.	3.5	·414	·462	·500	·528	·552	·589	·616	·650	·673
	4	·427	·476	·514	·544	·569	·606	·635	·668	·693
	5	·444	·496	·535	·567	·592	·631	·660	·696	·722
Light.	6	·458	·509	·550	·583	·610	·649	·679	·717	·750
	8	·474	·529	·571	·605	·632	·673	·704	·743	·770
	10	·490	·547	·590	·625	·654	·696	·720	·759	·797
Purpose.		Speed and passengers.			Freight and passengers.			Freight and slow speed.		

TABLE VII—*Length of Vessels = tabular number* $O\sqrt[3]{T}$.

	n'&n'' N	Proportion of draft aad length of vessels. 8	12	18	26	36	48	64	82	102
Sailing yachts.	·356	14·9	19·0	23·7	28·9	34·0	38·9	43·6	47·2	48·0
	·425	13·9	17·7	22·1	26·9	31·7	36·2	40·6	43·9	44·5
	·482	13·3	17·0	21·2	25·8	30·3	34·7	38·9	42·1	42·7
Ordinary vessels.	·528	12·9	16·5	20·5	25·0	29·4	33·7	37·7	40·8	41·3
	·569	12·5	16·0	20·0	24·4	28·7	30·8	36·8	39·8	40·3
	·631	12·1	15·5	19·4	23·5	27·6	31·6	35·4	38·3	38·8
Heavy Freight.	.679	11·8	15·1	18·8	22·9	26·9	30·8	34·6	37·4	38·0
	·723	11·6	14·8	18·5	22·5	26·5	30·3	34·0	36·8	37·2
	·797	11·2	14·3	17·9	21·8	25·6	29·3	32·9	35·6	36·0
Condition.		Vessels for deep water.			Ordinary navigation.			River steamers Light draft.		

ABCDEFGHIJKLMNO
PQRSTUVWXYZ.
abcdefghijklmnopqrstuvwxyz.

ABCDEFGHIJKLM
NOPQRSTUVWX
YZ abcdefghijklmnopqrstuvwxyz.

ABCDEFGHIJKLMNOPQRSTUVWX
YZ abcdefghijklmnopqrsſtuvwxyz

ABCDEFGHIJKLMNOPQRS
TUVWXYZ. 1234567890
abcdefghijklmnopqrstuvwxyz.

DEAF AND DUMB ALPHABET.

Keep your hand horizontal for the letters *a*, *g*, *h*, *r*, *s* and *t*.
For *j* and *z* describe the letter with the finger in the air.
For *x* make a motion up and down with the index finger.